Spring, 2007

EECS70A / CSE 70A Network Analysis I
Prof. Peter Burke

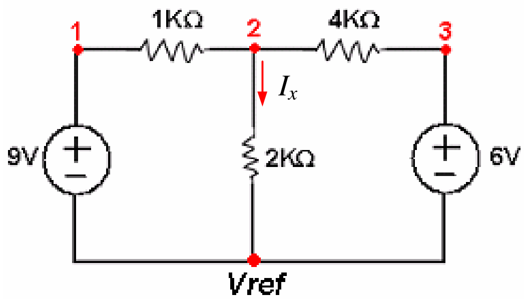
Homework # 2 solution

Q1. Considering that electric potential is defined as line integral of electric field, Kirchhoff's voltage law can be expressed equivalently with equation:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0,$$

Which states that line integral of electric field around closed loop C is zero.

Q2. Problem 3.1: (Using Nodal Analysis)



Voltage at node 1, $V_1 - V_{ref} = 9V$

Voltage at node 3, $V_3 - V_{ref} = 6V$

Apply KCL at node 2:

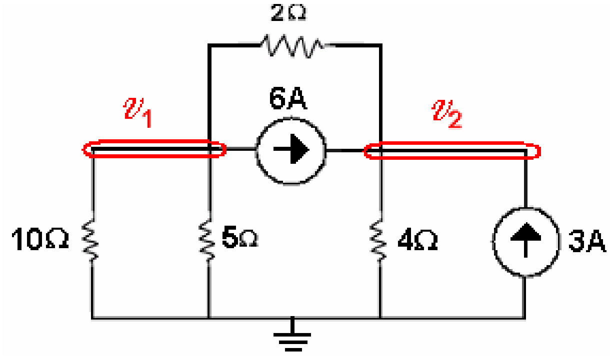
$(V_2 - V_1) / 1K\Omega + (V_2 - V_{ref}) / 2K\Omega + (V_2 - V_3) / 4K\Omega = 0$

$(V_2 - 9 - V_{ref}) / 1K\Omega + V_2 / 2K\Omega + (V_2 - 6 - V_{ref}) / 4K\Omega = 0$

$\therefore V_2 - V_{ref} = 6$ V

$\therefore I_x = (V_2 - V_{ref}) / 2K\Omega = 3 \times 10^{-3}$ A = 3 mA

Q3. Problem 3.2: (Using Nodal Analysis)



Apply KVL to node 1:

$V_1/10 + V_2/5 + (V_1 - V_2)/2 + 6 = 0$

$2V_1 - V_2 = -20$ ---- (1)

Apply KVL to node 2:

$(V_1 - V_2) / 2 + 6 + 3 = V_2 / 4$

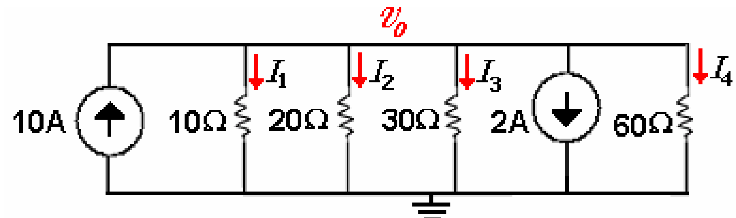
$2V_1 - 3V_2 = -36$ ---- (2)

Eqn 1 - eqn 2,

$2V_2 = 16 \Rightarrow \therefore V_2 = 8V$

$\Rightarrow \therefore V_1 = -6V$

Q4. Problem 3.3: (Using Nodal Analysis)



Apply KVL to the non-reference node:

$Vo / 10 + Vo / 20 + Vo / 30 + Vo / 60 + 2 = 10$

$6Vo + 3Vo + 2Vo + Vo = 480$

$\therefore Vo = 40 V$

Apply Ohm's law:

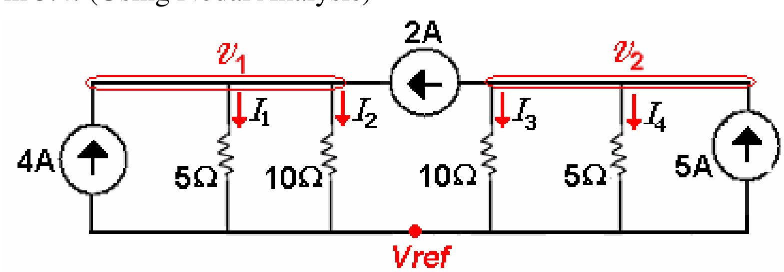
$I_1 = Vo / 10 = 4 A$

$I_2 = Vo / 20 = 2 A$

$I_3 = Vo/ 40 = 1.33 A$

$I_4 = Vo / 60 = 0.67 A$

Q5. Problem 3.4: (Using Nodal Analysis)



