ECE113LA Fall 2003 **Final Exam** Name:____

12-8-2003 ID no.:____

Sec.A: Peter Burke 10:30 am to 12:20 pm

1	2	3	4	5	6	7	8	9	10	Total
/10	/10	/10	/10	/10	/10	/10	/10	/10	/10	/100

DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

PROBLEM ONE:

10- points

A piece of Si at 300 K is doped with *both* As atoms at 5 10^{10} cm^{-3}, *and* Ga atoms at 10^{10} cm^{-3}. Please answer the question with *at least* 2 significant figures.

A) What is the density of electrons n.

N_D=5 10^{10} cm^{-3}
N_A=10^{10} cm^{-3}

Book equation 2.29a:

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$= \frac{5x10^{10}cm^{-3} - 10^{10}cm^{-3}}{2} + \left[\left(\frac{5x10^{10}cm^{-3} - 10^{10}cm^{-3}}{2}\right)^2 + \left(10^{10}cm^{-3}\right)^2\right]^{1/2}$$

$$= 4.236x10^{10}cm^{-3}$$

Grading criteria:
Write down equation 2.29a from book: 1 point
Write down equation 2.29a from book, and put in correct #s for N_A, N_D: 2 points
All of above and correct answer (range 4-4.5): 3 points
1A is worth a maximum of 3 points.

B) What is the density of holes p.

2 ways to do problem:
First way:
Book eq. 2.29b

$$p = \frac{N_A - N_D}{2} + \left[\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$= \frac{10^{10}cm^{-3} - 5x10^{10}cm^{-3}}{2} + \left[\left(\frac{10^{10}cm^{-3} - 5x10^{10}cm^{-3}}{2}\right)^2 + \left(10^{10}cm^{-3}\right)^2\right]^{1/2}$$

$$= 2.36x10^{9}cm^{-3}$$

If you did it the first way:
Grading criteria:
Write down equation 2.29b from book: 1 point
Write down equation 2.29b from book, and put in correct #s for N_A, N_D: 2 points
All of above and correct answer (range 2-2.5): 3 points
1B is worth a maximum of 4 points.

Second way:
Equation 2.22 of book:

$$p = \frac{n_i^2}{n} = \left(10^{10}cm^{-3}\right)^2 / 4.236x10^{10}cm^{-3} = 2.361x10^{9}cm^{-3}$$

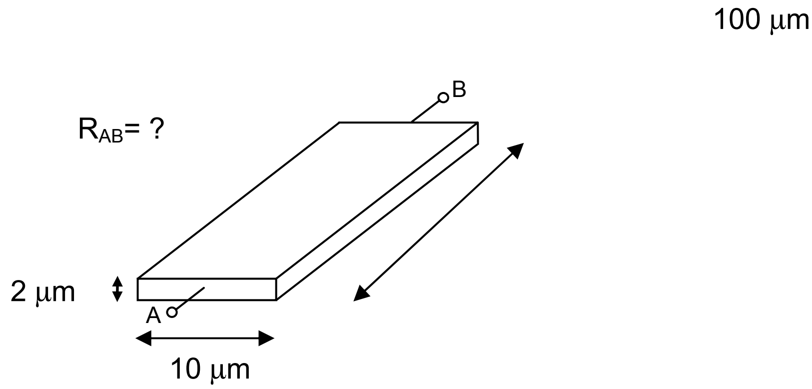
If you did it the second way:
Grading criteria:
Write down equation 2.22 from book: 1 point
All of above and correct answer (range 2-2.5): 3 points

If you did it both ways:
If both answers are consistent (i.e. both answers are between 2.2 and 2.5), you will be graded according to way #1.
If both answers are inconsistent, no credit at all for part 1B.

Maximum credit for 1B: 3 points.

PROBLEM ONE (CONTINUED):

C) What is R_{AB} for this structure given your answers to the questions above:



Doping density low, so mobility from fig. 3.5 of book is the low density limit, i.e.:

μ_n=1358 cm^2/V-s
μ_p=461 cm^2/V-s

Resistivity: Book equation 3.7

$$\rho = \frac{1}{q(n\mu_n + p\mu_p)} = \frac{1}{1.6x10^{-19}C(4.236x10^{10}cm^{-3}x1358cm^2/V-s + 2.361x10^9 cm^{-3}x461cm^2/V-s)}$$

$$= 1.07x10^5\Omega - cm$$

Resistance

$$R = \rho\frac{l}{A} = 1.07x10^5\Omega - cm\frac{100\mu m}{10\mu m x 2\mu m} = 1.07x10^5\Omega - (10^{-2}m)\frac{100x(10^{-6})m}{10(10^{-6})mx2(10^{-6})m}$$

$$= \frac{1.07}{2}10^{5-2+2-6-1+6+6}\Omega = 0.535x10^{10}\Omega = 5.35x10^9\Omega$$

Grading criteria:
Write down mobility correctly for both n (range 1300-1400) and p (range 400-500) : 1 point
Write down equation 3.7 from book: 1 point
Write down resistance from resitivity formula: 1 point
Correct answer (range 5-5.5) 1 point.
Max points for 1C: 4 points.

PROBLEM TWO:

10 points

Assume $\tau_p = \tau_n = 1\ \mu s$.

