EECS170a Homework Solutions 3

November 7, 2005

1. Answer a-d for the figure shown below:

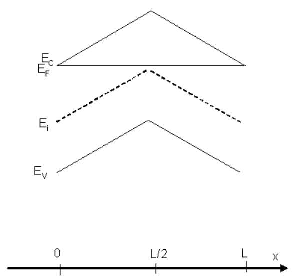

Figure 1: Problem 1

a. Do equilibrium conditions prevail? How do you know?

- The system is in equilibrium since the Fermi energy is flat (i.e. $dE_F/dx = 0$) and consequently there will be no net current flow ($J_{net} = 0$). Any drift currents (J_{drift}) created by internal electric fields will be exactly canceled out by an equal but opposite diffusion current, $J_{diffusion}$.

b. Sketch the electrostatic potential (V) inside the semiconductor as a function of x.

- Electrons in the conduction band will exist at an energy E_C as defined by the energy band diagram and experience a corresponding potential difference of $V = -E_C/q$. Remember, V is a relative quantity and can be scaled by any reference voltage.

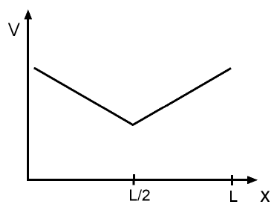

Figure 2: Problem 1b

c. Sketch the electric field, **E**, inside the semiconductor as a function of x.

- The electric field **E** is can be obtained from V by the relation $E = -dV/dx$. For $x \in (0, L/2)$, V has a negative slope ($dV/dx < 0$) so $E = -(-c) = +c$ where c is some positive constant. For $x \in (L/2, L)$, V has a positive slope ($dv/dx > 0$) so $E = -(+c) = +c$.

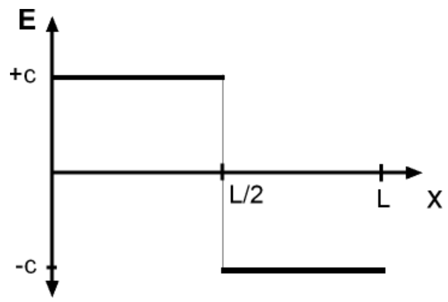

Figure 3: Problem 1c

d. Roughly sketch n and p versus x.

- Electrons will accumulate at the lowest energy possible relative to the Fermi energy. Consequently, the electron density n will be highest near $x = 0$ and $x = L$. Holes (*i.e.* electron vacancies) will accumulate at the highest possible energy relative to E_F which in this case is centered at $x = L/2$. The magnitude of p is small relative to n because even at $x = L/2$, where p is max, $E_F - E_V < E_C - E_F$.

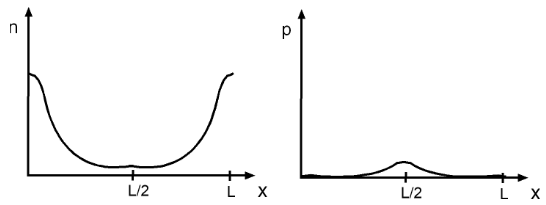

Figure 4: Problem 1d

2. Answer a-d for the figure shown below:

a. Do equilibrium conditions prevail? How do you know?

- The system is in equilibrium since the Fermi energy is flat (i.e. $dE_F/dx = 0$) and consequently, there will be no net current flow ($J_{net} = 0$). Any drift currents (J_{drift}) created by internal electric fields will be exactly canceled out by an equal but opposite diffusion current, $J_{diffusion}$.

b. Sketch the electrostatic potential (V) inside the semiconductor as a function of x.

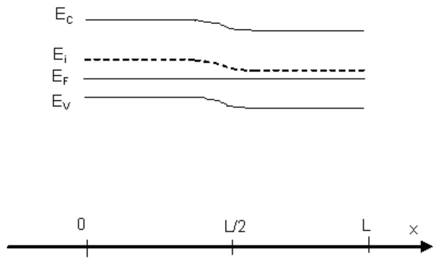

Figure 5: Problem 2

- Electrons in the conduction band will exist at an energy E_C as defined by the energy band diagram and experience a corresponding potential difference of $V = -E_C/q$. Remember, V is a relative quantity and can be scaled by any reference voltage.