Apply KVL to node 1:

$(V_1 - V_{ref}) / 5 + (V_1 - V_{ref}) / 10 = 4 + 2$

$\therefore (V_1 - V_{ref}) = 20 V$

Apply KVL to node 2:

$(V_2 - V_{ref}) / 5 + (V_2 - V_{ref}) / 10 + 2 = 5$

$\therefore (V_2 - V_{ref}) = 10 V$

Apply Ohm's law:

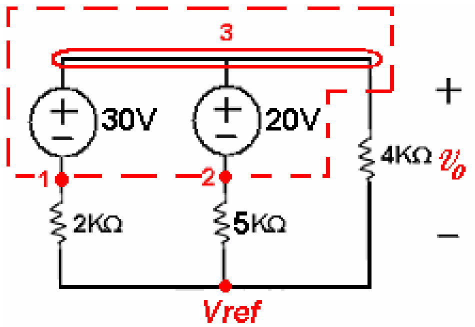
$I_1 = (V_1 - V_{ref}) / 5 = 4 A$

$I_2 = (V_1 - V_{ref}) / 10 = 2 A$

$I_3 = (V_2 - V_{ref}) / 10 = 1 A$

$I_4 = (V_2 - V_{ref}) / 5 = 2 A$

Q6. Problem 3.5: (Using Nodal Analysis)



Apply KCL at supernode:

$(V_1 - V_{ref}) / 2\text{K}\Omega + (V_2 - V_{ref}) / 5\text{K}\Omega + (V_3 - V_{ref}) / 4\text{K}\Omega = 0$

$10V_1 + 4V_2 + 5V_3 - 19V_{ref} = 0$ ---- (1)

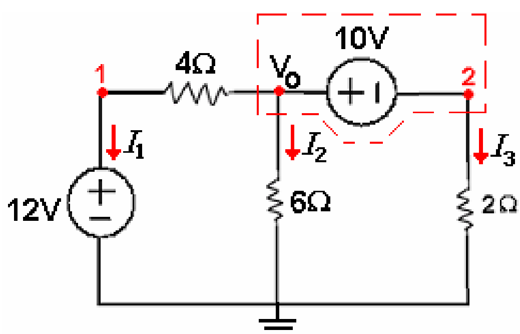
$V_3 - V_1 = 30$ ---- (2)

$V_3 - V_2 = 20$ ---- (3)

Substitute eqn 2 and 3 into eqn 1, we'll get

$\therefore V_3 - V_{ref} = V_o = 20\text{V}$

Q7.Problem 3.6: (Using Nodal Analysis)



$V_1 = 12\text{V}$

Apply KCL at the supernode:

$(Vo - 12) / 4 + Vo / 6 + V_2 / 2 = 0$

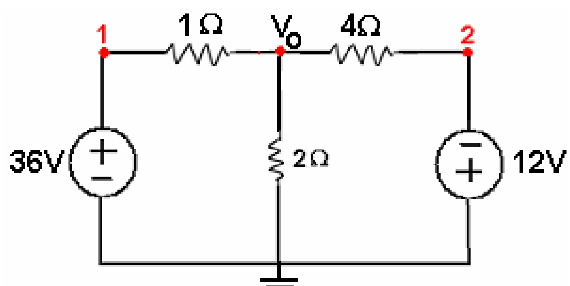
$3Vo - 36 + 2Vo + 6V_2 = 0$

$5Vo + 6V_2 = 36$ ---- (1)

$Vo - V_2 = 10$ ---- (2)

6 x (2) + (1) : $11Vo = 96$ ⇨ $\therefore Vo = 8.73$ V

Q8. Problem 3.11: (Using Nodal Analysis)



$V_1 = 36\text{V}$

$V_2 = -12\text{V}$

Apply KCL at the supernode:

$(Vo-36)/1+(Vo+12)/4 + Vo/2=0$

$4Vo - 144 + Vo + 12 + 2Vo = 0$

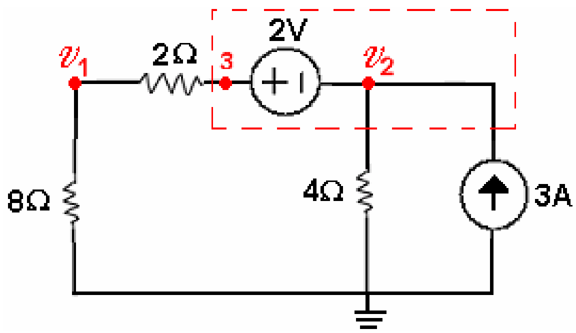
$7Vo = 132$ ⇨ $\therefore Vo = 18.86\text{V}$