There is a p-n diode from Si at 300 K, given $N_A=10^{16}\ cm^{-3}$ and $N_D=10^{14}\ cm^{-3}$.

The cross sectional area of the diode is 1 cm^2.

A) Calculate the I_0 that appears in $I = I_0\left(e^{qV_A/kT} - 1\right)$

Book equation 6.30:

$$I_0 = qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right) \approx qA\left(\frac{D_P}{L_P}\frac{n_i^2}{N_D}\right) = qA\left(\frac{D_P}{\sqrt{D_P\tau_p}}\frac{n_i^2}{N_D}\right)$$

$$= qA\left(\sqrt{\frac{D_P}{\tau_p}}\frac{n_i^2}{N_D}\right) = qA\left(\sqrt{\frac{\mu_p}{\tau_p}\frac{kT}{q}}\frac{n_i^2}{N_D}\right)$$

μ_p is the mobility of the holes on the ns-side.
The dopant density on the n-side is $10^{14}cm^{-3}$.
From book fig. 3.5, the mobility of the holes is 461 cm^2/V-s.
So:

$$I_0 = 1.6x10^{-19}Cx1cm^2\left(\sqrt{\frac{461cm^2/V-s}{1\mu s}\frac{0.0259eV}{q}}\frac{\left(10^{10}cm^{-3}\right)^2}{10^{14}cm^{-3}}\right)$$

$$= 1.6x10^{-19}Cx1cm^2\left(\sqrt{\frac{461cm^2/V-s}{10^{-6}s}0.0259V}\frac{\left(10^{10}cm^{-3}\right)^2}{10^{14}cm^{-3}}\right)$$

$$= 1.6\left(\sqrt{461x0.0259}\right)x10^{-19}\left(\sqrt{\frac{1}{10^{-6}}}\frac{\left(10^{10}\right)^2}{10^{14}}\right)\frac{C}{S} = 5.53x10^{-19+3+20-14}A = 5.53x10^{-10}A \approx 0.5nA$$

Grading criteria:
0.5 points if you only write down book equation 6.30 and nothing else.
1 point if you only write down book equation 6.30 and making the approximation that only the second term is important.
2 points if your write down 6.30, use the full formula including both terms, and get the mobility correct for μ_p (461, not 437), and get the mobility correct for μ_n (1248, not 1358), but don't do the calculation correctly.
2 points if your write down 6.30, use the approximate formula including one term, and get the mobility correct for μ_p (461, not 437), but don't do the calculation correctly.

2.5 points if your write down 6.30, use the full formula including both terms, and get the mobility correct for μ_p (461, not 437), and get the mobility correct for μ_n (1248, not 1358), and do the calculation correctly (range 0.4-0.6).
2.5 points if your write down 6.30, use the approximate formula including one term, and get the mobility correct for μ_p (461, not 437), and do the calculation correctly (range 0.4-0.6).

2 points if your write down 6.30, use the full formula including both terms, and get WRONG mobility for μ_p (437 or other, not 461), and get WRONG mobility for μ_n (1358 or other, not 1248), but get the final answer in range 0.4-0.6.

2 points if your write down 6.30, use the approximate formula including one term, and get WRONG mobility for μ_p (437 or other, not 461), but get the final answer in range 0.4-0.6.

Max points for 2A: 2.5 points.

B) At $V_A = 0.6$ V, calculate Δp on the n-side at the interface.

Book equation 6.18:

$$\Delta p_n(x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

$$= \frac{\left(10^{10}\,cm^{-3}\right)^2}{10^{14}\,cm^{-3}}\left(e^{0.6/0.0259} - 1\right) = \left(10^6\,cm^{-3}\right)\left(1.15x10^{10} - 1\right) = 1.15x10^{16}\,cm^{-3}$$

Grading criteria:
If you only write down equation 6.18, and nothing else right, 1 point.
If you write down equation 6.18 and get answer in range 1-1.3, 2.5 points.
Max. points for 2B: 2.5 points.

PROBLEM TWO (CONTINUED):

C) At V_A = 0.6 V, calculate Δn on the p-side at the interface.

Book equation 6.15:

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$$

$$= \frac{\left(10^{10}\,cm^{-3}\right)^2}{10^{16}\,cm^{-3}}\left(e^{0.6/0.0259} - 1\right) = \left(10^4\,cm^{-3}\right)\left(1.15x10^{10} - 1\right) = 1.15x10^{14}\,cm^{-3}$$

Grading criteria:
If you only write down equation 6.15, and nothing else right, 1 point.
If you write down equation 6.15 and get answer in range 1-1.3, 2.5 points.
Max. points for 2C: 2.5 points

D) Do low level injection conditions prevail for V_A=0.6 V and why?

No, because $\Delta p << n_0$ **is NOT true.**

Grading criteria: 0.5 point for "NO" only.
2.5 points for "NO" and correct explanation.

