CAPACITOR

$$i(t) = C \frac{dv(t)}{dt}$$

$$\Rightarrow \cancel{I} \Rightarrow V = I \frac{1}{j\omega C}$$

$$\frac{V}{I} = R \qquad j\omega L \qquad \frac{1}{j\omega C}$$

Impedance $Z \equiv \frac{V}{I}$

Admittance $Y \equiv \frac{1}{Z}$

Thevenin ~~Theorem~~ Thm still applies:

Source impedance

Input Impedance

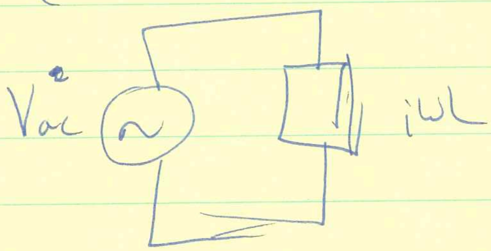
E.g. scope $1M\Omega \parallel 20pF$

Example

$v(t) = V_{ac}\cos(\omega t)$ phasor

$= Re(V_{ac}e^{i\omega t}) \Rightarrow V = V_{ac}$

$$Z = \frac{V}{I} = i\omega L$$



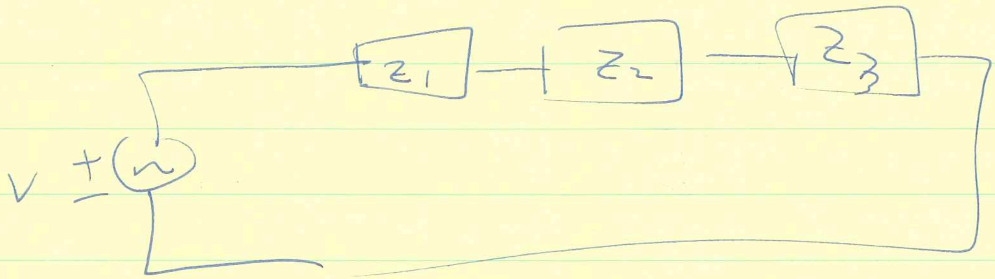
$$I(t) = Re(I e^{i\omega t})$$
$$V(t) = Re(V e^{i\omega t})$$

$$I(t) = Re\left(\frac{V}{Z} e^{i\omega t}\right)$$
$$= Re\left(\frac{V_{ac}}{j\omega L} e^{i\omega t}\right)$$
$$= \frac{V_{ac}}{\omega L} \cancel{-j}\, Re\left(-j e^{i\omega t}\right)$$
$$= \frac{-V_{ac}}{\omega L} Re \cancel{(j \sin\omega t + j\cos\omega t)}$$
$$(j(\cos\omega t + j\sin\omega t))$$
$$=$$
$$\frac{V_{ac}}{\omega L}\sin(\omega t) = I(t)$$

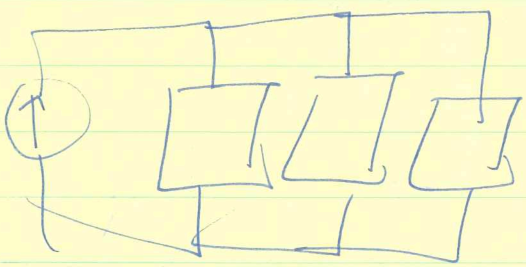
Could have done

$$v(t) = L\frac{dI(t)}{dt} \Rightarrow I(t) = \frac{1}{L}\int v(t)\,dt$$
$$= \frac{1}{L}\int V_{ac}\cos(\omega t)\,dt$$
$$= \frac{V_{ac}}{\omega L}\sin(\omega t)$$

Impedance in series



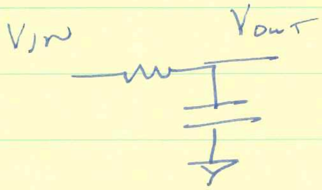
$$Z_{eq} = Z_1 + Z_2 + Z_3$$



$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

RC Filters
LR Filters
RLC Filters

RC LPF



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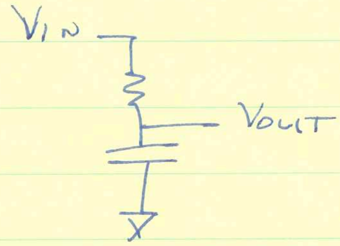