Power dissipated in 1Ω - resistor: $P = (Vo - 36)^2 / R = (18.86 - 36)^2 / 1 = 293.9$ W

Power dissipated in 2Ω - resistor: $P = Vo^2 / R = 18.86^2 / 2 = 177.8$ W

Power dissipated in 4Ω - resistor: $P = (Vo + 12)^2 / R = (18.86 + 12)^2 / 4 = 238$ W

Q9. Problem 3.13: (Using Nodal Analysis)



Apply KCL at supernode:

$(V_3 - V_1) / 2 + V_2 / 4 = 3$

$V_3 - V_2 = 2$

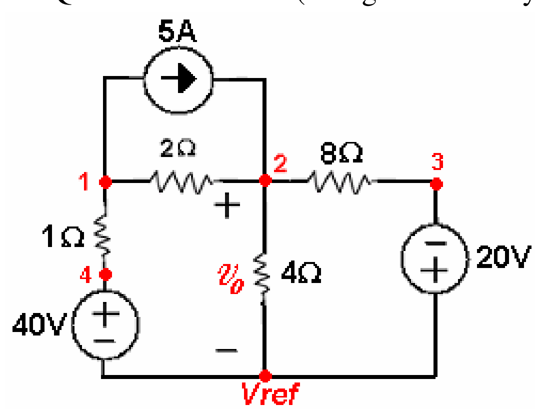
Applied KCL at node 1:

$(V_1 - V_3) / 2 + V_1 / 8 = 0$

Solve the three equations simultaneously, we'll get:

$V_1 = 8$ V and $V_2 = 8$ V

Q10. Problem 3.14: (Using Nodal Analysis)



$V_4 - V_{ref} = 40$V

$V_{ref} - V_3 = 20$V

Apply KCL at node 1:

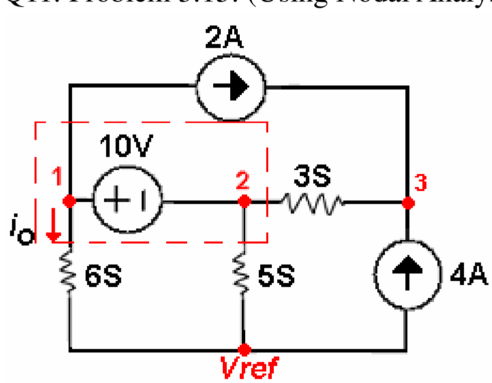
$(V_1 - V_4) / 1 + (V_1 - V_2) / 2 + 5 = 0$

Apply KCL at node 2:

$(V_2 - V_1)/2 + (V_2 - V_{ref})/4 + (V_2 - V_3)/8 = 5$

Solve simultaneously, ⟹ $\therefore V_2 - V_{ref} = V_o = 18.24$

Q11. Problem 3.15: (Using Nodal Analysis)



Apply KCL at node 3:

$3(V_3 - V_2) = 4 + 2 = 6$

$V_1 - V_2 = 10$

Apply KCL at supernode:

$6(V_1 - V_{ref}) + 5(V_2 - V_{ref}) + 3(V_2 - V_3) + 2 = 0$

Solve the equations simultaneously, we'll get:

$\therefore V_1 - V_{ref} = 4.91$ V

$V_2 - V_{ref} = -5.09$ V

$V_3 - V_{ref} = -3.09$ V

$\therefore i_o = 5 \,.\, (V_2 - V_{ref}) = 29.45$ A

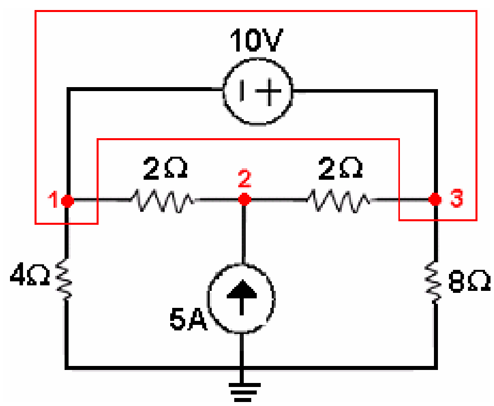
Power dissipated in each resistor:

