Spring 2008

EECS70A / CSE 70A Network Analysis I
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Final solution

Problem 1:

A) A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

B) A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the sort-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the terminals when the independent sources are turned off.

Grading criteria: 3 pts for equivalent circuit with a resistor and a source
5 pts for mentioning R_{Th} and V_{Th} in series
5 pts for mentioning R_N and I_N in parallel

Problem 2:

Complete response = transient response + steady-state response

A complete response consists of a transient response (temporary response) and a steady-state response (permanent response). The transient response is the circuit's temporary response that will die out with time. And the steady-state response is the behavior of the circuit a long time after an external excitation is applied.

Grading criteria: 5 pts for mentioning transient and steady-state responses for the complete response
5 pts for temporary responses for the transient response

Problem 3:

$$s^2 + 4s + 8 = 0 \text{ leads to } s = \frac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$$

$$v(t) \quad = \quad V_s + (A_1\cos 2t + A_2\sin 2t)e^{-2t}$$

$$8V_s = 24 \text{ means that } V_s = 3$$

$$v(0) = 0 = 3 + A_1 \text{ leads to } A_1 = -3$$

$$dv/dt = -2(A_1\cos 2t + A_2\sin 2t)e^{-2t} + (-2A_1\sin 2t + 2A_2\cos 2t)e^{-2t}$$

$$0 = dv(0)/dt = -2A_1 + 2A_2 \text{ or } A_2 = A_1 = -3$$

$v(t) =$ **$[3 - 3(\cos 2t + \sin 2t)e^{-2t}]$ volts**

Grading criteria: 5 pts for correct characteristic eq. with correct roots
5 pts for choosing a correct type of solution
15 pts showing correct 2nd order circuit process without including Vs

Problem 4:

For $t < 0$, the equivalent circuit is as shown below.

$$v(0) = -12V \quad \text{and} \quad i(0) = 12/2 = 6A$$

For $t > 0$, we have a series RLC circuit.

$$\alpha = R/(2L) = 2/(2x0.5) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.5x1/4} = 2\sqrt{2}$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{8-4} = 2$$

i(t) = (Acos2t + Bsin2t)e^{-2t}

i(0) = 6 = A

di/dt = -2(6cos2t + Bsin2t)e^{-2t} + (-2x6sin2t + 2Bcos2t)$e^{-\alpha t}$

di(0)/dt = -12 + 2B = -(1/L)[Ri(0) + v_C(0)] = -2[12 – 12] = 0

Thus, B = 6 and i(t) = **<u>(6cos2t + 6sin2t)e^{-2t} A</u>**

Grading criteria: 5 pts for correct equivalent circuit and initial values for t<0
10 pts for correct equivalent circuit and characteristic eq. and variables for t>0
3 pts for correct solution for i(t)
5 pts for correct process to get i(t) using initial value and derivative of i(t)

Problem 5:

$$\mathbf{Z}_{in} = j\omega L \,||\, \left(R + \frac{1}{j\omega C}\right)$$

$$\mathbf{Z}_{in} = \frac{j\omega L\left(R + \frac{1}{j\omega C}\right)}{R + j\omega L + \frac{1}{j\omega C}} = \frac{\frac{L}{C} + j\omega L R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\mathbf{Z}_{in} = \frac{\left(\frac{L}{C} + j\omega L R\right)\left(R - j\left(\omega L - \frac{1}{\omega C}\right)\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

To have a resistive impedance, $\text{Im}(\mathbf{Z}_{in}) = 0$. Hence,

$$\omega L R^2 - \left(\frac{L}{C}\right)\left(\omega L - \frac{1}{\omega C}\right) = 0$$

$$\omega R^2 C = \omega L - \frac{1}{\omega C}$$

$$\omega^2 R^2 C^2 = \omega^2 LC - 1$$

$$L = \frac{\omega^2 R^2 C^2 + 1}{\omega^2 C}$$

Now we can solve for L.

$$L = R^2 C + 1/(\omega^2 C)$$

$= (200^2)(50x10^{-9}) + 1/((2\pi x50{,}000)^2(50x10^{-9})$

$= 2x10^{-3} + 0.2026x10^{-3} =$ **<u>2.203 mH</u>**.

Grading criteria: 3 pts for correct w using w=2πf

10 pts for showing correct total impedance (Z_T) or (Z_{in}) equation

10 pts for showing imaginary part of total impedance is zero

Problem 6:

$$H(s) = \frac{V_o}{V_i}$$

$$V_o = \frac{2}{10 + 2 + \frac{8}{jw}} V_i = \frac{2jw}{12jw + 8} V_i$$

$$\frac{V_o}{V_i} = \frac{2jw}{12jw + 8} = \frac{2s}{12s + 8}$$

Grading criteria: 15 pts for correct voltage divider equation

10 pts for correct current for getting V_o using Ohm's law or KVL