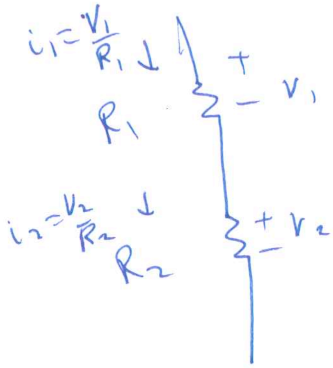
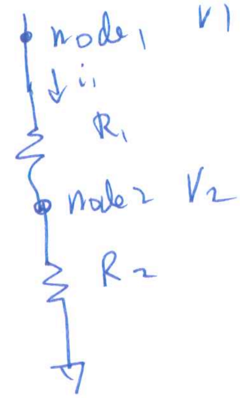
Ch3: Nodal Analysis

Ch 2 way



V is voltage drop

Ch 3 way



$$i_1 = \frac{V_1 - V_2}{R_1}$$

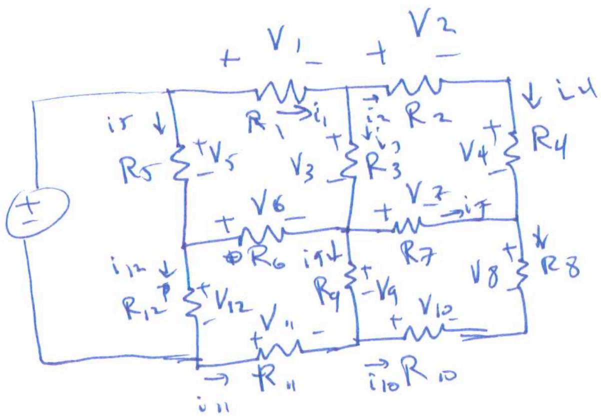
$$i_2 = \frac{V_2}{R_2}$$

V_1 is voltage of node 1 relative to node 0.

Two definitions, same symbol.

Sorry

Ch2 way



$i_1 = V_1 R_1$

$i_2 = V_2 R_2$

$i_4 = V_4 R_4$

$i_8 = V_8 R_8$

$\vdots$

Label voltages as drops accross resistors.

Typical notation:

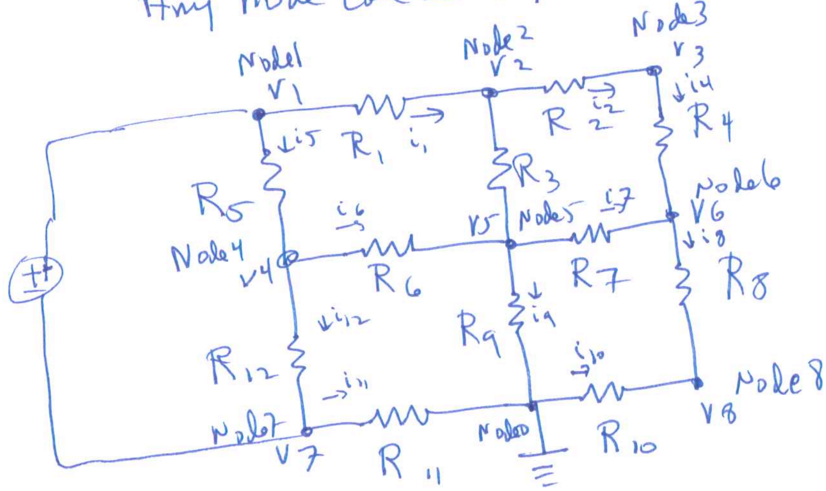
V_1 is voltage drop accross R_1.

i_1 is current through R_1

Ch 3 way (better)

Label voltages of each node relative to a "reference" node. "Node 0"

Any node can be ref node. Typically on bottom of diagram.



Typical notation: i_1 is current through R_1 (Same as Ch2.)

V_1 is voltage of node 1 with respect to node 0.

Different.

