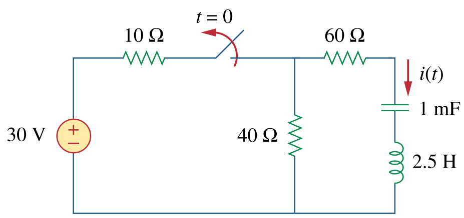
Spring, 2007

EECS70A / CSE 70A Network Analysis I
Prof. Peter Burke

Homework # 6 solution

Q1. Problem 8.16



At t = 0, i(0) = 0, $V_C(0)$ = 40x30/50 = 24V

For t > 0, we have a source-free RLC circuit.

$\alpha = R/(2L) = (40 + 60)/5 = 20$ and $\omega_o = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{10^{-3} x 2.5}} = 20$

$\omega_o = \alpha$ leads to critical damping

$i(t) = [(A + Bt)e^{-20t}]$, $i(0) = 0 = A$

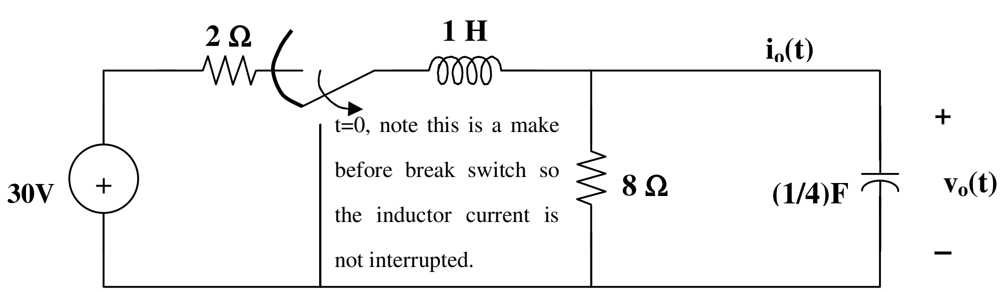
$di/dt = \{[Be^{-20t}] + [-20(Bt)e^{-20t}]\}$,

but $di(0)/dt = -(1/L)[Ri(0) + V_C(0)] = -(1/2.5)[0 + 24]$

Hence, B = -9.6 or i(t) = **<u>$[-9.6te^{-20t}]$ A</u>**

Q2. Problem 8.25:

In the circuit in Fig. 8.76, calculate $i_o(t)$ and $v_o(t)$ for t>0.



At $t = 0^-$, $v_o(0) = (8/(2 + 8)(30) = 24$

For $t > 0$, we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = 1/4$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/4} = 2$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4-(1/16)} = 1.9843$$

$$v_o(t) = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$$

$v_o(0) = 30(8/(2+8)) = 24 = A_1$ and $i_o(t) = C(dv_o/dt) = 0$ when $t = 0$.

$$dv_o/dt = -\alpha(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t} + (-\omega_d A_1\sin\omega_d t + \omega_d A_2\cos\omega_d t)e^{-\alpha t}$$

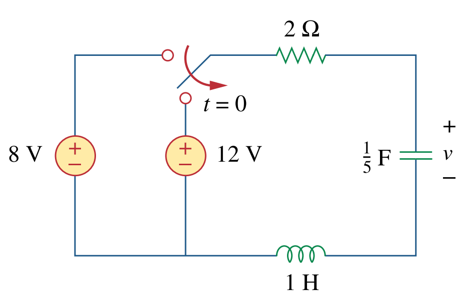
at $t = 0$, we get $dv_o(0)/dt = 0 = -\alpha A_1 + \omega_d A_2$

Thus, $A_2 = (\alpha/\omega_d)A_1 = (1/4)(24)/1.9843 = 3.024$

$v_o(t) =$ **<u>$(24\cos 1.9843t + 3.024\sin 1.9843t)e^{-t/4}$ volts</u>**.

$i_0(t) = Cdv/dt = 0.25[–24(1.9843)\sin 1.9843t +$
$3.024(1.9843)\cos 1.9843t – 0.25(24\cos 1.9843t) –$
$0.25(3.024\sin 1.9843t)]e^{–t/4}$
= **<u>$[0.000131\cos 1.9843t – 12.095\sin 1.9843t]e^{–t/4}$ A</u>**.

Q3. Problem 8.35



At t = 0-, $i_L(0) = 0$, $v(0) = v_C(0) = 8$ V

For $t > 0$, we have a series RLC circuit with a step input.

$\alpha = R/(2L) = 2/2 = 1$,

$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1/5} = \sqrt{5}$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \pm j2$$

$v(t) = V_s + [(A\cos 2t + B\sin 2t)e^{-t}]$, $V_s = 12$.

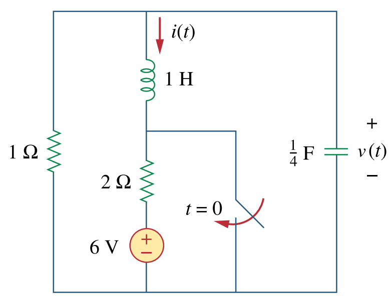
$v(0) = 8 = 12 + A$ or $A = -4$, $i(0) = Cdv(0)/dt = 0$.