PROBLEM THREE:

10 points

A new semiconductor material is discovered.
A team of scientists have determined the following parameters for the new material:

Band gap = 2 eV
Effective mass for holes = 10^{10} m_0
Effective mass for electrons = 1 m_0

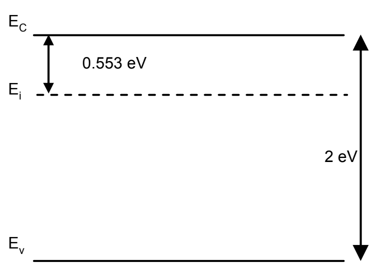
A) Draw E_C, E_V, E_F for the case of no doping for this new material. Label the distance between E_C, E_V, and E_F.

Book equation 2.36:

$$E_i = \frac{E_c + E_v}{2} + \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$E_c - E_i = E_c - \frac{E_c + E_v}{2} - \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{E_c - E_v}{2} - \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= 1eV - (0.75)x0.0259eV * \ln(10^{10}) = 1eV - 0.447eV = 0.553eV$$



Grading criteria:
1 point for only writing down equation 2.36.
2 points: calculate E_i location correctly and write down equation 2.36.
3 points: calculate E_i location correctly and write down equation 2.36, but draw E_i below midgap.
5 points: calculate E_i location correctly and write down equation 2.36, draw E_i above midgap (if labeled correctly).

PROBLEM THREE (CONTINUED):

B) Find p and n for the undoped semiconductor.

3 ways to do problem:
First way: Use initial equations 2.16a, 2.16b, 2.13a, 2.13b:

$$n = N_c e^{(E_F - E_C)/kT}$$

$$p = N_v e^{(E_v - E_F)/kT}$$

$$N_c = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2} \quad or = 2.51x10^{19}\,cm^{-3}(m_n^*/m_0^*)^{3/2}$$

$$N_v = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2} \quad or = 2.51x10^{19}\,cm^{-3}(m_p^*/m_0^*)^{3/2}$$

then explicitly calculate n,p using E_F from part A, and the other known constants.
(By the way, this is very tedious and time consuming.)
If you do that, the answer you get is:

$$n = p = 1.35x10^{10}\,cm^{-3}$$

Second way: Use initial equations 2.21, 2.13a, 2.13b:

$$n = p = n_i$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_G/2kT}$$

$$N_c = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2} \quad or = 2.51x10^{19}\,cm^{-3}(m_n^*/m_0^*)^{3/2}$$

$$N_v = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2} \quad or = 2.51x10^{19}\,cm^{-3}(m_p^*/m_0^*)^{3/2}$$

then explicitly calculate n,p using E_G and the other known constants.
(By the way, this is also very tedious and time consuming.)
If you do that, the answer you get is:

$$n = p = 1.35x10^{10}\,cm^{-3}$$

The third way is to use ratios:

Use initial equations

$$n = p = n_i$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_G/2kT}$$

$$N_c = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2} \propto \left(m_n^*\right)^{3/2}$$

$$N_v = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2} \propto \left(m_p^*\right)^{3/2}$$

then

$$\frac{n_{i,new}}{n_{i,Si}} = \frac{\sqrt{N_{c,new}N_{v,new}}\, e^{-E_{G,new}/2kT}}{\sqrt{N_{c,Si}N_{v,Si}}\, e^{-E_{G,Si}/2kT}} = \sqrt{\frac{N_{c,new}}{N_{c,Si}}}\sqrt{\frac{N_{v,new}}{N_{v,Si}}}\, e^{-(E_{G,new}-E_{G,Si})/2kT}$$

$$= \sqrt{\frac{\left(m_{n,new}^*\right)^{3/2}}{\left(m_{n,Si}^*\right)^{3/2}}}\sqrt{\frac{\left(m_{p,new}^*\right)^{3/2}}{\left(m_{p,Si}^*\right)^{3/2}}}\, e^{(E_{G,Si}-E_{G,new})/2kT} = \left(\frac{m_{n,new}^*}{m_{n,Si}^*}\right)^{3/4}\left(\frac{m_{p,new}^*}{m_{p,Si}^*}\right)^{3/4} e^{(E_{G,Si}-E_{G,new})/2kT}$$

$$= \left(\frac{1m_0}{1.18xm_0}\right)^{3/4}\left(\frac{10^{10}xm_0}{0.81xm_0}\right)^{3/4} e^{(E_{G,Si}-E_{G,new})/2kT} = \left(\frac{1}{1.18}\right)^{3/4}\left(\frac{10^{10}}{0.81}\right)^{3/4} e^{(-2+1.12)/2(0.0259)}$$

$$= 0.883x3.7x10^7x4.188x10^{-8} = 0.883x3.7x10^6x4.188x10^{-9} = 1.37 = \frac{n_{i,new}}{n_{i,Si}}$$

$$n_{i,new} = 1.37xn_{i,Si} = 1.37x10^{10}\,cm^{-3} = n = p$$

Grading criteria:
2 points for writing down ALL initial equations for the method you chose.
4 points for writing down intial equations, and getting n or p correct (range 1.2-1.5).
5 points for writing down intial equations, and getting n = p correct in range 1.2-1.5.

If you give inconsistent final answers, no credit at all thiis problem.
Max points this problem: 5 points.