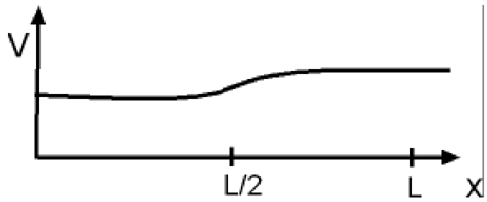

Figure 6: Problem 2b

c. Sketch the electric field, **E**, inside the semiconductor as a function of x.

- The electric field **E** is can be obtained from V by the relation $E = -dV/dx$. Around $x = L/2$ we see that $dV/dx = +$ so $\mathbf{E} = -(+) = -$.

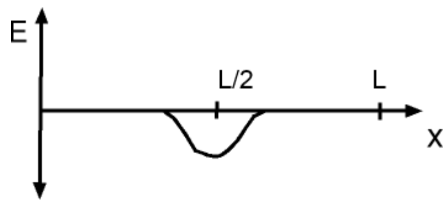

Figure 7: Problem 2c

d. Roughly sketch n and p versus x.

- Electrons will accumulate at the lowest energy possible relative to the Fermi energy. Consequently, the electron density n will be highest for $x \in (L/2, L)$. Holes (*i.e.* electron vacancies) will accumulate at the highest possible energy relative to E_F which in this case $x \in (0, L/2$. The magnitude of n is smaller compared to p because even at $x = L/2$, where p is max, $E_C - E_F > E_F - E_V$.

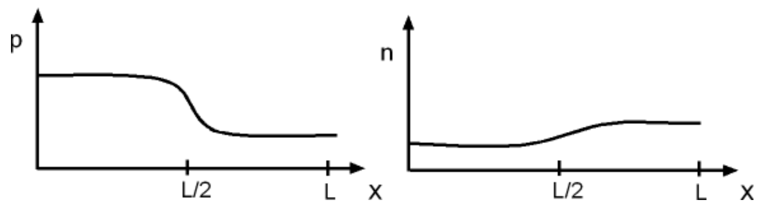

Figure 8: Problem 2d

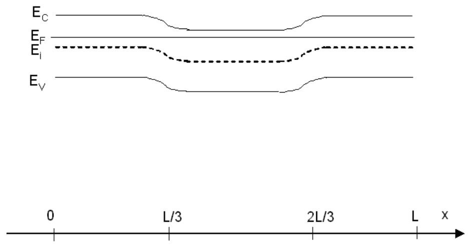

Figure 9: Problem 3

3. Answer a-d for the figure shown below:

a. Do equilibrium conditions prevail? How do you know?

- The system is in equilibrium since the Fermi energy is flat (i.e. $dE_F/dx = 0$). There will be no net current flow ($J_{net} = 0$). Any drift currents (J_{drift}) created by internal electric fields will be exactly canceled out by an equal but opposite diffusion current, $J_{diffusion}$.

b. Sketch the electrostatic potential (V) inside the semiconductor as a function of x.

- Electrons in the conduction band will exist at an energy E_C as defined by the energy band diagram and experience a corresponding potential difference of $V = -E_C/q$. Remember, V is a relative quantity and can be scaled by any reference voltage.

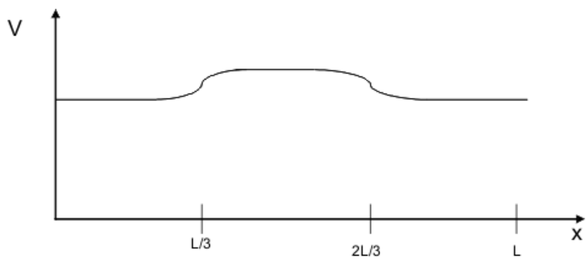

Figure 10: Problem 3b

c. Sketch the electric field, **E**, inside the semiconductor as a function of x.

- The electric field **E** is can be obtained from V by the relation $E = -dV/dx$.

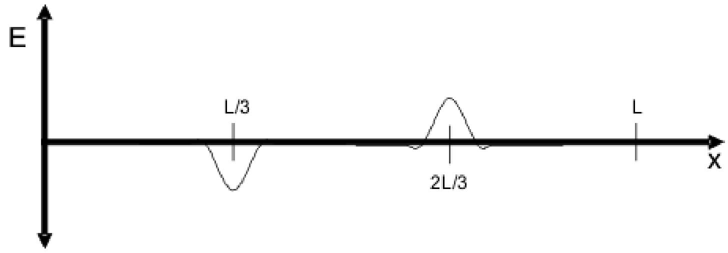

Figure 11: Problem 3c

d. Roughly sketch n and p versus x.

