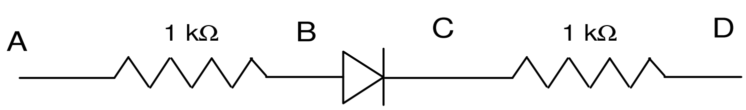
EECS 170A Section B
Homework Solution #4

1) In class we found:

$$I = I_0(e^{qV_{diode}/kT} - 1)$$

Take $I_0 = 10^{-14}$ A. For the circuit shown, fill in the following table:



Solution:

By applying Kirchoff's Law to this circuit, we obtain

$$V_{AD} = RI + V_{diode} + RI = 2RI + V_{diode} \qquad (2)$$

Substitute eqn (1) into (2), we get:

$$V_{AD} = V_{diode} + I_0(e^{qV_{diode}/kT} - 1) \ x \ 2R$$

Substitute $I_0 = 10^{-14}$ A and R = 1 KΩ, we get:

$$V_{AD} = V_{diode} + 10^{-14}A \ x \ (e^{qV_{diode}/kT} - 1) \ x \ 2K\Omega$$

$$V_{AD} = V_{diode} + 2 \ x \ 10^{-11} \ x \ (e^{qV_{diode}/kT} - 1)$$

$$(e^{qV_{diode}/kT} - 1) = (V_{AD} - V_{diode}) / \ 2 \ x \ 10^{-11}$$

$$V_{diode} = KT/q \ ln \ ((V_{AD} - V_{diode}) / 2 \ x \ 10^{-11} + 1)$$

(2pts for showing the equations)

Since we can not explicitly solve for V_{diode} the above transcendental equation can be solved in an iterative process. To begin with, lets take V_{diode} in the log as 0.6V for the condition that V_{AD} = 1V,

$$V_{diode} = 0.026V \ x \ ln \ ((1V - 0.6V) / 2 \ x \ 10^{-11} + 1)$$

$$V_{diode} = 0.6167V$$

For the second iterative step we can use the updated value of V_{diode} as 0.62V and further enhance our precision of V_{diode}. Now we get,

$$V_{diode} = 0.026V \ x \ ln \ ((1V - 0.62V) / 2 \ x \ 10^{-11} + 1)$$

$$V_{diode} = 0.6154V$$

For the third iterative step we can use the updated value of V_{diode} as 0.6156V and further enhance our precision of V_{diode}. Now we get,

$$V_{diode} = 0.026V \times \ln (1V - 0.6156V) / 2 \times 10^{-9} + 1)$$
$$V_{diode} = 0.6156V$$

If a further iterative step does not change V_{diode} appreciably then we can stop the iteration. In this case, the iterative results in the same value to the thousandths place so the final answer is V_{diode} = 0.6156V for V_{AD} = 1V.

Now that we know V_{diode} we can solve for the current I_{AD} and obtain:

$$I_{AD} = 10^{-14} A \times (e^{(0.6156V/0.026V)} - 1) = 2.71 \times 10^{-7} A$$

With the same approach, we can obtain the results for each V_{AD} value:

V_{AD} (V)	V_{diode} (V) = V_{BC}	I_{AD} (A)
0	0	0
0.5	0.4913	1.94×10^{-6}
1	0.6156	1.92×10^{-4}
1.5	0.6366	4.30×10^{-4}
2	0.6484	6.77×10^{-4}
2.5	0.654	9.23×10^{-4}
3	0.6626	1.17×10^{-3}
3.5	0.6676	1.41×10^{-3}
4	0.6718	1.66×10^{-3}
4.5	0.6754	1.91×10^{-3}
5	0.6786	2.16×10^{-3}
5.5	0.6814	2.41×10^{-3}
6	0.6840	2.66×10^{-3}
6.5	0.6863	2.91×10^{-3}
7	0.6884	3.15×10^{-3}
7.5	0.6904	3.40×10^{-3}
8	0.6922	3.65×10^{-3}
8.5	0.6939	3.90×10^{-3}
9	0.6956	4.15×10^{-3}
9.5	0.6971	4.40×10^{-3}
10	0.6985	4.65×10^{-3}

(0.25 pts for correct answers)

Now do the same, assuming the resistors are 1 MΩ instead of 1 kΩ.

Substitute $I_0 = 10^{-14}$ A and R = 1 MΩ, we get:

$$V_{diode} = KT/q \ln((V_{AD} - V_{diode}) / 2 \times 10^{-8} + 1)$$

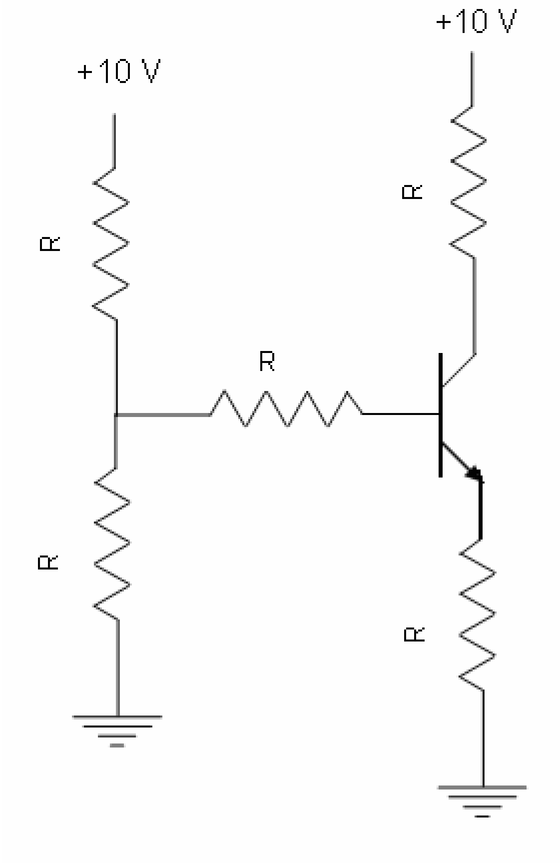
V_{AD} (V)	V_{diode} (V) = V_{BC}	I_{AD} (A)
0	0	0
0.5	0.4008	4.952×10^{-8}
1	0.4458	2.772×10^{-7}
1.5	0.4619	5.189×10^{-7}
2	0.4719	7.640×10^{-7}
2.5	0.4792	1.010×10^{-6}
3	0.4849	1.258×10^{-6}
3.5	0.4896	1.505×10^{-6}
4	0.4935	1.753×10^{-6}
4.5	0.4970	2.002×10^{-6}
5	0.5000	2.250×10^{-6}
5.5	0.5027	2.499×10^{-6}
6	0.5052	2.747×10^{-6}
6.5	0.5075	2.996×10^{-6}
7	0.5095	3.245×10^{-6}
7.5	0.5115	3.494×10^{-6}
8	0.5132	3.743×10^{-6}
8.5	0.5149	3.993×10^{-6}
9	0.5165	4.242×10^{-6}
9.5	0.5180	4.491×10^{-6}
10	0.5194	4.740×10^{-6}

(0.25 pts for correct answers)

How much does this effect the "on voltage" by?

The resistances differ by 3 order of magnitude (from 1kΩ to 1MΩ), but the on voltage only change by approximately 0.18V. The effect is relatively small.
(2pts)

2) For the circuit shown below, find I_B, I_E, I_C, V_C, V_B, V_E, V_{BE}, V_{CE}, V_{BC} defined in figure 10.2 of the text. Hints: the BE voltage drop is about 0.6 V. Take β = 100. Then I_C = 100 I_B. The rest is just applications of Kirchoff's current and voltage laws. Is the transistor biased in active mode? Assume R = 1 kΩ.



Solution:

Assuming the transistors is in active mode,

$V_{BE} = V_B - V_E = 0.6$ V (2.1) (4pts)
$I_C = \beta I_B = 100\ I_B$ (2.2) (4pts)
$I_E = I_B + I_C = 101\ I_B$ (2.3) (4pts)
$I_C = (10 - V_C) / R$ (2.4) (4pts)
$I_B = (5 - V_B) / (R + R/2)$ (2.5) (4pts)
$I_E = V_E / R$ (2.6) (4pts)

From eqn 2.3 and eqn 2.6:

$101\ I_B = V_E / 1\text{k}\Omega$ (2.7)

Substitute eqn 2.5 into eqn 2.7:

$101\ (5 - V_B) / 1.5\text{k}\Omega = V_E / 1\text{k}\Omega$ (2.8)

From eqn 2.1 and substitute into eqn 2.8:

$101\ (5 - V_B) / 1.5\text{k}\Omega = (V_B - 0.6) / 1\text{k}\Omega$
$101\ (5 - V_B) = 1.5 V_B - 0.9$

$$\therefore\ V_B = 4.936\ V \qquad (2.9)$$

Substitute into eqn (2.9) into eqn (2.1):

$$V_E = V_B - 0.6$$
$$= 4.336\ V \qquad (2.10)$$

From eqn (2.5) and eqn (2.9):

$$\therefore\ I_B = (5 - V_B)\ /\ 1.5R = 4.293 \times 10^{-5}\ A$$

From eqn (2.2):

$$I_C = 100\ I_B = 4.293 \times 10^{-3}\ A$$

From eqn (2.3):

$$I_E = 101\ I_B = 4.336 \times 10^{-3}\ A$$

From eqn (2.4):

$$V_C = 10 - R\ I_C$$
$$= 5.707\ V$$

Since $V_{BE} = V_{BE(on)}$ and $V_{BC} < 0$, the BJT is indeed in active mode. (4pts)

$I_E = 4.336 \times 10^{-3}$ A	(2pts)
$I_B = 4.293 \times 10^{-5}$ A	(2pts)
$I_C = 4.293 \times 10^{-3}$ A	(2pts)
$V_E = 4.336$ V	(2pts)
$V_B = 4.936$ V	(2pts)
$V_C = 5.707$ V	(2pts)
$V_{BE} = 0.6$ V	(2pts)
$V_{CE} = 1.371$ V	(2pts)
$V_{BC} = -0.771$ V	(2pts)