PROBLEM FOUR:

10 points

Given for Si at 300 K:

$E_C(x) = 1.1 \text{ eV} + 1 \text{ meV} \sin[10^6 \mu\text{m}^{-1}\, x]$

$E_F = 0.5$ eV (constant)

$E_V = E_C - E_G$

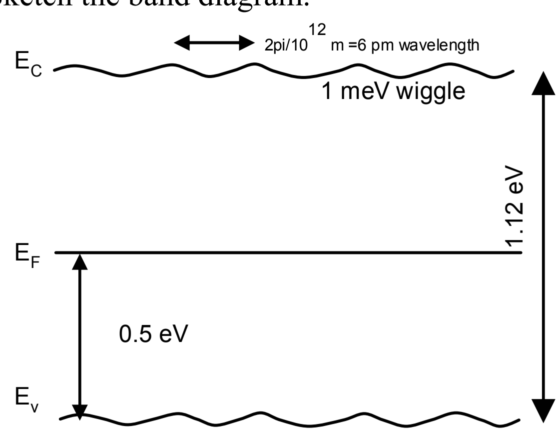
A) Is the system in equilibrium?

Yes.
Grading criteria: 1 point for yes.

B) Why?

E_F constant.
Grading criteria: 1 point for "E_F constant.".
No credit if A answer wrong, even if B answer right.

C) Sketch the band diagram.



Grading criteria: 2 points for 2 wiggly lines and E_F. 1 point for 1 wiggly line and E_F. E_F, E_C, E_V need to be labeled. Need to label E_F location, gap, in units.

PROBLEM FOUR (CONTINUED):

D) Calculate the electric field ε(x).

Book 3.15:

$$\varepsilon = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}$$

$$\varepsilon = \frac{1}{q}\frac{d}{dx}\left\{1meV \sin(10^6 \mu m^{-1} x)\right\} = \frac{1meV}{q} 10^6 \mu m^{-1} \cos(10^6 \mu m^{-1} x)$$
$$= mVx10^6 \mu m^{-1} \cos(10^6 \mu m^{-1} x) = 10^3 \cos(10^6 \mu m^{-1} x) \quad V/m$$

Write down eq. 3.15, one point.
Write down eq. 3.15, correct answer, 2 points. (Answer must be taken to completion. No qs.)

E) Calculate the electrostatic potential V(x).

Two ways to do problem:
First way:
Book 3.14:

$$\varepsilon = -\frac{dV}{dx}$$

$$V = -\int \varepsilon dx + \text{constant} = -10^{-3} \sin(10^6 \mu m^{-1} x) \quad V + \text{constant}$$

If you do it this way,
Write down eq. 3.14, one point.
Write down eq. 3.14, correct answer, 2 points. (Answer must be taken to completion. No qs.)
Second way:
Book 3.12:

$$V = -\frac{1}{q}\left(E_c - E_{ref}\right)$$

So:

$$V = -10^{-3} \sin(10^6 \mu m^{-1} x) \quad V + \text{constant}$$

If you do it this way,
Write down eq. 3.12, one point.
Write down eq. 3.12, correct answer, 2 points. (Answer must be taken to completion. No qs.)

F) Calculate the electron density n(x).

$$n = N_c e^{(E_F - E_C)/kT} = N_c e^{\left(-0.6\,eV - 1\,meV\ \sin(10^6\ \mu m^{-1} x)\right)/kT}$$

Grading criteria:
0 credit if only write down 2.16a.
1 point if correct.

1 point max this part.

G) Calculate the hole density p(x).

Two ways to do problem:
Either:

$$p = N_v e^{(E_v - E_F)/kT} = N_v e^{\left(-0.5eV + 1meV \sin(10^6\ \mu m^{-1} x)\right)/kT}$$

Grading criteria:
0 credit if only write down 2.16b.
1 point if correct.

Two ways to do problem:
Or

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_c e^{\left(-0.6eV - 1meV \sin(10^6\ \mu m^{-1} x)\right)/kT}}$$

Grading criteria:
1 point if only wrote down law of mass action and previous part correct.
1 point if got answer correct (RHS of above equation.)
Else 0.
1 point max this part.

PROBLEM FIVE:

10 points

Given for Si at 300 K:
Assume $\tau_p = \tau_n = 1$ µs.
A p-n diode where the p side is very heavily doped and the n side is very lighly doped, i.e. $N_A >> N_D$
The cross sectional area of the diode is 1 cm^2.
At an applied voltage of 0.775 V, a current of 1 mA flows.
What is N_D?
Hint: N_D is so small that the mobility (and hence diffusion coefficient) is independent of doping density, i.e. is in the flat region of fig. 3.5a of the book.