Note: Ohms law says $V = IR$

↑ Voltage drop accross resistor

E.g. $i_1 R_1 =$ voltage drop across R_1

$$= V_1 - V_2$$

$$i_2 R_2 = V_2 - V_3$$

$$i_4 R_4 = V_3 - V_6$$

$$i_8 = V_6 - V_8$$

$$i_{10} = V_0 - V_8 = -V_8$$

$$i_{11} = V_7 - V_0 = V_7$$

$$i_{12} = V_4 - V_7$$

$$i_9 = V_5 - V_0 = V_5$$

$$i_7 = V_5 - V_6$$

$$i_6 = V_4 - V_5$$

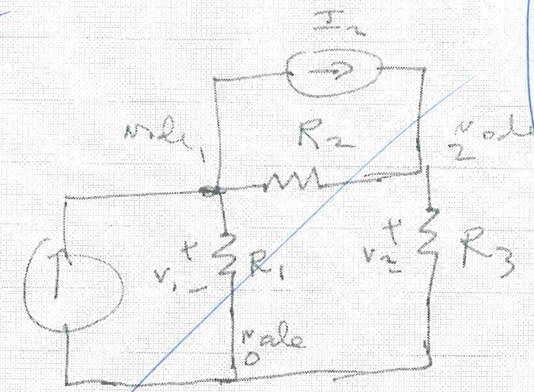
$$i_5 = V_1 - V_4$$

RCWs: absolute. Power supply 1

Nodal/Mesh analysis

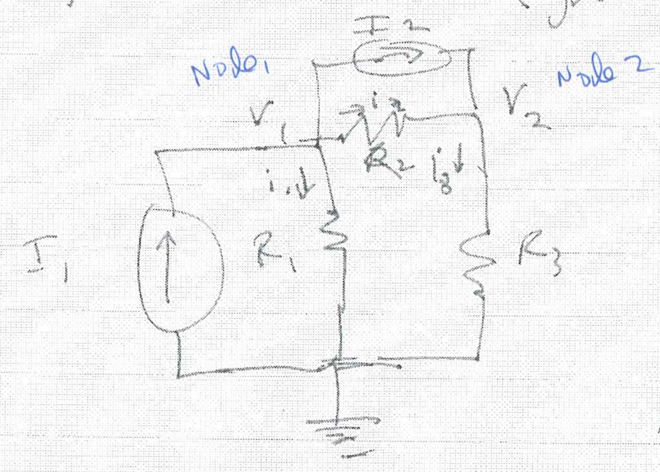
Nodal on
1) Select ref node
2) KCL + Ohm
3) Solve V_{ns}.

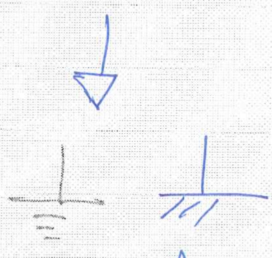
E.g.



n unknowns i_1, i_2, i_3, V_1, V_2

STEP 1 Define reference node (ground)





Chassis

Node 0: Reference

STEP 2 Apply KCL to all nodes (Not ref node.) And Ohm's Laws

$\cancel{I_1+I_2}$ $I_1 = I_2 + i_1 + i_2$

$I_2 + i_2 = i_3$

$$i_1 = \frac{V_1 - 0}{R_1}$$

$$i_2 = \frac{V_1 - V_2}{R_2}$$

$$i_3 = \frac{V_2 \cancel{-0}}{R_3}$$

5 Unknowns i_1, i_2, i_3, V_1, V_2

5 eqns.

Could do:

$(-1)\,i_1 \; (-1)\,i_2 + 0\,i_3 + 0\,v_1 + 0\,v_2 = I_2 - I_1$

$(1)\,'' + (0)\,'' + (0)\,'' + (0)\,'' + (0)\,'' = V_1/R_1$

$(0)\,'' \; (-1)\,'' \; (1)\,'' \; (0)\,'' \; (0)\,'' = I_2$

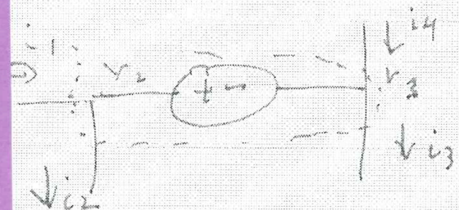
$(1)\,'' \; (1)\,'' \; (1)\,'' \; \left(-\frac{1}{R_2}\right)'' \; \left(+\frac{1}{R_2}\right)'' = 0$

$(0)\,'' \; (0)\,'' \; (1)\,'' \; (0)\,'' \; \left(-\frac{1}{R_3}\right)'' = 0$

~~fig 3~~ Solve for all 5. (Hard way)

After p.3

voltage source is present:



apply KCL, use "supernode" concept:

$i_1 + i_4 = i_2 + i_3$

Also need to apply KVL with a loop containing voltage source.

$\Rightarrow$ n equations, n unknowns.

~~Easier way Step 3~~ ~~Eaise~~ Easier way

Simplify   Express all i's in terms of V's:

$$I_1 = I_2 + i_1 + i_2$$

$$I_2 + i_2 = i_3$$

Express i's in terms of V's:

$$I_1 = I_2 + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

$$I_2 + \frac{V_1 - V_2}{R_2} = \frac{V_2}{R_3}$$

Now only 2 unknowns (# nodes) V_1 V_2

2 eqns.

Step 3 Solve for nodal voltages. Then ohm to solve for i's

Called "Nodal analysis"

Allows us to find V's everywhere.

Note: Only 2 eqns. instead of 5.

"Nodal analysis applies KCL to find unknown voltages in a given circuit."

Later

s KVL to find

Chapter 3, Problem 3.

