Chapter 8, Solution 17.

$$i(0) = I_0 = 0, \; v(0) = V_0 = 4x15 = 60$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0+60) = -240$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20, \text{ which is} > \omega_o.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.679, -37.32$$

$$i(t) = A_1 e^{-2.679t} + A_2 e^{-37.32t}$$

$$i(0) = 0 = A_1 + A_2, \; \frac{di(0)}{dt} = -2.679A_1 - 37.32A_2 = -240$$

$$\text{This leads to } A_1 = -6.928 = -A_2$$

$$i(t) = 6.928\left(e^{-37.32t} - e^{-2.679t}\right)$$

$$\text{Since, } v(t) = \frac{1}{C}\int_0^t i(t)dt + \text{const, and } v(0) = 60\text{V, we get}$$

$$v(t) = \underline{\mathbf{(64.65e^{-2.679t} - 4.641e^{-37.32t})\ V}}$$

We note that v(0) = 60.009V and not 60V. This is due to rounding errors since v(t) must go to zero as time goes to infinity. {In other words, the constant of integration must be zero.

Chapter 8, Solution 24.

When the switch is in position A, the inductor acts like a short circuit so

$$i(0^-) = 4$$

When the switch is in position B, we have a source-free parallel RCL circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2x10x10x10^{-3}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x10x10^{-3}}} = 20$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -5 + \sqrt{25 - 400} = -5 + j19.365$$

$$i(t) = e^{-5t}\left(A_1 \cos 19.365t + A_2 \sin 19.365t\right)$$

$$i(0) = 4 = A_1$$

$$v = L\frac{di}{dt} \quad \longrightarrow \quad \frac{di(0)}{dt} = \frac{v(0)}{L} = 0$$

$$\frac{di}{dt} = e^{-5t}\left(-5A_1 \cos 19.365t - 5A_2 \sin 19.365t - 19.365A_1 \sin 19.365t + 19.365A_2 \cos 19.365t\right)$$

$$0 = \frac{di(0)}{dt} = -5A_1 + 19.365A_2 \quad \longrightarrow \quad A_2 = \frac{5A_1}{19.365} = 1.033$$

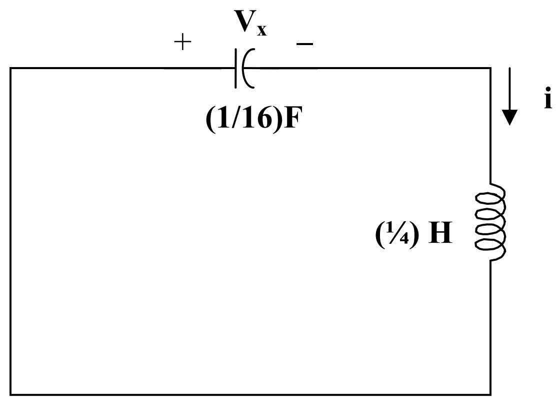
$$i(t) = \underline{e^{-5t}\left(4\cos 19.365t + 1.033\sin 19.365t\right)}$$

Chapter 8, Solution 34.

Before t = 0, the capacitor acts like an open circuit while the inductor behaves like a short circuit.

$$i(0) = 0,\ v(0) = 20\ \text{V}$$

For t > 0, the LC circuit is disconnected from the voltage source as shown below.



This is a lossless, source-free, series RLC circuit.

$$\alpha = R/(2L) = 0,\ \omega_o = 1/\sqrt{LC} = 1/\sqrt{\frac{1}{16} + \frac{1}{4}} = 8,\ s = \pm j8$$

Since α is less than ω_o, we have an underdamped response. Therefore,

$$i(t) = A_1\cos 8t + A_2\sin 8t \quad \text{where } i(0) = 0 = A_1$$

$$di(0)/dt = (1/L)v_L(0) = -(1/L)v(0) = -4x20 = -80$$

However, $di/dt = 8A_2\cos 8t$, thus, $di(0)/dt = -80 = 8A_2$ which leads to $A_2 = -10$

Now we have $\quad i(t) =$ **-10sin8t A**

Chapter 8, Solution 47.

At $t = 0-$, we obtain, $i_L(0) = 3x5/(10 + 5) = 1A$

and $v_o(0) = 0$.

For $t > 0$, the 20-ohm resistor is short-circuited and we have a parallel RLC circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.01) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.01} = 10$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -10$$

Thus, $i(t) = I_s + [(A + Bt)e^{-10t}], \quad I_s = 3$

$$i(0) = 1 = 3 + A \text{ or } A = -2$$

$$v_o = Ldi/dt = [Be^{-10t}] + [-10(A + Bt)e^{-10t}]$$

$$v_o(0) = 0 = B - 10A \text{ or } B = -20$$

Thus, $v_o(t) =$ **(200te^{-10t}) V**

Chapter 8, Solution 57.

(a) Let v = capacitor voltage and i = inductor current. At t = 0-, the switch is closed and the circuit has reached steady-state.

$$v(0-) = 16V \text{ and } i(0-) = 16/8 = 2A$$

At t = 0+, the switch is open but, v(0+) = 16 and i(0+) = 2.

We now have a source-free RLC circuit.

$$R\ 8 + 12 = 20 \text{ ohms},\ L = 1H,\ C = 4mF.$$

$$\alpha = R/(2L) = (20)/(2x1) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x(1/36)} = 6$$

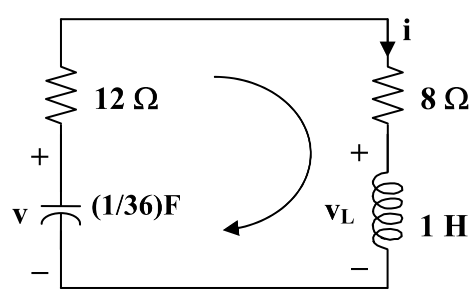
Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -18, -2$$

Thus, the characteristic equation is (s + 2)(s + 18) = 0 or <u>**s^2 + 20s +36 = 0**</u>.

(b) $$i(t) = [Ae^{-2t} + Be^{-18t}] \text{ and } i(0) = 2 = A + B \qquad (1)$$

To get di(0)/dt, consider the circuit below at t = 0+.



$$-v(0) + 20i(0) + v_L(0) = 0, \text{ which leads to,}$$

$$-16 + 20x2 + v_L(0) = 0 \text{ or } v_L(0) = -24$$

$$Ldi(0)/dt = v_L(0) \text{ which gives } di(0)/dt = v_L(0)/L = -24/1 = -24 A/s$$

$$\text{Hence } -24 = -2A - 18B \text{ or } 12 = A + 9B \qquad (2)$$

From (1) and (2), B = 1.25 and A = 0.75

$$i(t) = [0.75e^{-2t} + 1.25e^{-18t}] = -i_x(t) \text{ or } i_x(t) = \underline{\mathbf{[-0.75e^{-2t} - 1.25e^{-18t}]\ A}}$$

$$v(t) = 8i(t) = \underline{\mathbf{[6e^{-2t} + 10e^{-18t}]\ A}}$$