$\therefore P_{6S\text{-}resistor} = 6.(V_1 - V_{ref})^2 = 144.6$ W

$P_{5S\text{-}resistor} = 5.(V_2 - V_{ref})^2 = 129.6$ W

$P_{3S\text{-}resistor} = 3.(V_2 - V_3)^2 = 12$ W

Q12. Problem 3.18: (Using Nodal Analysis)



Apply KCL at supernode:

$(V_3 - V_2)/2 + (V_1 - V_2)/2 + V_1/4 + V_3/8 = 0$

$(V_3 - V_1) = 10$ V

Apply KCL at node 2:

$(V_2 - V_3) / 2 + (V_2 - V_1) / 2 = 5$

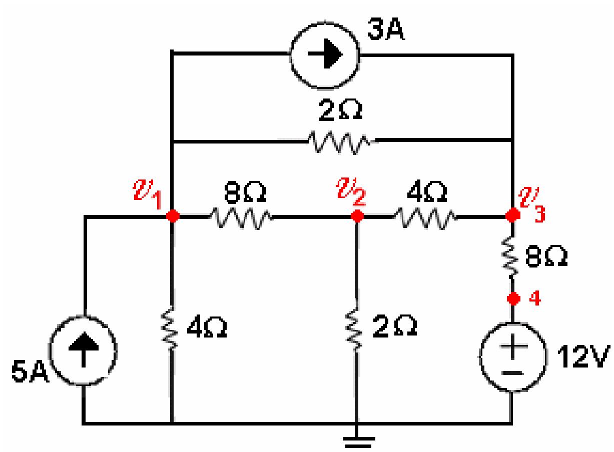
Solve simultaneously, we'll get:

$V_1 = 10$V

$V_2 = 20$V

$V_3 = 20$V

Q13. Problem 3.19: (Using Nodal Analysis)



Apply KCL at node 1:

$V_1 / 4 + (V_1 - V_2) / 8 + (V_1 - V_3) / 2 + 3 = 5$

Apply KCL at node 2:

$(V_2 - V_1) / 8 + (V_2 - V_3) / 4 + V_2 / 2 = 0$

Apply KCL at node 3:

$(V_3 - V_1) / 2 + (V_3 - V_2) / 4 + (V_3 - V_4) / 8 = 3$

Voltage at node 4:

$V_4 = 12$ V

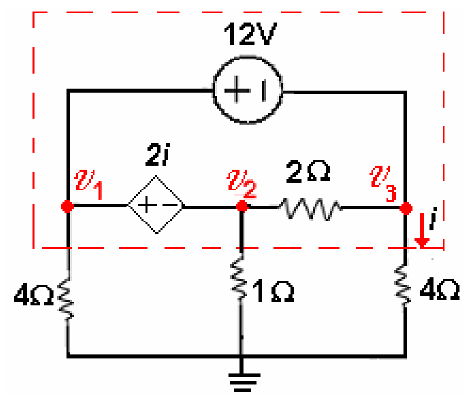
Solve the equations simultaneously, we'll get:

$\therefore V_1 = 10$ V

$V_2 = 4.933$ V

$V_3 = 12.267$ V

Q14. Problem 3.20:



Apply KCL at supernode:

$V_1 / 4 + V_2 / 1 + V_3 / 4 = 0$

$V_1 - V_3 = 12$

$V_1 - V_2 = 2i$

$V_3 / 4 = i$

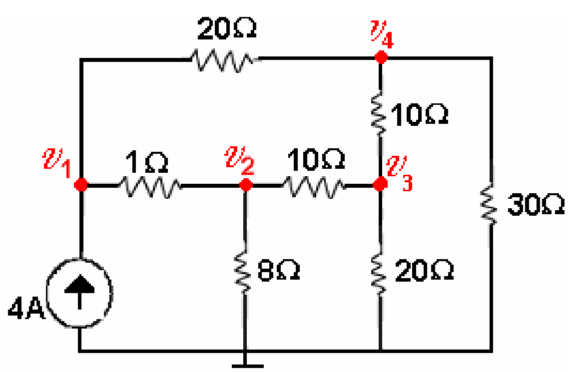
Solve simultaneously, we'll get:

$\therefore V_1 = -3$ V

$V_2 = 4.5$ V

$V_3 = -15$V

Q15. Problem 3.25: (Using Mesh Analysis)



Apply KCL at node 1:

$(V_1 - V_2) / 1 + (V_1 - V_4) / 20 = 4$

Apply KCL at node 2:

$(V_2 - V_1) / 1 + (V_2 - V_3) / 10 + V_2 / 8 = 0$

Apply KCL at node 31:

$(V_3 - V_2) / 10 + (V_3 - V_4) / 10 + V_3 / 20 = 0$

Apply KCL at node 4:

$(V_4 - V_1) / 20 + (V_4 - V_3) / 10 + V_4 / 30 = 0$

Solve equations simultaneously, we'll get:

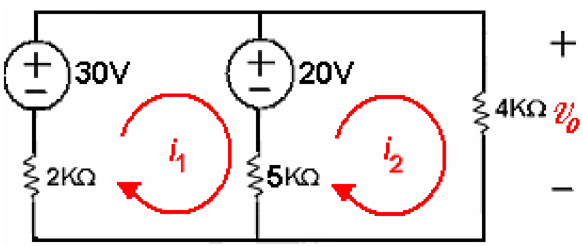
$\therefore V_1 = 25.52$ V

$V_2 = 22.05$ V

$V_3 = 14.842$ V

$V_4 = 15.055$ V

Q16. Problem 3.35:



Apply KVL to mesh 1:

$-30+20+(i_1-i_2).(5k)+i_1.(2k)=0$

Apply KVL to mesh 2:

$-20 + (i_2 - i_1).(5k) + i_2.(4k) = 0$

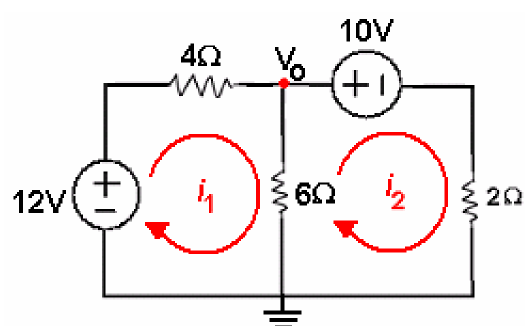
Solve equations simultaneously, we'll get:

$\therefore i_1 = 5$ mA

$i_2 = 5$ mA

$\therefore V_o = 4K\Omega \text{ x } i_2 = 20$ V

Q17. Problem 3.36: (Using Mesh Analysis)



Apply KVL to mesh 1:

$-12 + (i_1 - i_2).6 + i_1.4 = 0$

Apply KVL to mesh 2:

$10 + (i_2 - i_1).6 + i_2.2 = 0$

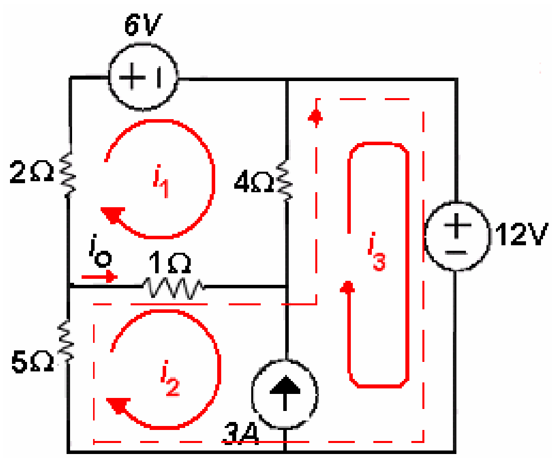
Solve equations simultaneously, we'll get:

$\therefore i_1 = 0.818$ A

$i_2 = -0.636$ A

$\therefore V_o = 6.(i_1 - i_2) = 8.73$ V

Q18. Problem 3.44: (Using Mesh Analysis)



Apply KVL to supermesh:

$12 + i_2.5 + (i_2 - i_1).1 + (i_3 - i_1).4 = 0$

Apply KVL to mesh 1:

$6 + (i_1 - i_3).4 + (i_1 - i_2).1 + i_1.2 = 0$

$i_3 - i_2 = 3$

Solve simultaneously, we'll get:

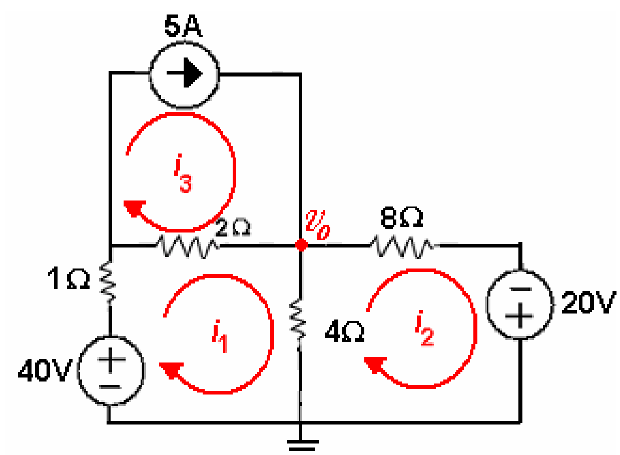
$\therefore i_1 = -1.33$ A

$i_2 = -3.07$ A

$i_3 = -0.07$A

$\therefore i_o = i_2 - i_1 = -1.73$ A

Q19. Problem 3.51: (Using Mesh Analysis)