Find the currents i_1 through i_4 and the voltage v_o in the circuit in Fig. 3.52.

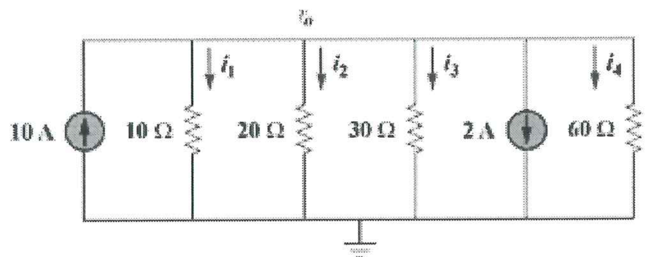


Figure 3.52

Unknowns i_1, i_2, i_3, i_4, V_o

Would need 5 eqns if wanted to solve all.

Nodal Anal:

Step 1 ✓

Step 2 KCL @ all non-ref nodes:

$$10A = i_1 + i_2 + i_3 + 2A + i_4$$

then Ohm

$V_o = i_1 \, 10\Omega$ $\quad i_1 = V_o/10$

$V_o = i_2 \, 20\Omega$ $\quad i_2 = V_o/20$

$i_3 = V_o/30$

$i_4 = V_o/60$

Sub into KCL

$$10A = \frac{V_o}{10\Omega} + \frac{V_o}{20\Omega} + \frac{V_o}{30\Omega} + 2A + \frac{V_o}{60\Omega}$$

Step 3 Solve for nodal voltages (Only 1 eqn!)

$$\Rightarrow V_o = \frac{8A}{\frac{1}{10\Omega} + \frac{1}{20\Omega} + \frac{1}{30\Omega} + \frac{1}{60\Omega}} = \frac{8A}{\frac{6+3+2+1}{60\Omega}}$$

$$= \frac{8 \times 60}{12} A\text{-}\Omega = 40V$$

Go back (Ohm) $i_1 = 4\Omega$ $\quad i_2 = 2A$ $\quad i_1 = 1.3A$ $\quad i_4 = 0.66A$

Bad news
Harder if voltage source present!

$(-1)\ i_1\ (\ldots \quad v_2 = I_2 - I_1$

$(1)\ \text{"}\ +(\ldots \quad \text{"} = V_1/R_1$

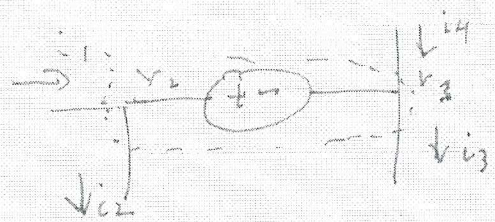
$(0)\ \text{"}\ (\ldots \quad \text{"} = I_2$

$(\ )\ \text{"}\ (\ldots)\ \text{"} = 0$

$(0)\ \text{"}\ (\ldots)\ \text{"} = 0$

Step 3 Solv…

If voltage source is present between 2 non-ref. nodes e.g.



Modify rules of nodal analysis:
When we apply KCL, use "supernode" concept:

$$i_1 + i_4 = i_2 + i_3$$

Also need to apply KVL with a loop containing voltage source.

~~⇒ n equations, n unknowns.~~

Chapter 3, Problem 15.

Apply nodal analysis to find i_o and the power dissipated in each resistor in the circuit of Fig. 3.64.

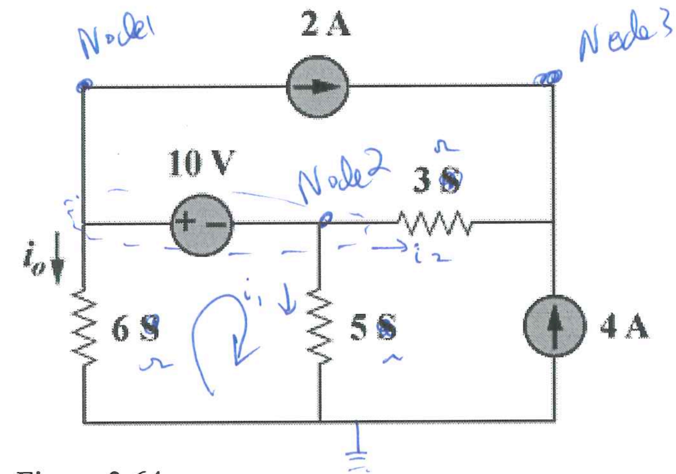


Figure 3.64

6 Unknowns

i_o i_1 i_2

V_1 V_2 V_3

Step 1 Ref node

Step 2 KCL @ all nodes *including "supernode"

KCL @ supernode IN = out

$0 = i_o + 2 + i_2 + i_1$

KCL @ Node3 IN = out

$2 + i_2 + 4 = 0$

* Need KVL w/ loop containing source (use i's not V's!)

