## Midterm 1 Solutions

1. 


3A
$\mathrm{I}_{3}$

At node 2,

$$
3+7+I_{2}=0 \quad \longrightarrow \quad I_{2}=-10 \mathrm{~A}
$$

At node 1,

$$
I_{1}+I_{2}=2 \longrightarrow I_{1}=2-I_{2}=12 \mathrm{~A}
$$

At node 4,

$$
2=I_{4}+4 \longrightarrow I_{4}=2-4=-2 \mathrm{~A}
$$

At node 3,

$$
7+I_{4}=I_{3} \quad \longrightarrow \quad I_{3}=7-2=5 \mathrm{~A}
$$

Hence,
$\underline{I_{1}=12 \mathrm{~A}, \quad I_{2}=-10 \mathrm{~A}, \quad I_{3}=5 \mathrm{~A}, \quad I_{4}=-2 \mathrm{~A}}$

Grading: " 5 " points for each node.
a. " 1 " point each sign/value of the current in each node.
2.


For mesh 1,
$-V_{4}+2+5=0 \longrightarrow \quad V_{4}=7 \mathrm{~V}$
For mesh 2,
$+4+V_{3}+V_{4}=0 \longrightarrow V_{3}=-4-7=-11 V$
For mesh 3,
$-3+V_{1}-V_{3}=0 \longrightarrow \quad V_{1}=V_{3}+3=-8 V$
For mesh 4,
$-V_{1}-V_{2}-2=0 \longrightarrow \quad V_{2}=-V_{1}-2=6 \mathrm{~V}$
Thus,
$V_{1}=-8 \mathrm{~V}, \quad V_{2}=6 \mathrm{~V}, \quad V_{3}=-11 \mathrm{~V}, \quad V_{4}=7 \mathrm{~V}$

Grading : " 5 " points for each mesh.
a. "1" point each sign/value of the voltage in each mesh.

## 3.

Starting from far right side of the network:

1. $1 \& 1$ in series resulting in 2 ohms.
2. $2 \& 1$ are in parallel resulting in $2 / 3$ ohm.
3. $2 / 3$ and 1 in series resulting in $5 / 3 \mathrm{ohm}$.
4. $5 / 3$ and 1 in parallel resulting in $5 / 8$ ohm.
5. $5 / 8$ and 1 in series resulting in Req as:

$$
\mathrm{R}_{\mathrm{eq}}=1+1 / /(1+1 / / 2)=1+1 / /(1+2 / 3)=1+1 / / 5 / 3=\underline{\mathbf{1} .625 \Omega}
$$

Grading: "2" points each part.
If you did Delta-Y:
" 2 " points for each part in the picture below.

$$
y \rightarrow \Delta
$$

1. 


2.


$$
\begin{aligned}
& 3+\frac{3}{4}=\frac{15}{4} \Omega \\
& \frac{15}{4} \| 3=\frac{5}{3} \Omega
\end{aligned}
$$

3. 



$$
\frac{5}{3} \| 1=\frac{5}{8} \Omega
$$

4. 



Grading: "5" points each part.
5. $\quad R_{\text {eq }}=1+\frac{5}{8}=\frac{13}{8} \Omega$
4.

1. 5 in parallel with 20 resulting in 4 ohms.
2. $15 \& 15 \& 15$ are all in parallel resulting in 5 ohms all together.
3. 24 and 8 in parallel resulting in 6 ohms.
4. All the above are in series with the $40 h m s$ and $50 h m s$ also resulting in Req as:

$$
R_{e q}=4+5 / / 20+\frac{1}{3} \times 15+5+24 / / 8=4+4+5+5+6=24
$$

5. $I=48 / 24=\underline{2 A}$

Grading: "2" points each part.
5.

Let $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ be in $\mathrm{k} \Omega$.

$$
\begin{align*}
& \mathrm{R}_{\mathrm{eq}}=\mathrm{R}_{1}+\mathrm{R}_{2} \| 5  \tag{1}\\
& \frac{\mathrm{~V}_{0}}{\mathrm{~V}_{\mathrm{S}}}=\frac{5 \| \mathrm{R}_{2}}{5 \| \mathrm{R}_{2}+\mathrm{R}_{1}} \tag{2}
\end{align*}
$$

From (1) and (2), $0.05=\frac{5 \| \mathrm{R}_{1}}{40} \longrightarrow 2=5 \| \mathrm{R}_{2}=\frac{5 \mathrm{R}_{2}}{5+\mathrm{R}_{2}}$ or $\mathrm{R}_{2}=3.333 \mathrm{k} \Omega$
From (1), $40=\mathrm{R}_{1}+2 \longrightarrow \mathrm{R}_{1}=38 \mathrm{k} \Omega$

Thus $\underline{\mathbf{R}}_{1}=38 \mathbf{k} \Omega, \mathbf{R}_{2}=3.333 \mathrm{k} \Omega$

Grading: "7" points for equation (1).
" 5 " points for coming up with the equation 2 .
" 3 " points for equation in the third line above ending with $2=5 \backslash \backslash R_{2}$.
" 5 " points calculating $\mathrm{R}_{2}$ and $\mathrm{R}_{1}$ finally.
If you go with KVL:
" 7 " points again for equation (1).
" 3 " points for writing down the correct KVL.
" 5 " points to end up with the equation (2) after the KVL or something like that to use the $V_{0} / V_{s}$ ratio.
" 5 " points to calculate both $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ finally.

## 6.

As it can be seen from the network, the equivalent resistance between node "a" and "b" is the combination of one resistor " R " (which connects node "a" to "b") in parallel with the rest of the network. Therefore, the result for $\mathrm{R}_{\mathrm{ab}}$ should be less than or equal to R .

Grading: "20" point for anything between " 0 " and " R ".
" 10 " points for any effort that leads to $\mathrm{R}_{\mathrm{ab}}$ greater than " R ".
No point for blank paper!