Step 1: Book equation 6.30:

$$I_0 = qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right)$$

Step 2: Since n-side lightly doped:

$$I_0 = qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right) \approx qA\left(\frac{D_P}{L_P}\frac{n_i^2}{N_D}\right)$$

Step 3: Einstein:

$$I_0 = qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right) \approx qA\left(\frac{D_P}{L_P}\frac{n_i^2}{N_D}\right)$$

$$= qA\left(\frac{D_P}{\sqrt{D_P\tau_p}}\frac{n_i^2}{N_D}\right) = qA\left(\sqrt{\frac{D_P}{\tau_p}}\frac{n_i^2}{N_D}\right) = qA\left(\sqrt{\frac{\mu_p}{\tau_p}\frac{kT}{q}}\frac{n_i^2}{N_D}\right)$$

Step 4: Solving for N_D:

$$I = I_0\left(e^{qV_A/kT} - 1\right) \approx I_0 e^{qV_A/kT} = qA\left(\sqrt{\frac{\mu_p}{\tau_p}\frac{kT}{q}}\frac{n_i^2}{N_D}\right)e^{qV_A/kT}$$

$$\Rightarrow N_D = qA\left(\sqrt{\frac{\mu_p}{\tau_p}\frac{kT}{q}}\frac{n_i^2}{I}\right)e^{qV_A/kT}$$

Step 5: From book fig. 3.5, the mobility of the holes is 461 cm^2/V-s.

Step 6:

$$N_D = 1.6x10^{-19}Cx1cm^2\left(\sqrt{\frac{461cm^2/V-s}{10^{-6}s}0.0259V}\,\frac{10^{20}cm^{-6}}{10^{-3}A}\right)e^{0.775/0.0259}$$

$$= 5.47x10^{20}cm^{-3}$$

Grading criteria:
Step 1 worth 1 point
Step 2 worth 1 point
Step 3 worth 1 point
Step 4 worth 5 point
Step 5 worth 1 point
Step 6 worth 1 point
For a total of 10 points.
Extra credit of 5 points if you note that the value of N_D calculated violates the hint.

PROBLEM SIX:

10 points

Given for Si at 300 K:

The average time between scattering events for electrons is 1 ps.
The average time between scattering events for holes is 1 ps.

A) Calculate the mobility for the electrons

From book equation on page 79,

$$\mu = \frac{q\tau}{m^*}$$

So:

$$\mu = \frac{q\tau}{m^*} = \frac{1.6x10^{-19}Cx1ps}{1.18x9.1x10^{-31}kg} = \frac{1.6x10^{-19}Cx10^{-12}s}{1.18x9.1x10^{-31}kg} = \frac{1.6}{1.18x9.1}\frac{10^{-19}x10^{-12}}{10^{-31}}\frac{Cs}{kg} = 0.149\frac{Cs}{kg} = 0.149\frac{m^2}{V-s}$$

Grading criteria:
Write down equation from page 79, 1 point.
Write down equation from page 79 and calculate answer correctly (range 0.13-0.16), 2.5 pts. (Units of Cs/kg acceptable, although not standard.)

B) Calculate the mobility for the holes

From book equation on page 79,

$$\mu = \frac{q\tau}{m^*}$$

So:

$$\mu = \frac{q\tau}{m^*} = \frac{1.6x10^{-19}Cx1ps}{0.81x9.1x10^{-31}kg} = \frac{1.6x10^{-19}Cx10^{-12}s}{0.81x9.1x10^{-31}kg} = \frac{1.6}{0.81x9.1}\frac{10^{-19}x10^{-12}}{10^{-31}}\frac{Cs}{kg} = 0.217\frac{Cs}{kg} = 0.217\frac{m^2}{V-s}$$

Grading criteria:
Write down equation from page 79, 1 point.
Write down equation from page 79 and calculate answer correctly (range 0.2-0.24), 2.5 pts. (Units of Cs/kg acceptable, although not standard.)

C) Calculate the diffusion coefficient for the electrons

Einstein:

$$D = \mu\frac{kT}{q}$$

So:

$$D = \mu\frac{kT}{q} = 0.149\frac{m^2}{V-s}x0.0259V = 3.86x10^{-3}\frac{m^2}{s} = 38.6\frac{(cm)^2}{s}$$

Grading criteria:
Write down Einstein equation, 1 point.
Write down Einstein equation and calculate answer correctly (range 35-40), 2.5 pts. (Units of VCs/kg acceptable, although not standard.)

D) Calculate the diffusion coefficient for the holes

Einstein:

$$D = \mu\frac{kT}{q}$$

So:

$$D = \mu\frac{kT}{q} = 0.217\frac{m^2}{V-s}x0.0259V = 5.62x10^{-3}\frac{m^2}{s} = 56.3\frac{(cm)^2}{s}$$

Grading criteria:
Write down Einstein equation, 1 point.
Write down Einstein equation and calculate answer correctly (range 50-60), 2.5 pts. (Units of VCs/kg acceptable, although not standard.)

PROBLEM SEVEN:

10 points

Given for Si at 300 K, doped with Ga atoms at a concentration of 10^{18} cm^{-3}.

A) Calculate the ratio of the number of electrons with energy 0.01 eV above the conduction band edge to the number of electrons with energy 0.02 eV above the conduction band edge.