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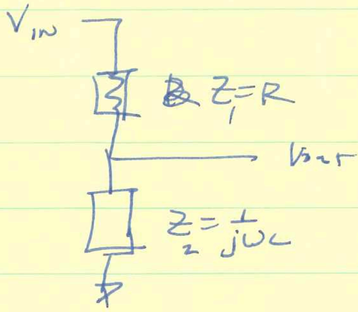


$$\frac{V_o}{V_1} = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \cdot \frac{j\omega C}{j\omega C}$$

$$= \frac{1}{1 + j\omega RC}$$

$$V_{IN}(t) = Re\left[V_{mi} e^{i\phi_i} e^{i\omega t} \right]$$

$$\Rightarrow V_{OUT}(t) = Re\left[\frac{1}{1 + j\omega RC} V_m e^{i\phi_i} e^{i\omega t} \right]$$

$$\cancel{\frac{V_{out}}{V_{IN}}} =$$

Want to express $\frac{1}{1+j\omega RC} V_m e^{i\phi_i}$

as $v e^{i\phi}$

$$\frac{1}{1+j\omega RC} \cdot \frac{1-j\omega RC}{1-j\omega RC} = \frac{1-j\omega RC}{1+(\omega RC)^2} = \frac{\cancel{1}}{1+(\omega RC)^2}$$

$$= \underbrace{\frac{1}{1+(\omega RC)^2}}_{\text{Real}} - \underbrace{\frac{j\omega RC}{1+(\omega RC)^2}}_{\text{Imag}}$$

$$\Rightarrow r = \sqrt{(\text{Real})^2 + (\text{Imag})^2}$$

$$= \sqrt{\left[\frac{1}{1+(\omega RC)^2}\right]^2 + \left[\frac{-j\omega RC}{1+(\omega RC)^2}\right]^2}$$

$$= \sqrt{\left[\frac{1}{1+(\omega RC)^2}\right]^2 + \left[\frac{(\omega RC)^2}{1+(\omega RC)^2}\right]^2}$$

$\sqrt{1+x^4}$

$= \sqrt{(1+x^2)(1-x^2)}$

$$= \sqrt{\left[\frac{1}{1+(\omega RC)^2}\right]^2 \left(1^2 + (\omega RC)^{2\,2}\right)}$$

