EECS170a Midterm Solutions

1. This is not a trick question. For this problem, use the approximation $\pi = 3$.

   The points on this plot are real data on very small cylindrical wires with a diameter of 2nm. All wires are made of the same material.

   The dotted line is the best fit to the data of $R = 6(k\Omega/\mu m) \times L$. Here R is the resistance, and L is the length. In other words, the longer the wire, the higher the resistance. Find the resistivity of the material used to make the wire. (No units, no credit)

   Solutions

   Resistance can be expressed in terms of resistivity as, $R = \frac{\rho}{A}l$ and comparing the linear fit of the data to the above equation we notice that $\frac{\rho}{A} = (6k\Omega/\mu m)$
   For a cylindrical wire, the cross-sectional area is $A = \pi r^2$ where $r = d/2 = 2nm/2 = 1nm$. Consequently,

$$\begin{aligned}
\rho &= (\frac{6k\Omega}{\mu m})(\pi r^2) \\
&\approx (\frac{6k\Omega}{\mu m})(3 \cdot 1nm^2) \\
&\approx (\frac{18k\Omega \cdot nm^2}{1000nm}) \\
\rho &\approx (18\Omega \cdot nm) = 1.8\mu\Omega \cdot cm = 1.8 \times 10^{-8}\Omega m
\end{aligned}$$

   Grading Criteria:

   - (5pts) for $\rho = \frac{R \cdot A}{l}$
   - (5pts) for $\rho = 6k\Omega/\mu m \cdot A$
   - (10pts) for $\rho = (\frac{6k\Omega}{\mu m})(3 \cdot 1nm^2)$
   - (30pts) for correct answer: range $\rho \in (10\Omega \cdot nm, 36\Omega \cdot nm)$ (w/ correct units).

2. Note: N_D on the left side: $N_A = 0$ on the right side.

a Is the system in equilibrium?

Yes, since $dE_F/dx = 0$.

b Determine n on the left side (far away from the junction region).

$$\begin{aligned} n_p &= n_i e^{E_i - E_F}/kT \\ &= (10^{10} cm^{-3}) e^{-0.259eV/0.0259eV} \\ &= 10^{10} cm^{-3} \cdot e^{-10} \\ &= 10^{10} cm^{-3} \cdot 4.5 \times 10^{-5} \\ n_p &= 4.5 \times 10^{5} cm^{-3} \end{aligned}$$

c Determine p on the left side (far away from the junction region).

$$\begin{aligned} p_p &= n_i e^{E_i - E_F}/kT \\ &= (10^{10} cm^{-3}) e^{+0.259eV/0.0259eV} \\ &= 10^{10} cm^{-3} \cdot e^{10} \\ &= 10^{10} cm^{-3} \cdot 2.2 \times 10^{4} \\ p_p &= 2.2 \times 10^{14} cm^{-3} \end{aligned}$$

d Determine N_A on the left side (far away from the junction region).

$$\begin{aligned} &\text{Since } N_A \gg N_D \text{and } p_p \gg n_i \\ &N_A \simeq p_p = 2.2 \times 10^{14} cm^{-3} \end{aligned}$$

e Determine n on the right side (far away from the junction region).

$$\begin{aligned} n_n &= n_i e^{E_F - E_i}/kT \\ &= (10^{10} cm^{-3}) e^{+0.259eV/0.0259eV} \\ &= 10^{10} cm^{-3} \cdot e^{10} \\ &= 10^{10} cm^{-3} \cdot 2.2 \times 10^{4} \\ n_n &= 2.2 \times 10^{14} cm^{-3} \end{aligned}$$

f Determine p on the right side (far away from the junction region).

$$\begin{aligned} p_n &= n_i e^{E_i - E_F}/kT \\ &= (10^{10} cm^{-3}) e^{-0.259eV/0.0259eV} \\ &= 10^{10} cm^{-3} \cdot e^{-10} \\ &= 10^{10} cm^{-3} \cdot 4.5 \times 10^{-5} \\ p_n &= 4.5 \times 10^{5} cm^{-3} \end{aligned}$$

g Determine N_D on the left side (far away from the junction region).

$$\begin{aligned} &\text{Since } N_D \gg N_A \text{and } n_n \gg n_i \\ &N_D \simeq n_n = 2.2 \times 10^{14} cm^{-3} \end{aligned}$$

Grading Criteria: 2

2a....
- (10pts) Correct answer

2b,c,e,f
- (5pts) for correct formula: either ${n_i}^2 = np$ or n, p vs. E_F.
- (10pts) correct answer: range $n, p \in (1.0 \times 10^{14} cm^{-3}, 10 \times 10^{14} cm^{-3})$ for **c, e** and $n, p \in (1.0 \times 10^{5} cm^{-3}, 10 \times 10^{5} cm^{-3})$ for **b,f** (w/ correct units).

2d,g
- (5pts) for correct formula ${n_i}^2 = np$ or $N_A - N_D - n + p = 0$ or any formula derivable from these.
- (10pts) correct answer: range $n, p \in (2.0 \times 10^{14} cm^{-3}, 2.4 \times 10^{14} cm^{-3})$ for **d,g** (w/ correct units).