

1A	1B	2	3A	3B	3C	3D	3E	Total
/15	/15	/20	/10	/10	/10	/10	/10	/100

**THREE PROBLEMS TOTAL.**

**DO NOT BEGIN THE EXAM  
UNTIL YOU ARE TOLD TO  
DO SO.**

**PROBLEM ONE: (30 points)**

- 1) For a piece of Si at 300 K,  $N_D = 10^{17} \text{ cm}^{-3}$ :
- Find the mobility  $\mu_n$  of electrons to within 10% (15 points)

$$\mu_n = 801 \text{ cm}^2/\text{V-s} \text{ (from the graph in book, figure 3.5)}$$

$$\text{Accepted answers: } 721 - 881 \text{ cm}^2/\text{V-s}$$

- Find the diffusion constant  $D_N$  of electrons to within 10% (15 points)

$$\begin{aligned} D_N &= \mu_n kT/q \\ &= (801 \text{ cm}^2/\text{V-s})(.0259\text{V}) \\ &= 20.7 \text{ cm}^2/\text{s} \end{aligned}$$

**PROBLEM TWO: (20 points)**

Some baseline temperatures in the three temperature scales:

temperature	kelvins	degrees Celsius	degrees Fahrenheit
symbol	K	°C	°F
boiling point of water	373.15	100.	212.
melting point of ice	273.15	0.	32.
absolute zero	0.	-273.15	-459.67

- 2) A p-n diode is reverse biased at  $-1$  V and cooled to the temperature of the melting point of ice ( $T = 273$  K). At that temperature, the current is 1 pA.

The diode is now put into a pot of boiling water, so that its temperature is 373 K.

What is the current now, assuming the voltage is still  $-1$  V?

**This problem cannot be solved exactly, since  $N_A$  and  $N_D$  are not given.**

**Therefore, an exact solution will not be required to get full credit on this problem.**

**Generally speaking, there are 2 ways to do this problem:**

**Use lecture notes, or use book.**

**METHOD I: USE LECTURE NOTES**

**In class we showed the**

$$I(V_A) = (\text{constant}) e^{-\frac{qV_{bi}}{k_B T}} \left( e^{\frac{qV_A}{k_B T}} - 1 \right)$$

**At  $V_A = -1$  V,**

$$\left( e^{\frac{qV_A}{k_B T}} - 1 \right) \approx 1$$

**for  $T = 273$  K and  $T = 373$  K.**

**Therefore,  $I(V_A = -1V) \approx -(\text{constant}) e^{-\frac{qV_{bi}}{k_B T}}$**

**Assuming the constant is independent of temperature (not exactly true but we didn't cover in lecture), we have:**

$$\frac{I(V_A = -1V)|_{273K}}{I(V_A = -1V)|_{373K}} = \frac{-(\text{constant})e^{-\frac{qV_{bi}}{k_B 273K}}}{-(\text{constant})e^{-\frac{qV_{bi}}{k_B 373K}}} = \frac{e^{-\frac{qV_{bi}}{k_B 273K}}}{e^{-\frac{qV_{bi}}{k_B 373K}}} = e^{-\frac{qV_{bi}}{k_B 273K} + \frac{qV_{bi}}{k_B 373K}}$$

Now,  $V_{bi}$  is not given so quantitatively this is as far as you can go.

But a typical value of  $V_{bi}$  is 0.5 V. So for a typical value:

$$\frac{I(V_A = -1V)|_{273K}}{I(V_A = -1V)|_{373K}} = e^{-\frac{q(0.5V)}{k_B 273K} + \frac{q(0.5V)}{k_B 373K}} = e^{-21.23+15} = e^{-5.75} = 0.003$$

So

$$I(V_A = -1V)|_{373K} = \frac{I(V_A = -1V)|_{273K}}{0.003} = 333 pA$$

## **METHOD II: USE BOOK**

Book eq. 6.29

$$I(V_A) = I_0 \left( e^{\frac{qV_A}{k_B T}} - 1 \right)$$

At  $V_A = -1$  V,

$$\left( e^{\frac{qV_A}{k_B T}} - 1 \right) \approx 1$$

for  $T=273$  K and  $T = 373$  K.