$$= \frac{1}{1+(\omega RC)^2} \sqrt{1+(\omega RC)^4}$$

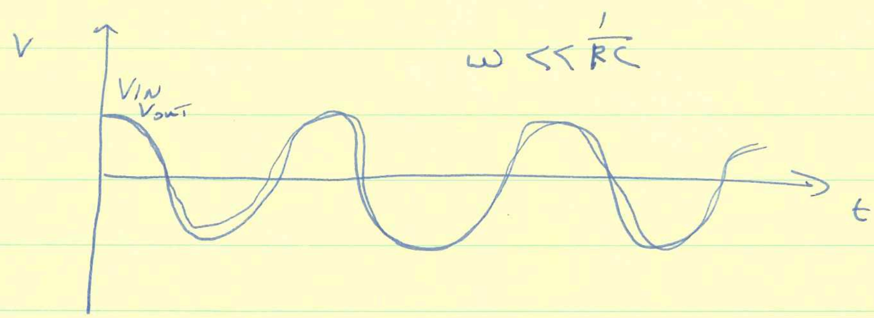
$=$

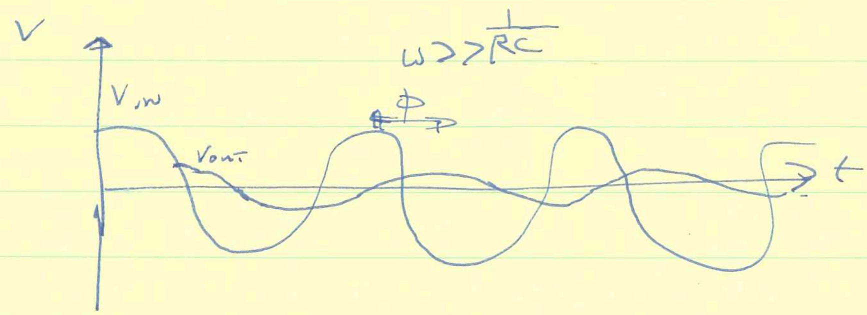
$$= \sqrt{\left(\frac{1}{1+(\omega RC)^2}\right)^2 + \left(\frac{-j\omega RC}{1+(\omega RC)^2}\right)^2}$$

$$= \frac{1}{1+(\omega RC)^2}\sqrt{1 + (-j\omega RC)^2}$$

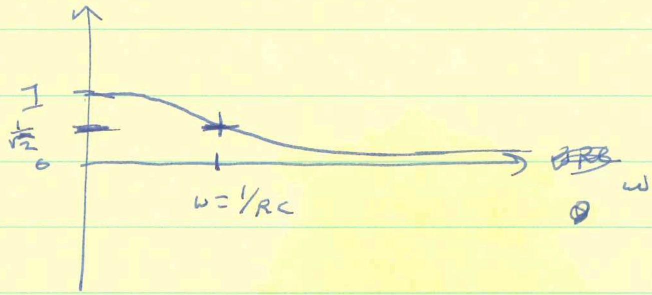
$$= \frac{1}{1+(\omega RC)^2}\sqrt{1 + (\omega RC)^2} = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\phi = -\tan^{-1}\frac{\mathrm{Im}}{\mathrm{Re}} = -\tan^{-1}(\omega RC)$$

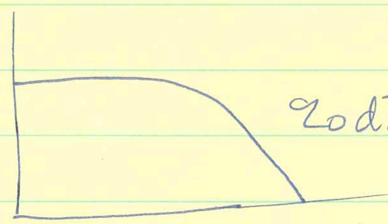




$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$



Called "Transfer Function"



20 dB/decade

Bode plots

dB scale.

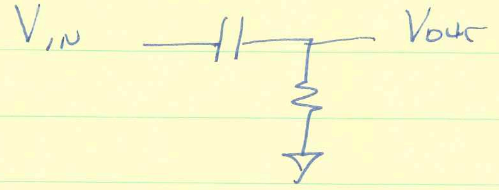
~~decade~~

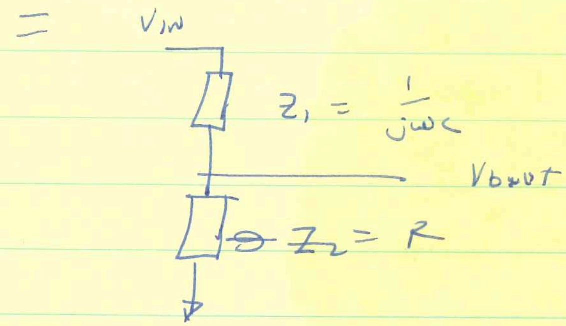
decade = 10x

$\frac{1}{(\omega\tau)^2} \downarrow 10$

$\Rightarrow$ dB $\downarrow$ 20 dB

RC HPF





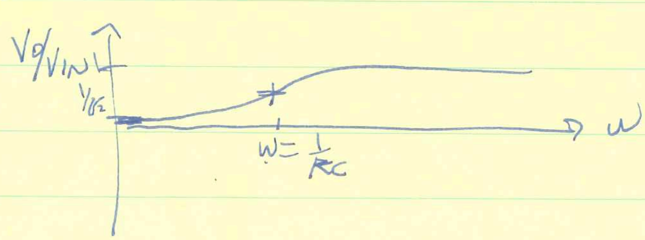
$$\frac{V_{out}}{V_{IN}} = \frac{R}{R + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = \frac{j\omega RC}{1 + j\omega RC}$$