Apply KVL to mesh 1:

$-40 + i_1.1 + (i_1 - i_3).2 + (i_1 - i_2).4 = 0$

Apply KVL to mesh 2:

$-20 + (i_2 - i_1).4 + i_2.8 = 0$

$i_3 = 5$A

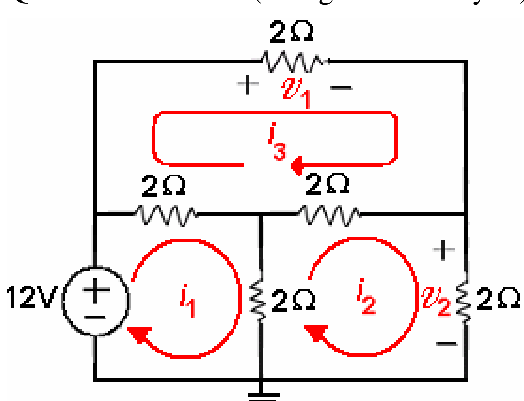
Solve simultaneously, we'll get:

$\therefore i_1 = 10$ A

$i_2 = 5$ A

$\therefore V_o = (i_1 - i_2).4 = 20$V

Q20. Problem 3.56: (Using Mesh Analysis)



Apply KVL to mesh 1:

$-12 + (i_1 - i_3).2 + (i_1 - i_2).2 = 0$

Apply KVL to mesh 2:

$(i_2 - i_3).2 + (i_2 - i_1).2 + i_2.2 = 0$

Apply KVL to mesh 3:

$(i_3 - i_2).2 + (i_3 - i_1).2 + i_3.2 = 0$

Solve simultaneously, we'll get:

$\therefore i_1 = 6$ A

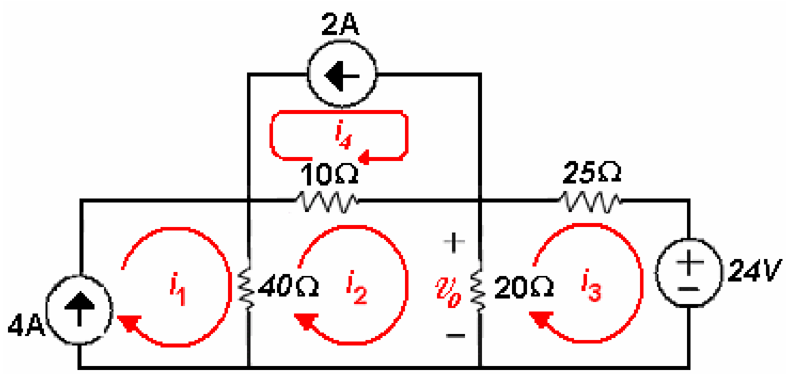
$i_2 = 3$ A

$i_3 = 3$ A

Therefore, $V_1 = i_3.2 = 6$ V

$V_2 = i_2.2 = 6$ V

Q21. Problem 3.68: (Using Mesh Analysis)



$i_1 = 4$ A

$i_4 = -2$ A

Apply KVL to mesh 2:

$(i_2 - i_3).20 + (i_2 - i_1).40 + (i_2 - i_4).10 = 0$

Apply KVL to mesh 3:

$(i_3 - i_2).20 + i_3 .25 + 24 = 0$

Solve simultaneously, we'll get:

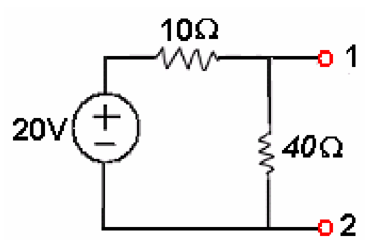
$\therefore i_2 = 2.12$ A

$i_3 = 0.41$A

Therefore, $V_o = (i_2 - i_3).20 = 34.2$ V

Q22. Problem 4.33: (Using Thevenin's Theorem)

(a)

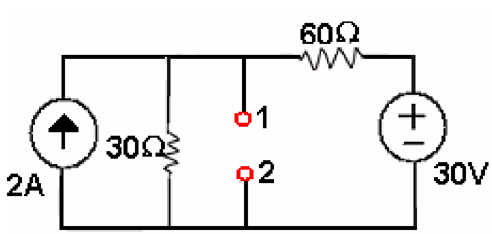


Voltage divider:

