EECS70A / CSE 70A Network Analysis I
Prof. Peter Burke

Homework # 6 solution

Chapter 8, Solution 17.

$$i(0) = I_0 = 0,\ v(0) = V_0 = 4x15 = 60$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0 + 60) = -240$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20,\ \text{which is} > \omega_o.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.679, -37.32$$

$$i(t) = A_1e^{-2.679t} + A_2e^{-37.32t}$$

$$i(0) = 0 = A_1 + A_2,\ \frac{di(0)}{dt} = -2.679A_1 - 37.32A_2 = -240$$

$$\text{This leads to } A_1 = -6.928 = -A_2$$

$$i(t) = 6.928\left(e^{-37.32t} - e^{-2.679t}\right)$$

$$\text{Since, } v(t) = \frac{1}{C}\int_0^t i(t)dt + \text{const, and } v(0) = 60\text{V, we get}$$

$$v(t) = \underline{\mathbf{(64.65e^{-2.679t} - 4.641e^{-37.32t})\ V}}$$

We note that v(0) = 60.009V and not 60V. This is due to rounding errors since v(t) must go to zero as time goes to infinity. {In other words, the constant of integration must be zero.

Chapter 8, Solution 24

When the switch is in position A, the inductor acts like a short circuit so

$$i(0^-) = 4$$

When the switch is in position B, we have a source-free parallel RCL circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2x10x10x10^{-3}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x10x10^{-3}}} = 20$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -5 + \sqrt{25-400} = -5 + j19.365$$

$$i(t) = e^{-5t}\left(A_1 \cos 19.365t + A_2 \sin 19.365t\right)$$

$$i(0) = 4 = A_1$$

$$v = L\frac{di}{dt} \longrightarrow \frac{di(0)}{dt} = \frac{v(0)}{L} = 0$$

$$\frac{di}{dt} = e^{-5t}\left(-5A_1 \cos 19.365t - 5A_2 \sin 19.365t - 19.365A_1 \sin 19.365t + 19.365A_2 \cos 19.365t\right)$$

$$0 = \frac{di(0)}{dt} = -5A_1 + 19.365A_2 \longrightarrow A_2 = \frac{5A_1}{19.365} = 1.033$$

$$i(t) = \underline{e^{-5t}\left(4\cos 19.365t + 1.033\sin 19.365t\right)}$$

Chapter 8, Solution 34.

Before t = 0, the capacitor acts like an open circuit while the inductor behaves like a short circuit.

i(0) = 0, v(0) = 20 V

For t > 0, the LC circuit is disconnected from the voltage source.
This is a lossless, source-free, series RLC circuit.

$$\alpha = R/(2L) = 0,\ \omega_o = 1/\sqrt{LC} = 1/\sqrt{\frac{1}{16}+\frac{1}{4}} = 8,\quad s = \pm j8$$

Since α is less than ω_o, we have an underdamped response. Therefore,

$$i(t) = A_1\cos 8t + A_2\sin 8t \quad \text{where } i(0) = 0 = A_1$$

$$di(0)/dt = (1/L)v_L(0) = -(1/L)v(0) = -4x20 = -80$$

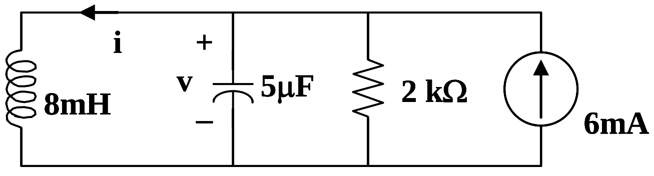
However, $di/dt = 8A_2\cos 8t$, thus, $di(0)/dt = -80 = 8A_2$ which leads to $A_2 = -10$

Now we have $\quad i(t) = $ **-10sin8t A**

Chapter 8, Solution 46.

For $t = 0-$, $u(t) = 0$, so that $v(0) = 0$ and $i(0) = 0$.

For $t > 0$, we have a parallel RLC circuit with a step input



$$\alpha = 1/(2RC) = (1)/(2x2x10^3 x5x10^{-6}) = 50$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{8x10^{-3}x5x10^{-6}} = 5{,}000$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \cong -50 \pm j5{,}000$$

Thus, $i(t) = I_s + [(A\cos 5{,}000t + B\sin 5{,}000t)e^{-50t}]$, $I_s = 6mA$

$$i(0) = 0 = 6 + A \quad \text{or} \quad A = -6mA$$

$$v(0) = 0 = L\,di(0)/dt$$

$$di/dt = [5{,}000(-A\sin 5{,}000t + B\cos 5{,}000t)e^{-50t}] + [-50(A\cos 5{,}000t + B\sin 5{,}000t)e^{-50t}]$$

$$di(0)/dt = 0 = 5{,}000B - 50A \quad \text{or} \quad B = 0.01(-6) = -0.06mA$$

Thus, $i(t) = $ **{6 – [(6cos5,000t + 0.06sin5,000t)e^{-50t}]} mA**

Chapter 8, Solution 57.

(a) Let v = capacitor voltage and i = inductor current. At t = 0-, the switch is closed and the circuit has reached steady-state.

$$v(0-) = 16V \quad \text{and} \quad i(0-) = 16/8 = 2A$$

At t = 0+, the switch is open but, v(0+) = 16 and i(0+) = 2.

We now have a source-free RLC circuit.

$$R \quad 8 + 12 = 20 \text{ ohms}, \quad L = 1H, \quad C = 4mF.$$

$$\alpha = R/(2L) = (20)/(2x1) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x(1/36)} = 6$$

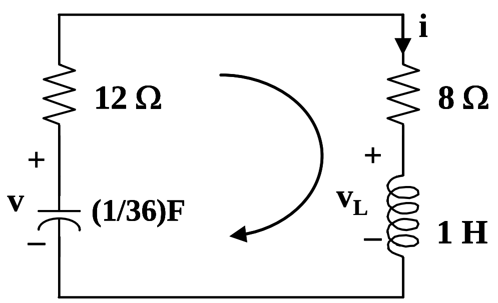
Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -18, -2$$

Thus, the characteristic equation is (s + 2)(s + 18) = 0 or **$s^2 + 20s + 36 = 0$**.

(b) $i(t) = [Ae^{-2t} + Be^{-18t}]$ and $i(0) = 2 = A + B$ (1)

To get di(0)/dt, consider the circuit below at t = 0+.



$-v(0) + 20i(0) + v_L(0) = 0$, which leads to,

$-16 + 20x2 + v_L(0) = 0$ or $v_L(0) = -24$

$Ldi(0)/dt = v_L(0)$ which gives $di(0)/dt = v_L(0)/L = -24/1 = -24$ A/s

Hence $-24 = -2A - 18B$ or $12 = A + 9B$ (2)

From (1) and (2), $B = 1.25$ and $A = 0.75$

$i(t) = [0.75e^{-2t} + 1.25e^{-18t}] = -i_x(t)$ or $i_x(t) =$ **$[-0.75e^{-2t} - 1.25e^{-18t}]$ A**

$v(t) = 8i(t) =$ **$[6e^{-2t} + 10e^{-18t}]$ A**