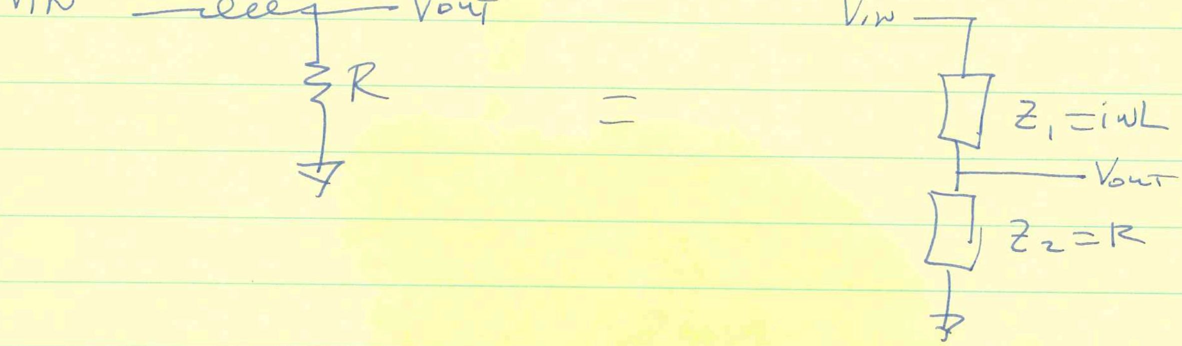
$$\cancel{\left|\frac{V_{out}}{V_{IN}}\right|} = \frac{j\omega RC\,(1 - j\omega RC)}{1 + (\omega RC)^2} = \frac{j\omega RC + (\omega RC)^2}{1 + (\omega RC)^2}$$

$$\cancel{= j\omega RC}$$

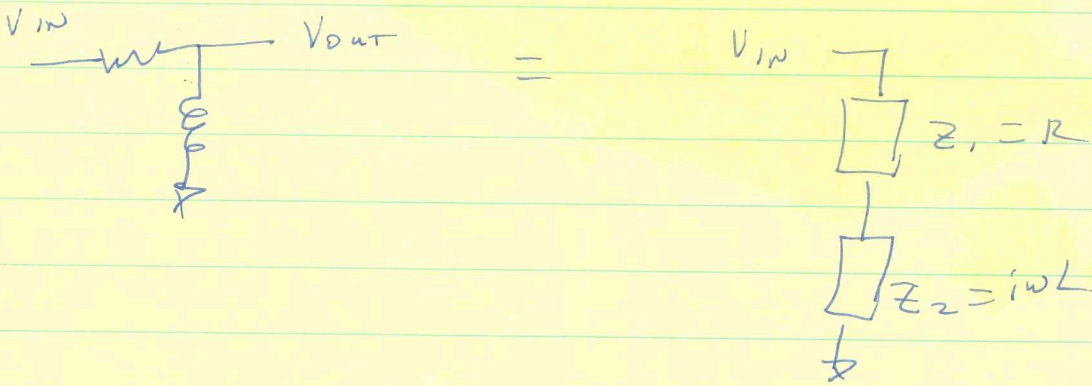
$$\left|\frac{V_o}{V_i}\right| = \frac{1}{1 + (\omega RC)^2}\sqrt{(\omega RC)^2 + (\omega RC)^4} = \frac{\sqrt{(\omega RC)^2}\sqrt{1 + (\omega RC)^2}}{1 + (\omega RC)^2}$$