$V_{th} = (40\Omega / (10\Omega + 40\Omega)) \times 20V = 16V$

To find R_{th}, replacing the voltage source with a short circuit: $R_{th} = 40\Omega \parallel 10\Omega = 8\Omega$

(b)



Apply KCL at terminal 1:

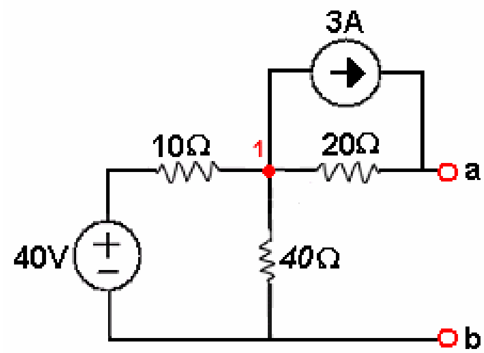
$V_{th} / 30\Omega + (V_{th} - 30) / 60 = 2$

$\therefore V_{th} = 50V$

To find R_{th}, replacing the voltage source with a short circuit and a current source with a open circuit:

$\therefore R_{th} = 30\Omega \parallel 60\Omega = 20\Omega$

Q23. Problem 4.34: (Using Thevenin's Theorem)



To find V_{th}, apply KCL at node 1:

$(V_1 – 40) / 10 + V_1 / 40 + (V_1 – V_{th}) / 20 + 3 = 0$

$7V_2 – 2V_{th} = 40$

Apply KCL at terminal a:

$(V_{th} – V_1) / 20 = 3$

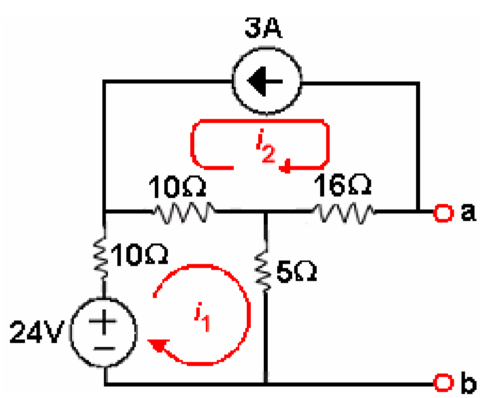
Solve equations simultaneously, we'll get:

$\therefore V_{th} = 32V$

To find R_{th}, replacing the voltage source with a short circuit and a current source with a open circuit:

$\therefore R_{th} = 20\Omega + 40\Omega \parallel 10\Omega = 28\Omega$

Q24. Problem 4.39: (Using Thevenin's Theorem)



Apply KVL to mesh1:

$-24 + i_1.10 + i_1.5 + (i_1 – i_2).10 = 0$

$i_2 = -3$ A

Solve simultaneously, we'll get:

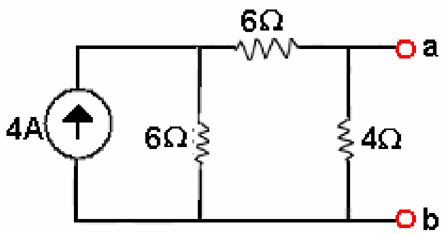
$\therefore i_1 = -0.24$ A

$\therefore V_{th} = i_2.16 + i_1.5 = - 49.2$ V

To find R_{th}, replacing the voltage source with a short circuit and a current source with an open circuit:

$\therefore R_{th} = 16\Omega + (5\Omega \parallel (10\Omega + 10\Omega)) = 20\Omega$

Q25. Problem 4.45: (Using Norton's Theorem)



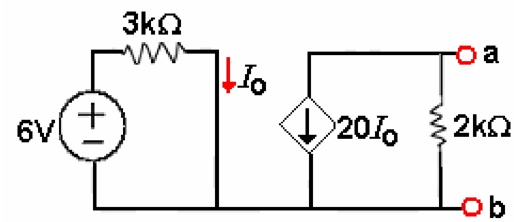
To find I_N, short terminals a and b:

$\therefore I_N = 2A$

To find R_N, replacing the current source with an open circuit:

$\therefore R_N = 4\Omega \parallel (6\Omega + 6\Omega) = 3\Omega$

Q26. Problem 4.52: (Using Thevenin's Theorem)