$-i_o 6 + 10 + i_1 5 = 0$

Now Ohm $i_o = V_1/6$ $i_1 = V_2/5$ $i_2 = \frac{V_2 - V_3}{3}$

$0 = \frac{V_1}{6} + 2 + \frac{V_2 - V_3}{3} + \frac{V_2}{5}$

$2 + \frac{V_2 - V_3}{3} + 4 = 0$

$-\frac{V_1}{6} \times 6 + 10 + \frac{V_2}{5} 5 = 0$

3 eqns. 3 unknowns. ~~Solve Vn~~

Step 3 Solve Vns the ohm ins.

Fundamentals of Electric Circuits, 3/e, Charles Alexander, Matthew Sadiku

Chapter 3, Problem 2.

For the circuit in Fig. 3.51, obtain v_1 and v_2.

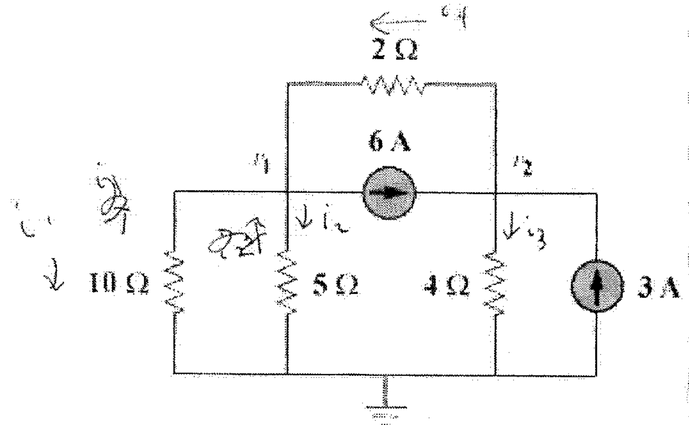


Figure 3.51

Solve voltages @ , currents everywhere

Unknowns

$i_1 \quad i_2 \quad i_3 \quad v_1 \quad v_2$

ohm reduces to:

$V_1 \ V_2$ unknowns

KCL used to solve

1) Ref

2) KCL + ohm

$i_1 = \frac{V_1}{10}$

$i_2 = \frac{V_1}{5}$

$i_3 = \frac{V_2}{4}$

$i_4 = \frac{V_2 - V_1}{2}$

ohm

KCL

$+i_1 + i_2 + 6 = i_4$

$\cancel{-i_4 + 6 - i_3 + 3 = 0}$

$i_3 + i_4 = 6 + 3$

$\Rightarrow$ ① $+\frac{V_1}{10} + \frac{V_1}{5} + 6 = \frac{V_2 - V_1}{2}$

$\cancel{\frac{V_2 - V_1}{4}}$

② $\frac{V_2}{4} + \frac{V_2 - V_1}{4} = 9$

① ② 2 eqns.

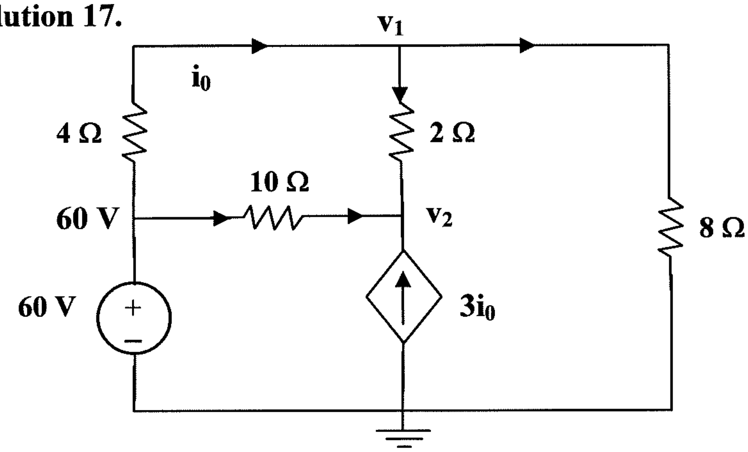
2 unknowns

V_1, V_2 solve.

Kramer???

Then find i's.

Chapter 3, Solution 17.



At node 1, $\frac{60 - v_1}{4} = \frac{v_1}{8} + \frac{v_1 - v_2}{2}$ $\qquad$ $120 = 7v_1 - 4v_2$ $\qquad$ (1)

At node 2, $3i_0 + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0$

But $i_0 = \frac{60 - v_1}{4}$.

Hence

$$\frac{3(60 - v_1)}{4} + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0 \longrightarrow 1020 = 5v_1 + 12v_2 \qquad (2)$$

Solving (1) and (2) gives $v_1 = 53.08$ V. Hence $i_0 = \frac{60 - v_1}{4} =$ **1.73 A**