$$= \frac{(\omega RC)}{\sqrt{1 + (\omega RC)^2}}$$

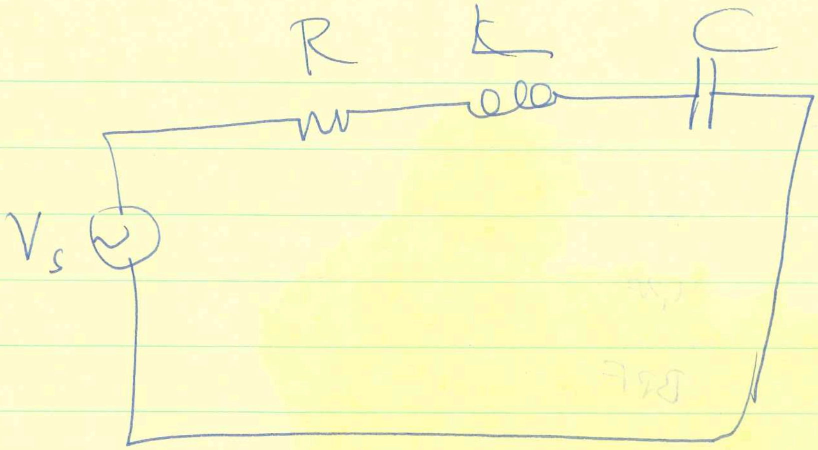




$\frac{V_{out}}{V_{IN}}$ as RC LPF but $\tau = L/R$
instead of $\tau = RC$



$\frac{V_{OUT}}{V_{IN}}$ as RC HPF but $\tau = L/R$
instead of $\tau = RC$



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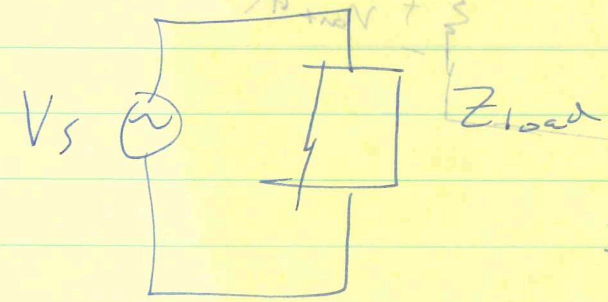


$Z_{load} = R + i\omega L + \frac{1}{j\omega C}$

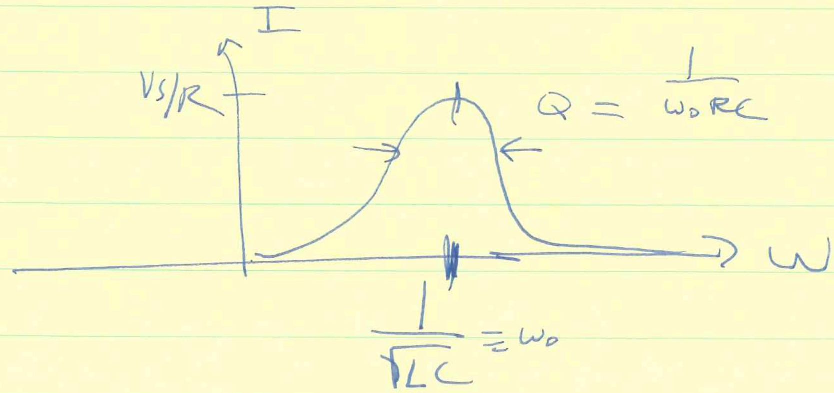
$I = \frac{V_s}{Z_{load}} = \frac{V_s}{R + i\omega L + \frac{1}{j\omega C}}$

At $\omega = \frac{1}{\sqrt{LC}} \Rightarrow Z_{load} = R$

$I = \frac{V_s}{R}$

$\omega \to \infty$ open circuit $I \to 0$

$\omega \to 0$ open circuit $I \to 0$



→

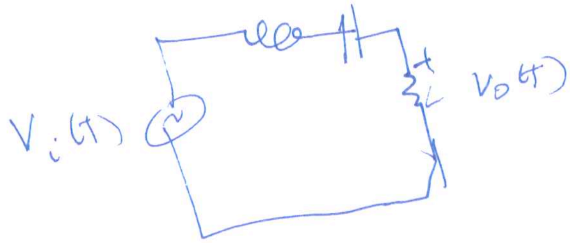
$$I = |I| = \left| \frac{V_s}{Z} \right|$$

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$|I| = \frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$|I|$ max @ $\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega = \frac{1}{\sqrt{LC}}$

