EECS170a Homework Solutions 2

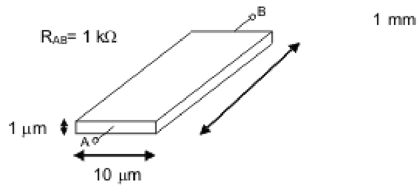
October 18, 2005

1. In modern integrated circuits, copper is used as the interconnect material. Ideally, the interconnect wiring would have zero resistance. In this exercise, we will see how low the resistance really is. Calculate the resistance of a typical copper trace. Assume the dimension is 0.1μm wide, 0.1μm high, and 1cm long. Pure, bulk copper (Cu) has a resistivity (ρ) of 1.7$\mu\Omega$−cm

$$\begin{aligned} L =& 1cm \\ W =& 0.1\mu\text{m} \\ H =& 0.1\mu\text{m} \\ \rho_{Cu} =& 1.7\mu\Omega - \text{cm} = 1.7x10^{-8}\Omega - \text{m} \end{aligned}$$

$$\begin{aligned} R_{wire} =& \frac{\rho L}{WH} = \frac{(1.7 \cdot 10^{-8}\Omega - \text{m})(1 \cdot 10^{-2}\text{m})}{(1 \cdot 10^{-7}\text{m})(1 \cdot 10^{-7}\text{m})} \\ R_{wire} =& 17k\Omega \end{aligned}$$

2. A thin metal film resistor as shown in the figure below has a resistance of 1$k\Omega$. It is 1mm long, 10μm wide, and 1μm thick.



a. Calculate the resistivity (ρ), in units of $\Omega - \text{m}$.

$$\rho = \frac{RA}{l} = \frac{1k\Omega \cdot (10\mu m \cdot 1\mu m)}{1mm} = 10^{-5}\Omega - m$$

b. Now express the resistivity in units of $\mu\Omega - cm$, a more common unit.

$$\begin{aligned} \rho =& 10^{-5}\Omega - m = (10^{-5}\Omega - m)\left(\frac{10^6\mu\Omega}{1\Omega}\right)\left(\frac{100cm}{1m}\right) \\ =& 10^3\mu\Omega - cm \end{aligned}$$

3. For Si at 300 K, do the following: (Use cm^{-3} as your units.)

a. $N_D = 10^{18}cm^{-3}$; $N_A \ll N_D$. Calculate the equilibrium electron concentration (n) and hole concentration (p)

Since $N_A \ll N_D$ and $n_i \ll N_D$, the concentration of electrons

$$
\begin{aligned}
n =& \frac{1}{2}\left[(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4{n_i}^2}\right] \\
\simeq& N_D \\
n =& 10^{18}cm^{-3}
\end{aligned}
$$

The corresponding hole concentration is,

$$
\begin{aligned}
p =& \frac{{n_i}^2}{n} \\
=& \frac{(10^{10})^2}{10^{18}} \\
p =& 10^2 cm^{-3}
\end{aligned}
$$

b. $N_D = 10^{12}cm^{-3}$; $N_A \ll N_D$. Calculate the equilibrium electron concentration (n) and hole concentration (p)

Again, $N_A \ll N_D$ and $n_i \ll N_D$ so the concentration of electrons (n),

$$
\begin{aligned}
n =& \frac{1}{2}\left[(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4{n_i}^2}\right] \\
\simeq& N_D \\
n =& 10^{12}cm^{-3}
\end{aligned}
$$

and the hole density (p) is,

$$
\begin{aligned}
p =& \frac{{n_i}^2}{n} \\
=& \frac{(10^{10})^2}{10^{12}} \\
p =& 10^8 cm^{-3}
\end{aligned}
$$

c. $N_A = 10^{18}cm^{-3}$; $N_A \gg N_D$. Calculate the equilibrium electron concentration (n) and hole concentration (p)

Since this semiconductor is p-typed (i.e. $N_A \gg N_D$ and $n_i \ll N_D$) the majority charge carriers are holes and we begin by calculating the hole concentration (p)

$$
\begin{aligned}
p =& \frac{1}{2}\left[(N_A - N_D) + \sqrt{(N_A - N_D)^2 + 4{n_i}^2}\right] \\
\simeq& N_A \\
p =& 10^{18}cm^{-3}
\end{aligned}
$$

and the electron density (n) is,

$$n = \frac{{n_i}^2}{p}$$
$$= \frac{(10^{10})^2}{10^{18}}$$
$$n = 10^2 cm^{-3}$$

d. $N_A = 10^{12} cm^{-3}$; $N_A \gg N_D$. Calculate the equilibrium electron concentration (n) and hole concentration (p)

$$p = \frac{1}{2}\left[(N_A - N_D) + \sqrt{(N_A - N_D)^2 + 4{n_i}^2}\right]$$
$$\simeq N_A$$
$$p = 10^{12} cm^{-3}$$

and the electron density (n) is,

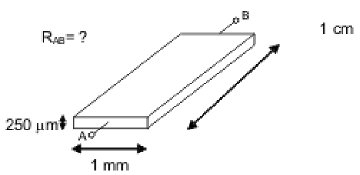
$$n = \frac{{n_i}^2}{p}$$
$$= \frac{(10^{10})^2}{10^{12}}$$
$$n = 10^2 cm^{-3}$$

4. For the silicon sample at $T = 300K$ shown below, given $N_A = 10^{20} cm^{-3}$, $N_D \ll N_A$,

a. Find the resistivity ρ of the Si to within 10% For units, use $\Omega - cm$.

From figure 3.8 we find that $\rho = 10^{-3}\Omega - cm$

b. Calculate the resistance R_{AB} in units of Ω, for the following geometry:



$$R_{AB} = \frac{\rho l}{A}$$
$$= \frac{10^{-3}\Omega - cm \cdot 1cm}{0.025cm \cdot 0.1cm}$$
$$R_{AB} = 0.4\Omega$$