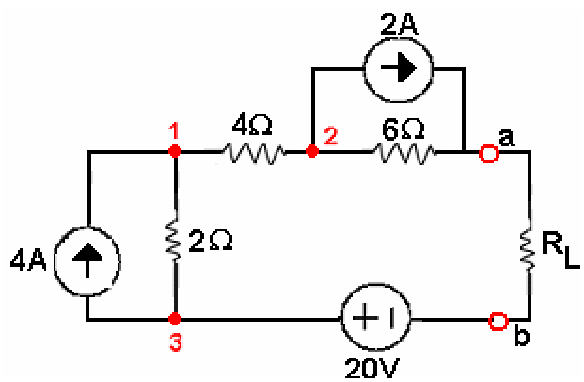
$I_o = 6V / 3k\Omega = 2mA$

$V_{th} = -20I_o \times (2k\Omega) = -80V$

To find R_{TH}, replacing the voltage source with a short circuit:

$\therefore R_{th} = 2k\Omega$

Q27. Problem 4.72: (Using Thevenin's Theorem)



(a) To find V_{th}:

Apply KCL at node a:

$I_{6\Omega,\text{ from node a to node 2}} = 2A$

Apply KCL at node 2:

$I_{4\Omega} = 0A$

Apply KCL at node 1:

$I_{2\Omega,\text{ from node 1 to node 3}} = 4A$

$\therefore V_{th} = 12 + 8 + 20 = 40V$

To find R_{th}, replacing the voltage source with a short circuit and a current source with an open circuit:

$\therefore R_{th} = 2 + 4 + 6 = 12\Omega$

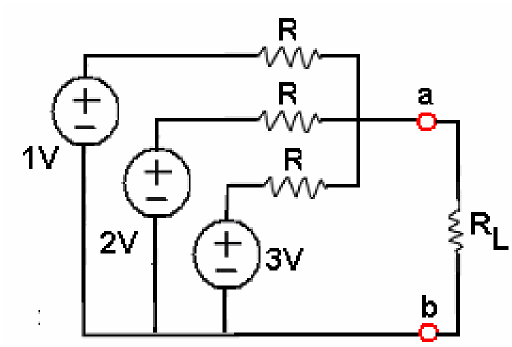
(b) When $R_L = 8\Omega$:

Current in $R_L = 40V / (12+8) = 2$ A

(c) To have maximum power deliverable, $R_L = R_{th} = 12\Omega$

(d) $P_{max} = V_{th}^2 / 4R_{th} = (40)^2 / (4 \times 12) = 33.3$ W

Q28. Problem 4.75



To find R_{th}, replacing the voltage sources with short circuits:

$\therefore R_{th} = R \parallel R \parallel R = R / 3$

To have maximum power, $R_L = R_{th} = R/3$

Apply KCL at node a:

$(V_{th} - 1)/R + (V_{th} - 2)/R + (V_{th} - 3)/R = 0$

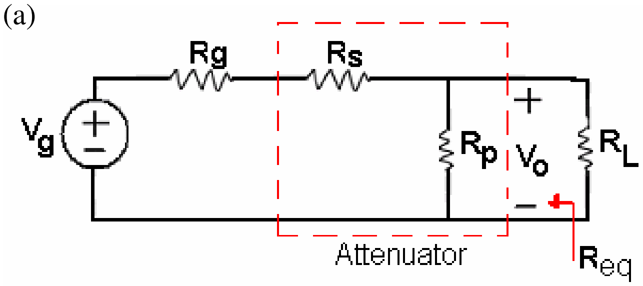
$\therefore V_{th} = 2V$

$\therefore$ To have $P_{max} = 3$ mW, we'll set $P_{max} = V_{th}^2 / 4R_{th} = (2)^2 / (4 \times R/3) = (3/R) = 3$ mW

$\therefore R = 1k\Omega$

Q29. Problem 4.94:

(a)



Given: $V_o / V_g = 0.125$

$R_{eq} = R_{th} = R_g = 100\ \Omega$

To find R_{th}, replacing the voltage source with a short circuit:

$R_{th} = R_p \parallel (R_s + R_g) = 100$

$V_o = R_p \,.\, V_g / (R_s + R_p + R_g)$

$V_o / V_g = R_p / (R_s + R_p + R_g) = 0.125$

Solve the equations simultaneously, we'll get:

$\therefore R_p = 114.3\ \Omega$

$R_s = 700\ \Omega$

(b) Given $R_g = 50\ \Omega$ and $V_g = 12V$

$I = V_{th} / (R_{th} + R_L) = V_o / (R_{th} + R_L) = 0.125V_g / (R_{th} + R_L) = 0.01$ A