From page 47 of book, the number of electrons with energy between E and E+dE is given by:

$$g_c(E)f(E)$$

where E is the energy, and f(E) is the Fermi-Dirac distribution function.
The formula for these is 2.6a:

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E-E_c)}}{\pi^2 \hbar^3}$$

and 2.7:

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

Now, from figure 2.15 for our case:

$$f(E) \approx e^{-(E-E_F)/kT}$$

We only want the ratios. Let E_1=E_C+0.01 eV, E_2=E_C+0.02 eV = E_1 + 0.01 eV, then the requested ratio is:

$$\frac{g_c(E_1)f(E_1)}{g_c(E_2)f(E_2)} = \frac{g_c(E_C+0.01eV)f(E_C+0.01eV)}{g_c(E_C+0.02eV)f(E_C+0.02eV)} = \frac{\frac{m_n^*\sqrt{2m_n^*(0.01eV)}}{\pi^2\hbar^3}e^{-(E_C+0.01eV-E_F)/kT}}{\frac{m_n^*\sqrt{2m_n^*(0.02eV)}}{\pi^2\hbar^3}e^{-(E_C+0.02eV-E_F)/kT}}$$

$$= \sqrt{\frac{0.01}{0.02}}e^{[-(E_C+0.01eV-E_F)/kT]-[-(E_C+0.02eV-E_F)/kT]} = \frac{1}{\sqrt{2}}e^{0.01eV/kT} = \frac{1}{\sqrt{2}}e^{0.01/0.0259} = \frac{1}{\sqrt{2}}x1.47 = 1.04$$

Grading criteria:
1 point of you only write down statement that # of electrons at energy E is given by g*f.
2 points if you write down statement that # of electrons at energy E is given by g*f and write down formula 2.6a and 2.7.
3 points if you write down statement that # of electrons at energy E is given by g*f, write down formula 2.6a and 2.7, and write approximation given in figure 2.15.
5 points if you write down statement that # of electrons at energy E is given by g*f, write down formula 2.6a and 2.7, and write approximation given in figure 2.15, and get correct answer (range 1.0-1.1).

There is a hard way to do this problem, which uses the exact formula equation 2.7. If you did problem the hard way, then you need to calculate EF, which takes a lot more time. If you did the problem the hard way, then the grading criteria is:
1 point of you only write down statement that # of electrons at energy E is given by g*f.
2 points if you write down statement that # of electrons at energy E is given by g*f and write down formula 2.6a and 2.7.
5 points if you write down statement that # of electrons at energy E is given by g*f, write down formula 2.6a and 2.7, and write approximation given in figure 2.15, and get correct answer (range 1.0-1.1).

B) Calculate the ratio of the number of holes with energy 0.01 eV below the valence band edge to the number of holes with energy 0.02 eV below the valence band edge.

From page 47 of book, the number of holes with energy between E and E+dE is given by:

$$g_v(E)\left[1-f(E)\right]$$

where E is the energy, and f(E) is the Fermi-Dirac distribution function.
The formula for these is 2.6b:

$$g_v(E)=\frac{m_p^*\sqrt{2m_p^*(E_v-E)}}{\pi^2\hbar^3}$$

and 2.7:

$$f(E)=\frac{1}{1+e^{(E-E_F)/kT}}$$

Now, from figure 2.15 for our case:

$$1-f(E)\approx 1-\left[1-e^{(E-E_F)/kT}\right]=e^{(E-E_F)/kT}$$

We only want the ratios. Let E_1=E_v-0.01 eV, E_2=E_v-0.02 eV = E_1 - 0.01 eV, then the requested ratio is:

$$\frac{g_v(E_1)\left[1-f(E_1)\right]}{g_v(E_2)\left[1-f(E_2)\right]}=\frac{g_c(E_v-0.01eV)\left[1-f(E_v-0.01eV)\right]}{g_c(E_v-0.02eV)\left[1-f(E_v-0.02eV)\right]}=\frac{\dfrac{m_n^*\sqrt{2m_n^*(0.01eV)}}{\pi^2\hbar^3}e^{(E_v-0.01eV-E_F)/kT}}{\dfrac{m_n^*\sqrt{2m_n^*(0.02eV)}}{\pi^2\hbar^3}e^{(E_v-0.02eV-E_F)/kT}} \text{ G}$$

$$=\sqrt{\frac{0.01}{0.02}}e^{0.01eV/kT}=\frac{1}{\sqrt{2}}e^{0.01eV/kT}=\frac{1}{\sqrt{2}}e^{0.01/0.0259}=\frac{1}{\sqrt{2}}x1.47=1.04$$

Grading criteria:
1 point of you only write down statement that # of holes at energy E is given by g*(1-f).
2 points if you write down statement that # of holes at energy E is given by g*(1-f) and write down formula 2.6b and 2.7.
3 points if you write down statement that # of holes at energy E is given by g*(1-f), write down formula 2.6b and 2.7, and write approximation given in figure 2.15.
5 points if you write down statement that # of electrons at energy E is given by g*(1-f), write down formula 2.6b and 2.7, and write approximation given in figure 2.15, and get correct answer (range 1.0-1.1).

There is a hard way to do this problem, which uses the exact formula equation 2.7. If you did problem the hard way, then you need to calculate EF, which takes a lot more time. If you did the problem the hard way, then the grading criteria is:
1 point of you only write down statement that # of holes at energy E is given by g*(1-f).
2 points if you write down statement that # of holes at energy E is given by g*(1-f) and write down formula 2.6b and 2.7.
5 points if you write down statement that # of holes at energy E is given by g*(1-f), write down formula 2.6b and 2.7, and get correct answer (range 1.0-1.1).

There is a much easier way to do the problem, which is to note the symmetry between the 2 cases:
The answer to part B is the same as part A!
This takes no time at all.
If you did it this way, 5 points credit.