Therefore,  $I(V_A = -1V) \approx I_0$

Book eq. 6.30

$$* I_0 = qAn_i^2 \left( \frac{D_N}{L_N} \frac{1}{N_A} + \frac{D_p}{L_p} \frac{1}{N_D} \right)$$

$q, A, N_A, N_D$  independent of temperature.

Book figure 2.20:

$$n_i (T=273 K) = 10^9 \text{ cm}^{-3}$$

$$n_i (T=373 K) = 10^{12} \text{ cm}^{-3}$$

$$\frac{D_N}{L_N} = \frac{D_N}{\sqrt{D_N \tau_n}} = \frac{\sqrt{D_N}}{\sqrt{\tau_n}}$$

Approximately,  $\tau_n$  independent of temperature.

By Einstein (book eqn. 3.25)

$$\sqrt{D_N} = \sqrt{\frac{k_B T \mu_n}{q}}$$

$\mu_n$  depends on temperature, as per book figure 3.7:  $\mu_n$  decreases with temperature in a way that depends on  $N_A, N_D$ .

**The most it decreases by is about a factor of 2. So  $\sqrt{D_N}$  changes some with temperature, but not by more than a factor of 10.**

**Similar arguments hold for  $\sqrt{D_P}$**

**So the factor in parenthesis in equation \* above changes by a small amount.**

**However, the  $n_i^2$  factor changes by a large amount, and this dominates.**

**So if we neglected the factor in parentheses we have:**

$$\frac{I(V_A = -1V)|_{273K}}{I(V_A = -1V)|_{373K}} = \frac{I_0|_{273K}}{I_0|_{373K}} = \frac{n_i^2|_{273K}}{n_i^2|_{373K}} = \frac{10^{18}}{10^{24}} = 10^{-6}$$

**So**

$$I(V_A = -1V)|_{373K} = \frac{I(V_A = -1V)|_{273K}}{0.001} = 10^6 \text{ pA} = 1\mu\text{A}$$

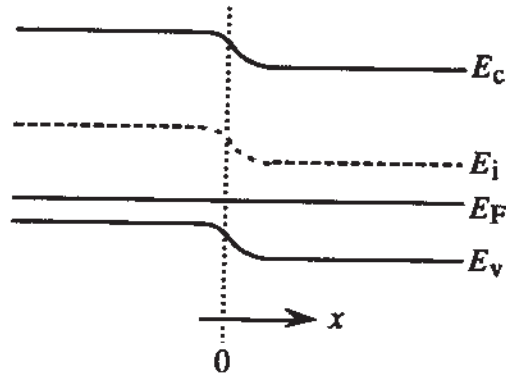
**Using either method, the conclusion is that the current at 373 K is much larger than the current at 273 K for reverse bias, by a large factor.**

**PROBLEM THREE: (50 points)**

Came right out of book (chapter 3 problem 5.3). Solution set from author follows.

### 5.3

(a) Because  $N_{A1} > N_{A2}$  and  $p = n_i \exp[(E_i - E_F)/kT] \cong N_A$  far from the junction, it follows that  $(E_i - E_F)_{x < 0} > (E_i - E_F)_{x > 0}$ . The energy band diagram must therefore be of the form



(b) Given that the dopings are nondegenerate, the same development leading to Eq. (5.10) can be employed, except

$$n(x_n) \cong n_i^2/N_{A2}$$

$$n(-x_p) \cong n_i^2/N_{A1}$$

which when substituted into Eq. (5.8) yields

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_{A1}}{N_{A2}}\right)$$

Alternatively, one can write

$$\begin{aligned} V_{bi} &= \frac{1}{q} [E_i(-\infty) - E_i(+\infty)] = \frac{1}{q} [(E_i - E_F)_{p1\text{-side}} - (E_i - E_F)_{p2\text{-side}}] \\ &= \frac{1}{q} [kT \ln(N_{A1}/n_i) - kT \ln(N_{A2}/n_i)] = \frac{kT}{q} \ln(N_{A1}/N_{A2}) \end{aligned}$$

Note that, as must be the case,  $V_{bi} \rightarrow 0$  if  $N_{A1} = N_{A2}$ .