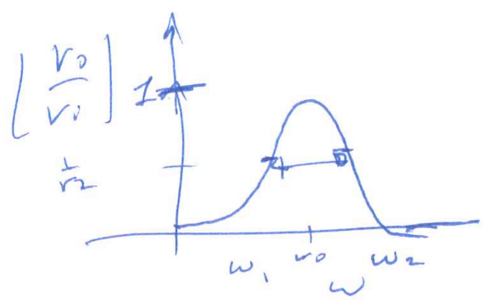
Bandpass Filter



$$\frac{V_o}{V_i} = \frac{R}{R + i\left(\omega L - \frac{1}{\omega C}\right)}$$

Show $\equiv H(\omega)$ Transfer Function

$$H(\omega) \equiv \left|\frac{V_o}{V_i}\right|$$



$$\left|\frac{R}{R + i\left(\omega_1 L + \frac{1}{\omega_1 C}\right)}\right| = \frac{R}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 2R = R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2$$

$$2R - R^2 = \omega_1^2 L^2 - 2\frac{L}{C} + \frac{1}{\omega_1^2 C^2}$$

$$\Rightarrow \omega_1^2 C^2 (2R - R^2) = \omega_1^4 L^4 - 2\frac{L}{C}\omega_1^2 + \frac{1}{C^2}$$

$$L^4 \omega_1^4 - \left[2\frac{L}{C} + C^2(2R - R^2)\right]\omega_1^2 + \frac{1}{C^2} = 0$$

$$\Rightarrow \omega_1^2 = 2\frac{L}{C}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

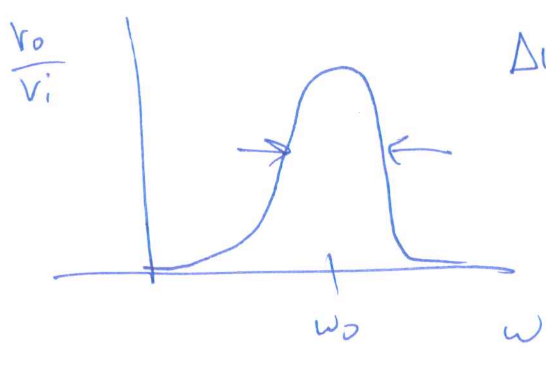
$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$B \equiv \omega_2 - \omega_1$$

$$B = \frac{R}{L} = \frac{\omega_0}{Q} = \omega_0^2 RC$$

$$Q \equiv \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

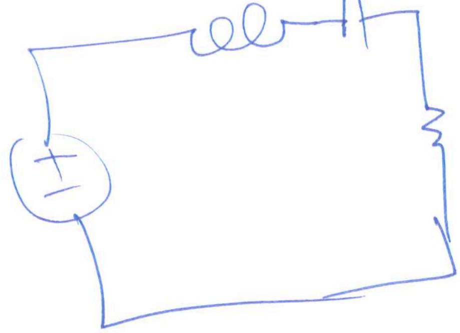
$$\frac{\Delta\omega}{\omega_0} \equiv \frac{1}{Q}$$



$$\Delta\omega = \frac{1}{Q}\omega_0$$

want large Q

$H(\omega) = \frac{V_o}{V_{in}}$ (3)



$V_o(t)$

$V_{in}(t)$
$= \cancel{V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t)}$

$= \mathrm{Re}[V_1 e^{j\omega_1 t}] + \mathrm{Re}[V_2 e^{j\omega_2 t}]$

$$V_o(t) = \mathrm{Re}[H(\omega_1) V_1 e^{j\omega_1 t}] + \mathrm{Re}[H(\omega_2) V_2 e^{j\omega_2 t}]$$

If $H(\omega_2) \approx 0 \Rightarrow V_o(t) = \mathrm{Re}[V_1 e^{j\omega_1 t}]$
$H(\omega_1) \approx 1$ $\cancel{\mathrm{Re}\ H(\omega}$

Only single sine wave come out
even though ~~two~~ two sine waveforms

FILTER

OLD FASHIONED TV

100-800MHz

TV cable

TV

~~Ch han~~
Channel 2-1000

700MHz