PROBLEM EIGHT:

10 points

Given the following for Si npn bipolor junction transistor at 300 K:

Emitter	Base	Collector
$N_E = 10^{17}$ cm^{-3}	$N_B = 10^{17}$ cm^{-3}	$N_C = 10^{17}$ cm^{-3}
$\mu_E = 200$ cm^2/V-s	$\mu_B = 800$ cm^2/V-s	$\mu_C = 200$ cm^2/V-s
D_E=5.18 cm^2/s	D_B=20.7 cm^2/s	D_C=5.18 cm^2/s
$\tau_E = 10^{-6}$ s	$\tau_B = 10^{-6}$ s	$\tau_C = 10^{-6}$ s
	W_B=0.1 μm	

Calculate β. Please carry out your calculation to *at least* 2 significant figures.

Book equation 11.44:

$$\beta = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

Now, some of the parameters are known from the table, namely D_E, D_B, N_B, N_E, W.
However, we are not given L_E and L_B, so we must calculate these from the known parameters:

$$L_E \equiv \sqrt{D_E \tau_E} = \sqrt{5.18\frac{cm^2}{V-s}x10^{-6}s} = 2.27596x10^{-3}cm$$

Similarly:

$$L_B \equiv \sqrt{D_B \tau_B} = \sqrt{20.7\frac{cm^2}{V-s}x10^{-6}s} = 4.549725x10^{-3}cm$$

So:

$$\beta = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2} = \frac{1}{\frac{5.18\frac{cm^2}{s}}{20.7\frac{cm^2}{s}}\frac{10^{17}cm^{-3}}{10^{17}cm^{-3}}\frac{0.1\mu m}{2.27596x10^{-3}cm} + \frac{1}{2}\left(\frac{0.1\mu m}{4.549725x10^{-3}cm}\right)^2}$$

$$= \frac{1}{\frac{5.18\frac{cm^2}{s}}{20.7\frac{cm^2}{s}}\frac{10^{17}cm^{-3}}{10^{17}cm^{-3}}\frac{0.01x10^{-3}cm}{2.27596x10^{-3}cm} + \frac{1}{2}\left(\frac{0.01x10^{-3}cm}{4.549725x10^{-3}cm}\right)^2}$$

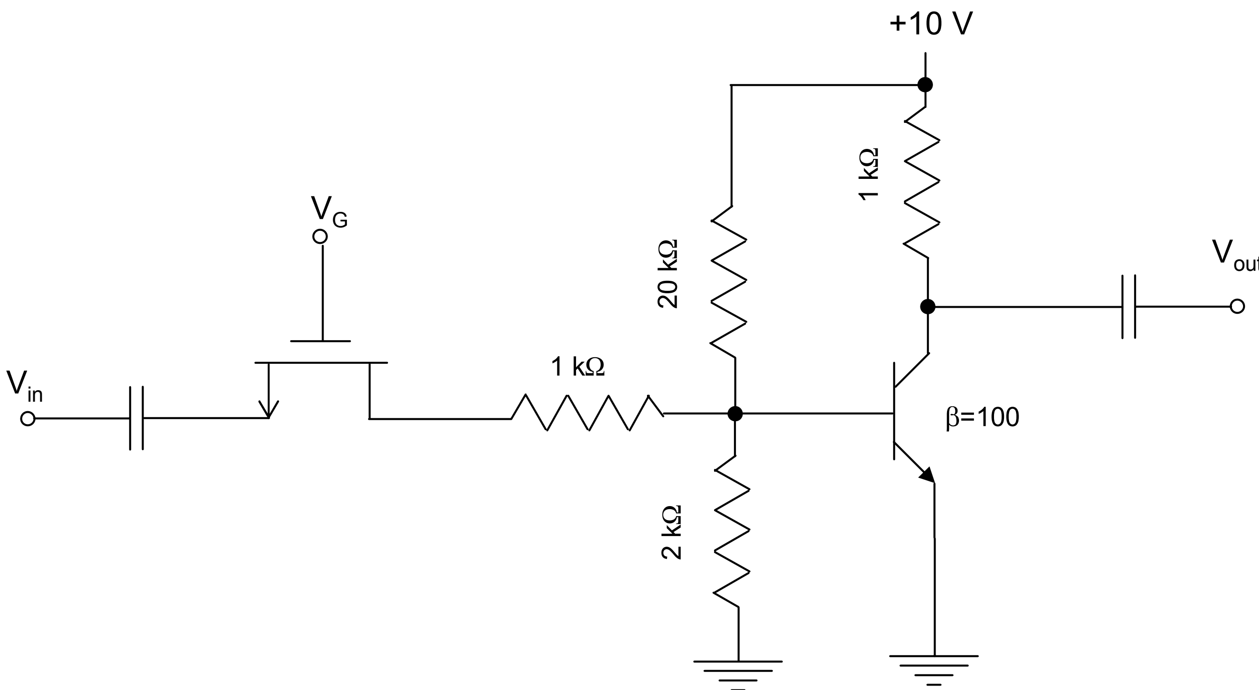
$$= \frac{1}{\frac{5.18}{20.7}\frac{0.01}{2.27596} + \frac{1}{2}\left(\frac{0.01}{4.549725}\right)^2} = \frac{1}{0.001099 + 0.0000024154592} = 908$$

Grading criteria:
Equation 11.44, 1 point.
Correct formula for L_E, 2 point.
Correct formula for L_B, 2 point.
If all of above are correct and plug in #s correctly but don't calculate correctly, 6 points for problem.
If all of above are correct and plug in #s correctly and calculate correctly (range 900-1000), 10 points for problem.