- Electrons will accumulate at the lowest energy possible relative to the Fermi energy.

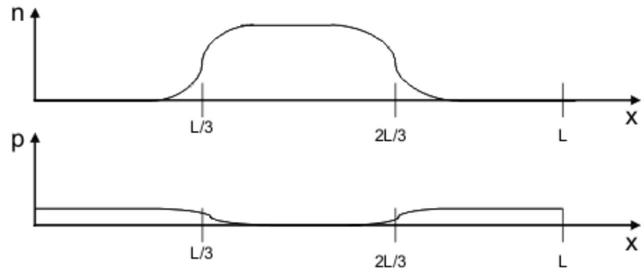

Figure 12: Problem 3d

4. For Si at 300 K, calculate $E_C - E_F$ and sketch E_C, E_F, E_i, and E_V as in figure 2.18 of the book for the following cases:

a. $N_D = 10^{17} cm^{-3}, N_A \ll N_D$

- Starting with $n = N_C e^{-(E_C - E_F)/kT}$ and solving for $E_C - E_F$ we obtain

$$E_C - E_F = -kT \ln(\frac{n}{N_C})$$

Since $N_D \gg N_A$ and $N_D \gg n_i$ we can approximate the electron concentration as $n \simeq N_D = 10^{17} cm^{-3}$. The value for kT and N_C are given in the book: $kT = 0.026eV$ and $N_C = (2.51 \times$

$10^{19}/cm^{-3})(m^*{}_n/m_0)^{3/2}$ (pg51) where $m^*{}_n/m_0 = 1.18$ (pg34) for Si at $T = 300K$ giving us

$$\begin{aligned} N_C &= (2.51 \times 10^{19}/cm^{-3})(m^*{}_n/m_0)^{3/2} \\ &= (2.51 \times 10^{19}/cm^{-3})(1.18)^{3/2} \\ &= 3.217 \times 10^{19} cm^{-3} \end{aligned}$$

$$\begin{aligned} E_C - E_F &= -kT \ln(\frac{n}{N_C}) \\ &= -(0.026eV) \ln(\frac{10^{17} cm^{-3}}{3.217 \times 10^{19} cm^{-3}}) \\ &= 0.15eV \end{aligned}$$

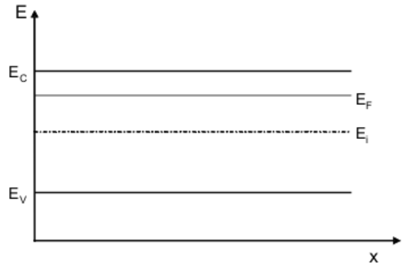

Figure 13: Problem 4a

b. $N_D = 10^{18} cm^{-3}; N_A \ll N_D$

- Since $N_D \gg N_A$ and $N_D \gg n_i$ we can approximate the electron concentration as $n \simeq N_D = 10^{18} cm^{-3}$. $kT = 0.026eV$ and $N_C = 3.217 \times 10^{19} cm^{-3}$ giving us

$$\begin{aligned} E_C - E_F &= -kT \ln(\frac{n}{N_C}) \\ &= -(0.026eV) \ln(\frac{10^{18} cm^{-3}}{3.217 \times 10^{19} cm^{-3}}) \\ &= 0.09eV \end{aligned}$$

c. $N_A = 10^{19} cm^{-3}; N_A \gg N_D$

- Starting with $p = N_V e^{-(E_F - E_V)/kT}$ and solving for $E_F - E_V$ we obtain

$$E_F - E_V = -kT \ln(\frac{p}{N_V})$$

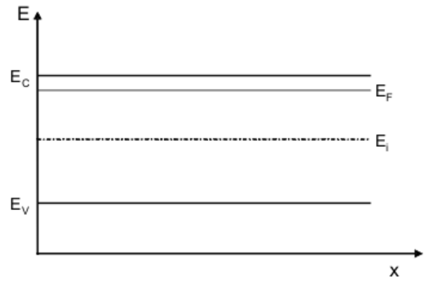

Figure 14: Problem 4b

Since $N_D \ll N_A$ and $N_A \gg n_i$ we can approximate the hole concentration as $p \simeq N_D = 10^{19} cm^{-3}$. The value for kT and N_V are given in the book: $kT = 0.026eV$ and $N_V = (2.51{\times}10^{19}/cm^{-3})({m^*}_n/m_0)^{3/2}$ (pg51) where ${m^*}_p/m_0 = 0.81$ (pg34) for Si at $T = 300K$ giving us

