EECS170a Final Solutions

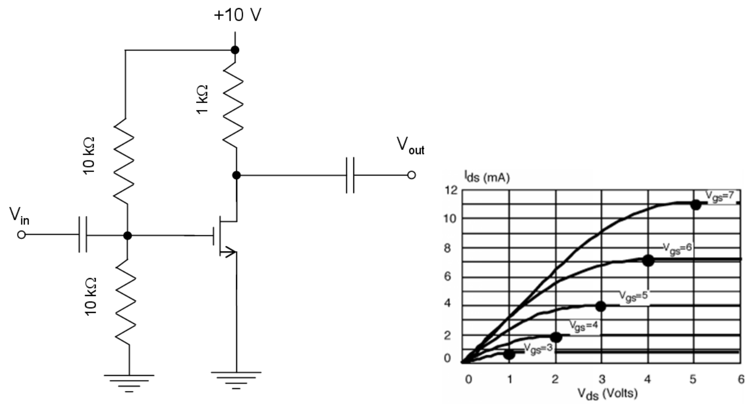


Figure 1: Question 1

1. A) (10 points) Find the value of Ids and Vds at dc for this circuit. (The graph shows the MOSFET I-V characteristics.)

   Solution:
   The voltage divider on the gate will maintain a gate voltage of $V_g = V_{gs} = +5V$ which indicates that the values of I_{ds} and V_{ds} will lay on the $V_{gs} = 5V$ curve. If we assume that the FET is operating in saturation then $I_{ds} = 4mA$ (this is an assumption that we will have to confirm at the end). Under this assumption,

$$\begin{aligned} V_{ds} &= 10V - I_{ds}R_d \\ &= 10V - (4mA \cdot 1k\Omega) \\ V_{ds} &= 6V \end{aligned}$$

   With $V_{ds} = 6V$ we can refer back to the I_{ds} vs V_{ds} plot and confirm our original assumption that the FET is in saturation.
   Grading Criteria

   - 5pts for $I_{ds} = 4mA$
   - 5pts for $V_{ds} = 6V$

   B) (15 points) Find the voltage gain of this circuit (V_{out}/V_{in}) at ac. Hint: An ac voltage changes on the gate causes an ac source-drain current.

Solution:

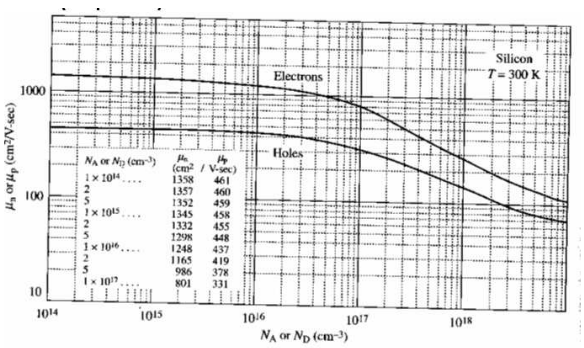
By applying an ac signal V_{in} to the gate, the gate voltage will fluctuate up and down about the dc biased value of $V_{gs} = 5V$. If we take this fluctuation to have a max/min values of $V_{in} = \pm 1V$ then $V_{GS} = V_{gs} + V_{in} = 4V$ or $6V$. The corresponding fluctuations to I_{ds} can be read off the plot to be $\Delta I_{ds} = 2mA$ and $\Delta I_{ds} = 3mA$, for $V_{ds} = 4V$ and $V_{ds} = 6V$, respectively. Using $\Delta V_{out} = 10V - \Delta I_{ds} R_d$ we obtain $\Delta V_{out} = 2V$ for the negative swings in V_{ds} and $\Delta V_{out} = 3V$ for positive unit swings in gate-source voltage. Thus, the voltage gain V_{out}/V_{in} is

$$\left|\frac{V_{out}}{V_{in}}\right| = V_{ds} = 10V - I_{ds} R_d$$
$$= 2 \text{ or } 3$$

Grading Criteria:

- 15pts for $|V_{out}/V_{in}| \in (2, 3)$ and correct work shown. In other words, correct answer but incorrect approach, no credit.

