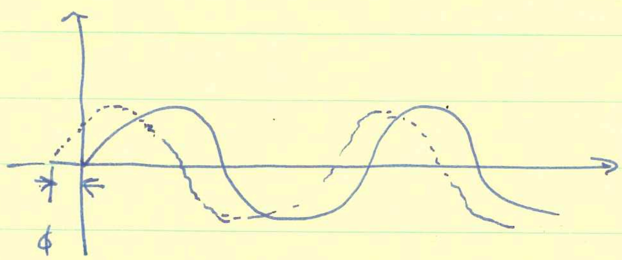
Phasors

$v(t) = V_m \sin(\omega t)$ $\qquad T = \frac{2\pi}{\omega}$

$v(t) = v(t+T)$

$F = \frac{1}{T}$

$v(t) = V_m \sin(\omega t \pm \phi)$



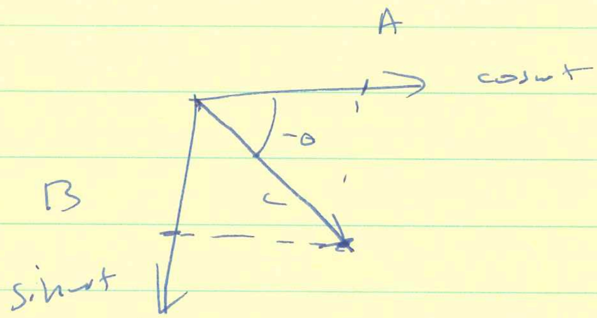
$\sin(A \pm B) = \cancel{SB}\ SA\ CB \pm CA\ SB$

$\cos A \pm B = CA\ CB \mp SA\ SB$

a

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta)$$

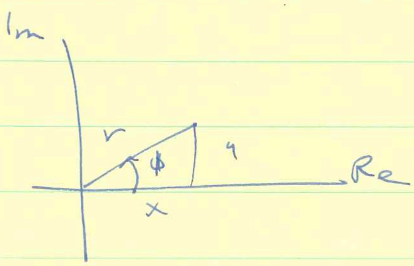
$$C = \sqrt{A^2 + B^2} \qquad \theta = \tan^{-1} \frac{B}{A}$$



Phasor — complx # represents amplitude and phase of a sinusoid

$$z = x + jy \qquad j = \sqrt{-1}$$

$$z = r\angle\phi = re^{j\phi}$$



$$r = \sqrt{x^2+y^2}$$

$$\phi = \tan^{-1} y/x$$

$$z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$$

$$z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$$

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$$

$$\frac{1}{z} = \frac{1}{r} \angle -\phi \qquad z^* = x - jy = re^{-j\phi}$$

$$\sqrt{z} = \sqrt{r} \angle \phi/2$$

$$e^{j\phi} = \cos\phi + j\sin\phi$$

$$\cos\phi = Re\left(e^{j\phi}\right)$$

$$\sin\phi = Im\left(e^{j\phi}\right)$$

$$v(t) = V_m \cos(\omega t + \phi)$$

$$= \cancel{Re}\, V_m\, Re\left(e^{j(\omega t + \phi)}\right)$$

$$= Re\left(V_m e^{j(\omega t + \phi)}\right) = Re\left(V_m e^{j\phi} e^{j\omega t}\right)$$

$$\Rightarrow v(t) = Re\left(V e^{j\omega t}\right)$$

$$V = V_m e^{j\phi}$$

RESISTOR

$I = I_m \angle \phi = I_m e^{j\phi}$

$$V(t) = i(t) R$$
$$= I_m (\cos \omega t + \phi) R$$
$$= Re \left(I e^{j\omega t} R \right)$$

$$V(t) = V_m (\cos \omega t + \phi)$$
$$V_m = I_m R$$

$$V(t) = Re \left(V e^{j\omega t} \right) \qquad V = V_m \angle \phi = V_m e^{j\phi}$$

$$V = I R$$

$\uparrow$ phasor $\quad \uparrow$ phasor

INDUCTOR

$$V(t) = L \frac{di(t)}{dt} = L \frac{d}{dt} I_m \cos(\omega t + \phi) \cancel{R}$$
$$= L I_m (-\omega) \sin(\omega t + \phi)$$
$$= \omega L I_m \cos(\omega t + \phi + 90^\circ)$$
$$= \omega L I_m \, re\left(e^{j\,\phi + 90^\circ} \right)$$
$$= \omega L \; Re\left(I_m e^{j\phi} e^{j90^\circ} \right)$$
$$= \omega L \, Re\left(I e^{j90^\circ} \right) \qquad e^{j90^\circ} = j$$
$$= \frac{\cancel{j\omega t}}{\omega L} Re\left(I j \right)$$
$$= Re\left(j\omega L I \cancel{)} e^{j\omega t)} \right.$$

$$\Rightarrow V(t) = Re\left(V e^{j\omega t} \right) \Rightarrow \boxed{V = j\omega L I}$$