$$\begin{aligned} N_V &= (2.51 \times 10^{19} cm^{-3})({m^*}_p/m_0)^{3/2} \\ &= (2.51 \times 10^{19} cm^{-3})(0.81)^{3/2} \\ &= 1.83 \times 10^{19} cm^{-3} \end{aligned}$$

$$\begin{aligned} E_F - E_V &= -kT \ln(\frac{p}{N_V}) \\ &= -(0.026eV) \ln(\frac{10^{19} cm^{-3}}{1.83 \times 10^{19} cm^{-3}}) \\ &= 0.0157eV \end{aligned}$$

To solve $E_C - E_F$ we note that

$$\begin{aligned} E_C - E_F &= E_G - (E_F - E_V) \\ &= (1.12eV) - (0.0157eV) \\ &= 1.104eV \end{aligned}$$

d. $N_A = 10^{15} cm^{-3}; N_A \gg N_D$

- Since $N_D \ll N_A$ and $N_A \gg n_i$ we can approximate the hole concentration as $p \simeq N_D = 10^{15} cm^{-3}$.

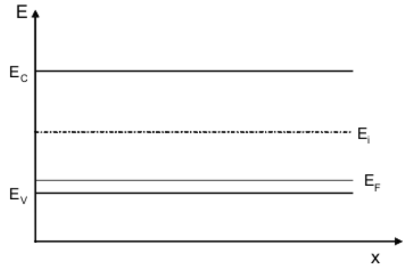

Figure 15: Problem 4c

The value for kT and N_V are given in the book: $kT = 0.026eV$ and $N_V = 1.83 \times 10^{19}cm^{-3}$

$$\begin{aligned} E_F - E_V &= -kT\ln(\frac{p}{N_V}) \\ &= -(0.026eV)\ln(\frac{10^{15}cm^{-3}}{1.83 \times 10^{19}cm^{-3}}) \\ E_F - E_V &= 0.255eV \end{aligned}$$

To solve $E_C - E_F$ we note that

$$\begin{aligned} E_C - E_F &= E_G - (E_F - E_V) \\ &= (1.12eV) - (0.255eV) \\ &= 0.8654eV \end{aligned}$$

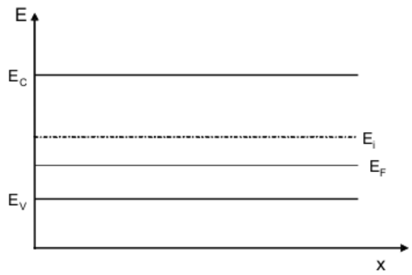

Figure 16: Problem 4d

e. $N_A = N_D = 10^{11}cm^{-3}$.

- In this case, $N_A \approx N_D$ so we need to use the more explicit equation to solve for either n or p. Solving for n we get,

$$n = \frac{N_D - N_A}{2} + [(\frac{N_D - N_A}{2})^2 + {n_i}^2]^{1/2}$$
$$= \frac{10^{11} - 10^{11}}{2} + [(\frac{10^{11} - 10^{11}}{2})^2 + {10^{10}}^2]^{1/2}$$
$$n = 10^{10} cm^{-3}$$

And like the previous problems we can solve for $E_C - E_F$ using,

$$E_C - E_F = -kT \ln(\frac{n}{N_C})$$
$$= -(0.026eV) \ln(\frac{10^{10} cm^{-3}}{3.217 \times 10^{19} cm^{-3}})$$
$$= 0.55eV$$

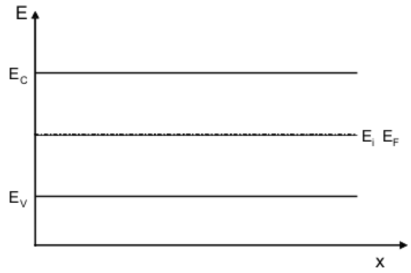

Figure 17: Problem 4e