EECS170a Homework Solutions 4

November 30, 2005

1. In class we found

$$I = I_0(e^{qV_{diode}/kT} - 1). \quad (1)$$

Take $I_0 = 10^{-14}A$. For the circuit shown in Fig 1, fill in the following table:

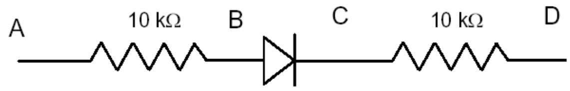

Figure 1: Question 1

Solution:

Applying Kirchoff's Law to this circuit we obtain

$$V_{AD} = RI_{AD} + V_{diode} + RI_{AD}$$

and solving for I_{AD} we get,

$$I_{AD} = \frac{V_{AD} - V_{diode}}{2R} \quad (2)$$

where $R = 10k\Omega$.

In order to solve for I_{AD} we must obtain V_{diode} which can be derived from (1) giving us

$$\begin{aligned} V_{diode} &= \frac{kT}{q} \ln\left(\frac{I_{AD}}{I_0} + 1\right) \\ &= \frac{kT}{q} \ln\left(\frac{(V_{AD} - V_{diode})/2R}{I_0} + 1\right) \end{aligned}$$

Since we can not explicitly solve for V_{diode} the above transcendental equation can be solved in an iterative process. To begin off lets take V_{diode} in the log as $0.6V$ for the condition that $V_{AD} = 1.0V$,

$$\begin{aligned} V_{diode} &= 0.026V \ln\left(\frac{(1V - 0.6V)/(2 \cdot 10k\Omega)}{10^{-14}A} + 1\right) \\ &= 0.557V \end{aligned}$$

For the second iterative step we can use the updated value of V_{diode} as $0.557V$ and further enhance our precision of V_{diode}. Now we get,

$$\begin{aligned} V_{diode} &= 0.026V \ln\left(\frac{(1V - 0.557V)/(2 \cdot 10k\Omega)}{10^{-14}A} + 1\right) \\ &= 0.559V \end{aligned}$$

If a further iterative step does not change V_{diode} appreciably then we can stop the iteration. In this case, a third iterative step results in the same value to the thousandths place so the final answer is $V_{diode} = 0.559V$ for $V_{AD} = 1.0V$. Now that we know V_{diode} we can solve for the current I_{AD} from (2) and obtain

$$\begin{aligned} I_{AD} &= \frac{V_{AD} - V_{diode}}{2R} \\ &= \frac{1V - 0.559V}{2 \cdot 10k\Omega} \\ &= 22\mu A \end{aligned}$$

The final solution table is,

V_{AD}	$V_{diode} = V_{BC}$	I_{AD}
0.0	0	0
0.5	$0.4795V$	$1.02\mu A$
1.0	$0.559V$	$22\mu A$
1.5	$0.579V$	$46\mu A$
2.0	$0.590V$	$70\mu A$
2.5	$0.597V$	$95\mu A$
3.0	$0.603V$	$120\mu A$
3.5	$0.608V$	$145\mu A$
4.0	$0.612V$	$169\mu A$
4.5	$0.616V$	$194\mu A$
5.0	$0.619V$	$219\mu A$
5.5	$0.622V$	$244\mu A$
6.0	$0.624V$	$269\mu A$
6.5	$0.627V$	$294\mu A$
7.0	$0.629V$	$319\mu A$
7.5	$0.631V$	$343\mu A$
8.0	$0.633V$	$368\mu A$
8.5	$0.634V$	$393\mu A$
9.0	$0.636V$	$418\mu A$
9.5	$0.637V$	$443\mu A$
10	$0.639V$	$468\mu A$

One can confirm that theses solutions are correct by plugging them into the two equations to determine if the they both produce the same results.