2. Use these graphs for the problem state on the next page.



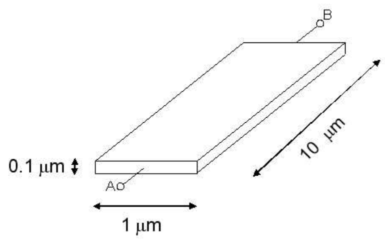


Figure 2: Question 2

A) (10 points) Find the resistance RAB of this piece of silicon, assuming it is n-type doped at $10^{17} cm^{-3}$.

Solution:
The value of μ_n for $N_D = 10^{17} cm^{-3}$ can be read off the graph to be $\mu_n = 801 cm^2/V - sec$.

$$\begin{aligned} R_{AB} &= \rho_n \frac{l}{A} \\ &= \frac{1}{q\mu_n N_D} \frac{l}{A} \\ &= \frac{1}{(1.6 \times 10^{-19} C)(801 cm^2/V - sec)(10^{17} cm^{-3})} \frac{10\mu m}{1\mu m \cdot 0.1\mu m} \\ &= \frac{1}{12.8} \frac{V \cdot sec \cdot cm}{C} \cdot 10^6 cm^{-1} \\ &= 78k\Omega \end{aligned}$$

Grading Criteria:

(a) 2pts for $R_{AB} = \rho_n \frac{l}{A}$

(b) 3pts for μ_n $801 cm^2/V - sec$

(c) 2pts for $R_{AB} = \frac{1}{(1.6\times10^{-19}C)(801cm^2/V-sec)(10^{17}cm^{-3})} \frac{10\mu m}{1\mu m \cdot 0.1\mu m}$. The length units can be cm, μm, m units alright as long as it is equivalent t o the above.

(d) 3pts for $R \in (70k\Omega, 100k\Omega)$ or a form where by a simplification effort was made to obtain this result from item (3).

B) (5 points) Find the current density if $V_{AB} = 1V$ for the doping specified in part A.
Solution:

$$\begin{aligned} J_{AB} = \frac{I_{AB}}{A} &= \frac{V_{AB}}{A \cdot R_{AB}} \\ &= \frac{1V}{1\mu m \cdot 0.1\mu m \cdot 78k\Omega} \\ &= 1.28 \times 10^4 A/cm^2 = \frac{10^9}{7.8} A/m^2 = \frac{10^5}{7.8} A/cm^2 = 1.28 \times 10^8 A/m^2 = 1.28 \times 10^{-4} A/\mu m^2 \end{aligned}$$

Grading Criteria:

- 3pts for $J_{AB} = \frac{1V}{1\mu m \cdot 0.1\mu m \cdot 78k\Omega}$
- 2pts for $J_{AB} \in (1.0 \times 10^4 A/cm^2, 1.6 \times 10^4 A/cm^2)$

C) (5 points) Find the resistance RAB of this piece of silicon, assuming it is p-type doped at $10^{17} cm^{-3}$.

Solution:
The value of μ_n for $N_A = 10^{17}cm^{-3}$ can be read off the graph to be $\mu_n = 801cm^2/V - sec$.

$$\begin{aligned} R_{AB} &= \rho_p \frac{l}{A} \\ &= \frac{1}{q\mu_p N_A}\frac{l}{A} \\ &= \frac{1}{(1.6\times 10^{-19}C)(321cm^2/V\cdot sec)(10^{17}cm^{-3})}\frac{10\mu m}{1\mu m\cdot 0.1\mu m} \\ &= \frac{1}{5.12}\frac{V\cdot sec\cdot cm}{C}\cdot 10^6 cm^{-1} \\ &= 195k\Omega \end{aligned}$$

Grading Criteria:

- 1pts for $R_{AB} = \rho_n \frac{l}{A}$
- 1pts for $\mu_n = 321cm^2/V - sec$
- 1pts for $R_{AB} = \frac{1}{(1.6\times 10^{-19}C)(321cm^2/V\cdot sec)(10^{17}cm^{-3})}\frac{10\mu m}{1\mu m\cdot 0.1\mu m}$
- 2pts for $R \in (150k\Omega, 250k\Omega)$

D) (5 points) Find the current density if $V_{AB} = 1V$ for the doping specified in part C.
Solution:

$$\begin{aligned} J_{AB} = \frac{I_{AB}}{A} &= \frac{V_{AB}}{A\cdot R_{AB}} \\ &= \frac{1V}{1\mu m\cdot 0.1\mu m\cdot 195k\Omega} \\ &= 5.12\times 10^3 A/cm^2 = 5.12\times 10^7 A/m^2 = 5.12\times 10^{-5}A/\mu m^2 \end{aligned}$$

Grading Criteria:

- 3pts for $J_{AB} = \frac{1V}{1\mu m\cdot 0.1\mu m\cdot 195k\Omega}$
- 2pts for $J_{AB} \in (4.5\times 10^3 A/cm^2, 5.5\times 10^3 A/cm^2)$

3. A) (12.5 points) Is this transistor biased in active mode?

Solution:

Since the transistor is pnp, it is active biased if the emitter-base junction is forward biased (i.e. $V_{EB} > 0$) and the collector-base junction is reverse biased (i.e. $V_{CB} < 0$). In the figure we have $V_B > V_E$ so $V_{EB} < 0$ and the transistor is NOT in active mode.
Grading Criteria:

- 12.5pts for Not active biased

B) (12.5 points) Is this transistor biased in active mode?

If so, find: (Assume $\beta = 100$)

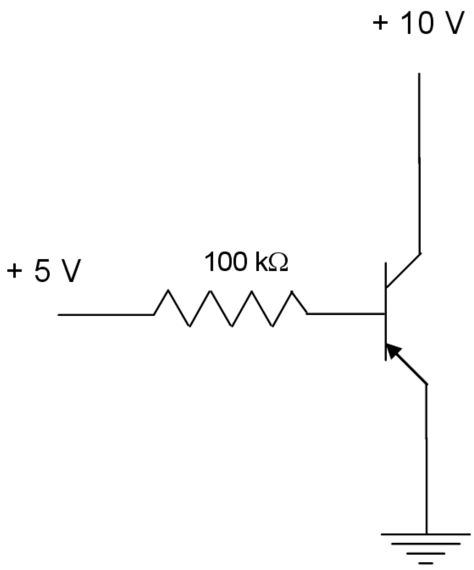


Figure 3: Question 2

Solution:
Since the transistor is npn, it is active biased if the emitter-base junction is forward biased (i.e. $V_{EB} < 0$) and the collector-base junction is reverse biased (i.e. $V_{CB} > 0$). In the figure we have $V_B > V_E$ so $V_{EB} < 0$ and the transistor is in active mode.
Since we now know the transistor is active biased, we can take $V_{BE} = 0.6V$ and we immediately see that $V_C = +10V$ and $V_E = 0V$.

$$\begin{aligned} V_B &= V_E + V_{BE} \\ &= 0V + 0.6V \\ &= 0.6V \end{aligned}$$

$$\begin{aligned} I_B &= \frac{R_B}{5V - V_B} \\ &= \frac{100k\Omega}{5V - 0.6V} \\ &= 44\mu A \end{aligned}$$

$$\begin{aligned} I_C &= \beta I_B \\ &= 100 \cdot 44\mu A \\ &= 4.4mA \end{aligned}$$

$$\begin{aligned} I_E &= I_C + I_B \\ &= 4.4mA + 44\mu A \\ &= 4.444mA \end{aligned}$$

Grading Criteria:

I_E	$\in (4.04, 4.545)mA$	$2pts$
I_B	$\in (40, 45)\mu A$	$2pts$
I_C	$\in (4.0, 4.5)mA$	$2pts$
V_E	$0V$	$1pts$
V_B	$\in (0.5V, 0.8V)$	$2pts$
V_C	$10V$	$1pts$
$active?$	Yes	$2.5pts$