PROBLEM NINE:

10 points

For the MOSFET in the circuit below, $V_T = 2$ V.
The circuit is an amplifier, so in actual use V_{in} will be restricted to be a very small ac voltage, say less than 1 mV.



A) What is the gain if V_G is set to 0 V?

This is the amplifier you built in lab. It is on the cover of you lab book. The only difference is the introduction of the MOSFET, which is a variable resistor in this circuit because the voltages are limited to below 1 mV.
In class, we found the gain for the circuit on the cover of the lab book to be:

$$G = \frac{R_C}{R_B}\beta$$

For this problem, R_B=1 kΩ + R_{SD} so the gain is given by:

$$G = \frac{R_C}{R_B}\beta = 100\frac{1k\Omega}{1k\Omega + R_{SD}}$$

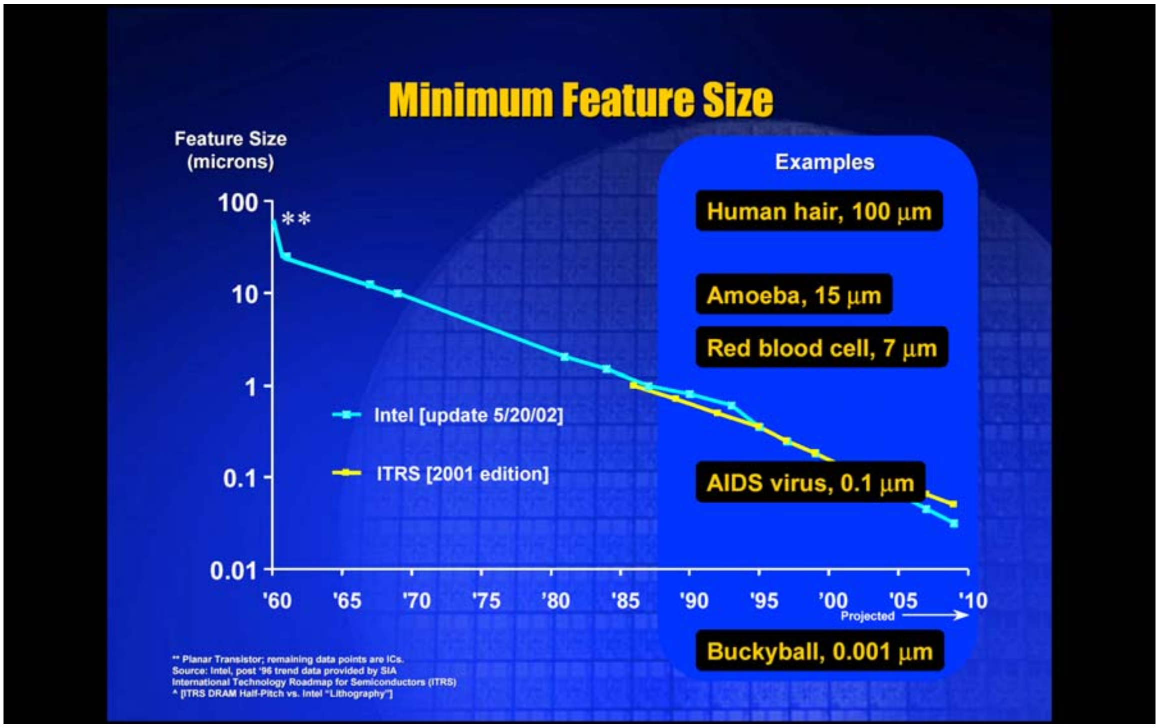
If V_G is set to 0 V, R_{SD} is very large, so G=0.
Grading criteria:
Formula for the gain, 1 point.
Say R_B=1 kΩ + R_{SD}, 1 point.
At 0V, R_{SD} large, 1 point.
All of above correct, and answer (0), 5 points.

B) What is the gain if V_G is set to +5 V?

If V_G is set to +5 V, R_{SD} is very small, so G=100.
Grading criteria:
Formula for the gain, 1 point.
Say R_B=1 kΩ + R_{SD}, 1 point.
At 0V, R_{SD} small, 1 point.
All of above correct, and answer (100), 5 points.

PROBLEM TEN:

10 points



This slide is from a presentation by Gordon Moore this year. He is the CEO of INTEL. Estimate the year in which the size of a transistor will be the size of a single atom. Do you expect this to occur in your lifetime? The study of circuits on this small length scale is called *nanotechnology*.

The size of an atom is about 1 Angstrom.

Extrapolating the curve, gives year 2045-2060.

Grading critera: Say size of atom 1 Angstrom (range 0.5-5 Angstroms), 2 points.
Give year 2040-2070, and say correct size of atom: 10 points.