2. For the circuit shown below, find I_B,I_E, I_C, V_C, V_B, V_E, V_{BE}, V_{CE}, V_{BC} defined in Figure 10.2 of the text. Take $\beta = 100$. Then $I_C = 100I_B$. The rest is just applications of Kirchoff's current and voltage laws. Is the transistor biased active mode?

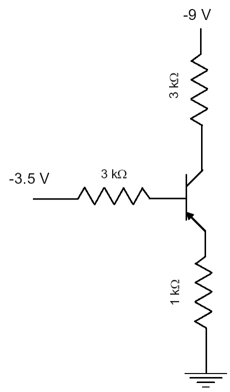

Figure 2: Question 2

Solution:

The system of equations that describe this circuit are

$$
\begin{aligned}
&V_C - V_1 = I_C R_1 \\
&0 - V_E = I_E R_3 \\
&V_B - V_2 = I_B R_2 \\
&V_B - V_E = V_d \\
&I_C = 100 I_B \\
&I_E = I_B + I_C
\end{aligned}
$$

where $V_1 = -9V$, $V_2 = -3.5V$, $R_1 = R_2 = 3k\Omega$, $R_3 = 1k\Omega$, and $V_d = 0.6V$. We can compactly express this system of equations as an array

$$
\begin{pmatrix} V_1 \\ 0 \\ V_2 \\ 0 \\ V_d \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -R_1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -R_3 & 0 \\ 0 & 0 & 1 & 0 & 0 & -R_2 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -100 \end{pmatrix} \begin{pmatrix} V_C \\ V_E \\ V_B \\ I_C \\ I_E \\ I_B \end{pmatrix}.
$$

Using Matlab or Mathematica we can invert the $6x6$ matrix and obtain,

$$
\begin{pmatrix} V_C \\ V_E \\ V_B \\ I_C \\ I_E \\ I_B \end{pmatrix} = \frac{1}{R_2 + 101R_3} \begin{pmatrix} R_2 + 101R_3 & -100R_1 & -100R_1 & 100R_1R_3 & 100R_1 & R_1R_2 + R_1R_3 \\ 0 & -R_2 & 101R_3 & R_2R_3 & -101R_3 & -R_2R_3 \\ 0 & -R_2 & 101R_3 & R_2R_3 & R_2 & -R_2R_3 \\ 0 & -100 & -100 & 100R_3 & 100 & R_2 + R_3 \\ 0 & -101 & -101 & -R_2 & 101 & R_2 \\ 0 & -1 & -1 & R_3 & 1 & -R_3 \end{pmatrix} \begin{pmatrix} V_1 \\ 0 \\ V_2 \\ 0 \\ V_d \\ 0 \end{pmatrix}
$$

$$\begin{pmatrix} V_C \\ V_E \\ V_B \\ I_C \\ I_E \\ I_B \end{pmatrix} = \frac{1}{R_2 + 101R_3} \begin{pmatrix} (R_2 + 101R_3)V_1 - 100R_1V_2 + 100R_1V_d \\ 101R_3V_2 - 101R_3V_d \\ 101R_3V_2 + R_2V_d \\ 100V_d - 100V_2 \\ 101V_d - 101V_2 \\ V_d - V_2 \end{pmatrix} = \begin{pmatrix} 2.83V \\ -3.98V \\ -3.38V \\ 3.94mA \\ 3.98mA \\ 39.4\mu A \end{pmatrix}.$$

Noting that $V_{BC} = V_B - V_C = -V_{CB}$ and that an active biased *pnp* transistor is defined to have $V_{BC} > 0$ (reverse biased) and $V_{BE} < 0$ (forward biased) we obtain the values,

I_E	$3.98mA$
I_B	$39.4\mu A$
I_C	$3.94mA$
V_E	$-3.98V$
V_B	$-3.38V$
V_C	$2.83V$
V_{BE}	$0.6V$
V_{CE}	$6.81V$
V_{BC}	$-6.21V$
Activebiased?	No

Again, we can check that these equations are correct by substituting them back into the original equations.