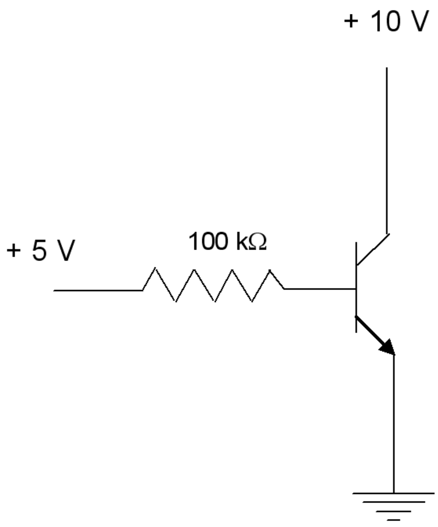


Figure 4: Question 3

4. Use the band diagram above for the MOS structure for the question stated on the following page.

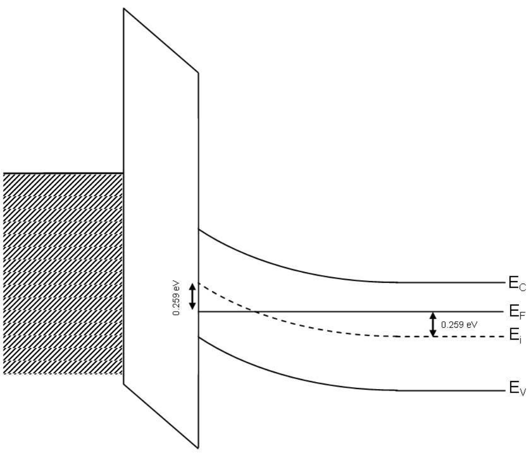


Figure 5: Question 4

A) (5 points) Find n far away from the junction region.

$$
\begin{aligned}
n &= n_i e^{(E_F - E_i)/kT} \\
&= 10^{10} cm^{-3} e^{0.259eV/0.0259eV} \\
&= 10^{10} cm^{-3} \cdot 2.2 \times 10^4 \\
&= 2.2 \times 10^{14} cm^{-3}
\end{aligned}
$$

B) (5 points) Find p far away from the junction region.

$$\begin{aligned} p &= n_i e^{(E_i - E_F)/kT} \\ &= 10^{10} cm^{-3} e^{-0.259eV/0.0259eV} \\ &= 10^{10} cm^{-3} \cdot 4.5 \times 10^{-5} \\ &= 4.5 \times 10^{5} cm^{-3} \end{aligned}$$

C) (10 points) Find n inside the semiconductor for the region close to the oxide.

$$\begin{aligned} n &= n_i e^{(E_i - E_F)/kT} \\ &= 10^{10} cm^{-3} e^{-0.259eV/0.0259eV} \\ &= 10^{10} cm^{-3} \cdot 4.5 \times 10^{-5} \\ &= 4.5 \times 10^{5} cm^{-3} \end{aligned}$$

D) (5 points) Find p inside the semiconductor for the region close to the oxide.

$$\begin{aligned} p &= n_i e^{(E_i - E_F)/kT} \\ &= 10^{10} cm^{-3} e^{-0.259eV/0.0259eV} \\ &= 10^{10} cm^{-3} \cdot 2.2 \times 10^{4} \\ &= 2.2 \times 10^{14} cm^{-3} \end{aligned}$$

Grading Criteria:

- 1pts for $n = n_i e^{(E_F - E_i)/kT}$ (parts A) and $p = n_i e^{(E_i - E_F)/kT}$ (parts B,D) or $np = {n_i}^2$ (parts B,D).
- 4pts for $2.2 \times 10^{14} cm^{-3}$ (parts A,D) and $4.5 \times 10^{5} cm^{-3}$ (parts B)
- 2pts for $n = n_i e^{(E_F - E_i)/kT}$ (parts C).
- 8pts for $4.5 \times 10^{5} cm^{-3}$ (parts C)