But $dv/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [2(-A\sin 2t + B\cos 2t)e^{-t}]$

$0 = dv(0)/dt = -A + 2B$ or $2B = A = -4$ and $B = -2$

$v(t) =$ $\underline{\mathbf{\{12 - (4\cos 2t + 2\sin 2t)e^{-t}\ V}}$.

Q4. Problem 8.48



For $t = 0^-$, we obtain $i(0) = -6/(1 + 2) = -2$ and $v(0) = 2\times 1 = 2$.

For $t > 0$, the voltage is short-circuited and we have a source-free parallel RLC circuit.

$\alpha = 1/(2RC) = (1)/(2\times 1\times 0.25) = 2$

$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1\times 0.25} = 2$

Since $\alpha = \omega_o$, we have a critically damped response.
$s_{1,2} = -2$

Thus, $i(t) = [(A + Bt)e^{-2t}]$, $i(0) = -2 = A$

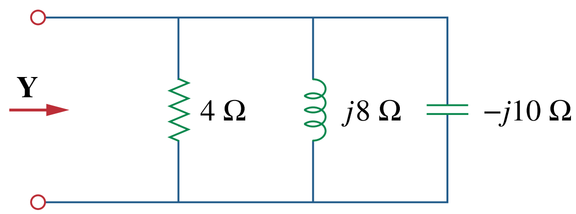
$v = Ldi/dt = [Be^{-2t}] + [-2(-2 + Bt)e^{-2t}]$

$v_o(0) = 2 = B + 4$ or $B = -2$

Thus, $i(t) =$ $\underline{\mathbf{[(-2 - 2t)e^{-2t}]\ A}}$

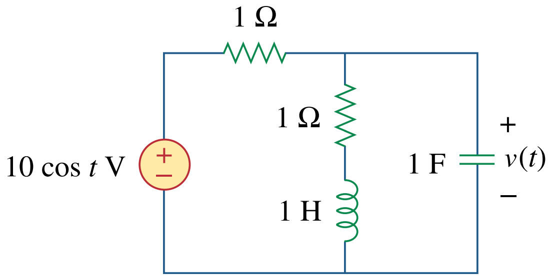
And $v(t) =$ $\underline{\mathbf{[(2 + 4t)e^{-2t}]\ V}}$

Q5. Problem 9.37



$$Y = \frac{1}{4} + \frac{1}{j8} + \frac{1}{-j10} = \underline{0.25 - j0.025\ \text{S}} = \underline{\mathbf{250–j25\ mS}}$$

Q6. Problem 9.41



$\omega = 1$,

$$1\,\text{H} \longrightarrow \text{j}\omega\text{L} = \text{j}(1)(1) = \text{j}$$

$$1\,\text{F} \longrightarrow \frac{1}{\text{j}\omega\text{C}} = \frac{1}{\text{j}(1)(1)} = \text{-j}$$

$$\mathbf{Z} = 1 + (1+\text{j}) \parallel (\text{-j}) = 1 + \frac{\text{-j}+1}{1} = 2 - \text{j}$$

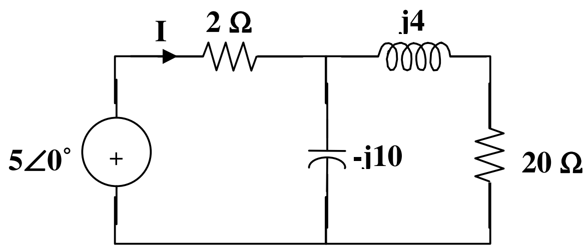
$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10}{2-\text{j}}, \qquad \mathbf{I}_c = (1+\text{j})\,\mathbf{I}$$

$$\mathbf{V} = (\text{-j})(1+\text{j})\,\mathbf{I} = (1-\text{j})\,\mathbf{I} = \frac{(1-\text{j})(10)}{2-\text{j}} = 6.325\angle\text{-}18.43°$$

Thus, v(t) = **<u>6.325 cos(t – 18.43°) V</u>**

Q6. Problem 9.47

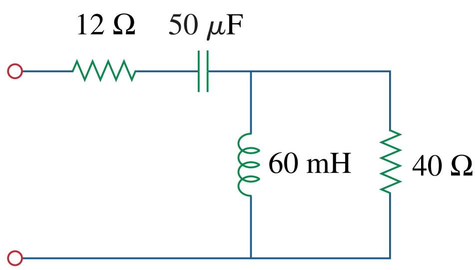
First, we convert the circuit into the frequency domain.



$$I_x = \frac{5}{2 + \frac{-j10(20+j4)}{-j10+20+j4}} = \frac{5}{2+4.588-j8.626} = \frac{5}{10.854\angle -52.63°} = 0.4607\angle 52.63°$$

$i_s(t)$ = **<u>460.7cos(2000t +52.63°) mA</u>**

Q6. Problem 9.56



$$50\mu F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j377x50x10^{-6}} = -j53.05$$

$$60mH \quad \longrightarrow \quad j\omega L = j377x60x10^{-3} = j22.62$$

$$Z_{in} = 12 - j53.05 + j22.62 // 40 = \underline{21.692 - j35.91\ \Omega}$$