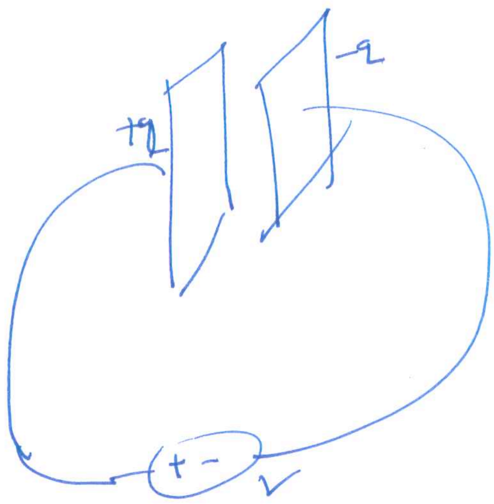


A capacitor consists of 2 metal plates separated by an insulator (or dielectric).

$q = Cv$

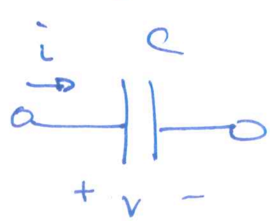
capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in Farad (F).

1 Farad = 1 Coulomb / 1 volt.

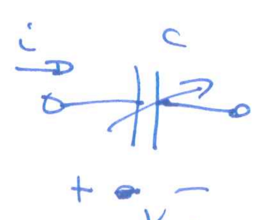
Vacuum => $k=1$ "High k" => $k>10$

$$C = \frac{k\varepsilon A}{d}$$

$\varepsilon = 8.85\times10^{-12}$ F/m
constant of nature.



Fixed



Variable.

HH page

Applications

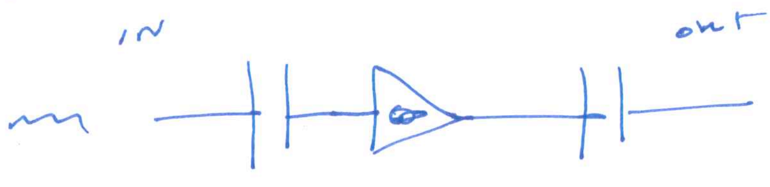
Filters (all communications equipment)

Smooth out voltage spikes on power lines
On every circuit board (should be)

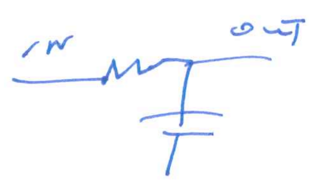
Memory. (Picture?)

1) Capacitor is an open circuit at dc.

eg. amplifier "dc block"



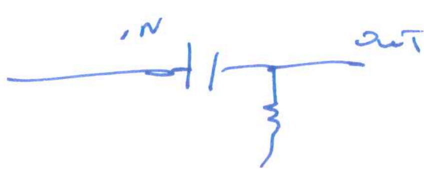
LPF



$\omega < \frac{1}{RC}$ pass

$\omega > \frac{1}{RC}$ block

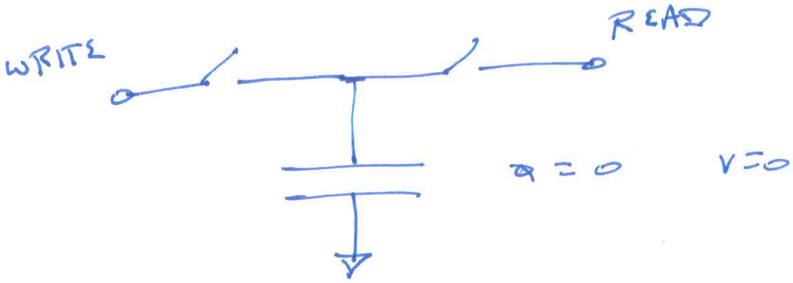
HPF

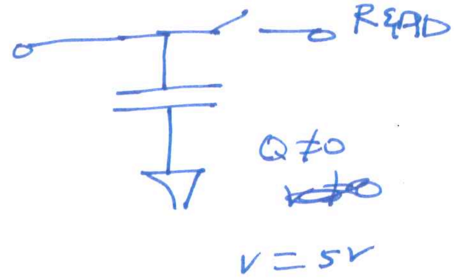


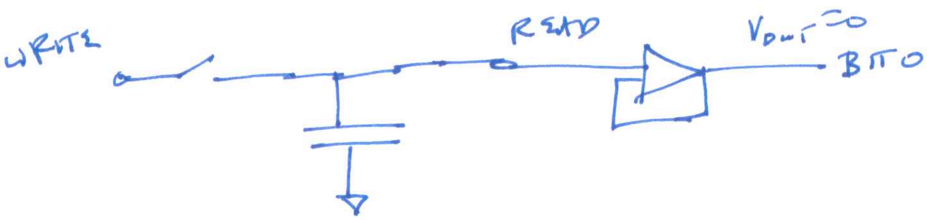
$\omega > \frac{1}{RC}$ pass

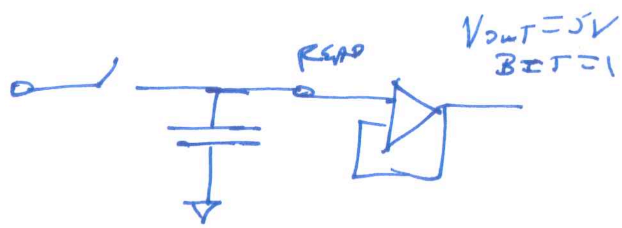
$\omega < \frac{1}{RC}$ block

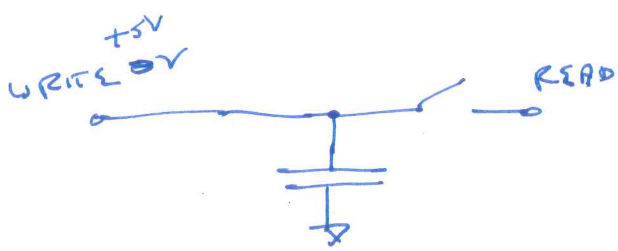
1bit Memory

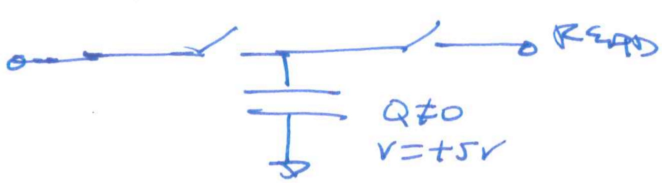

WRITE
READ
Q = 0
V=0


READ
Q≠0
V = 5V


WRITE
READ
VOUT =0
BIT 0


READ
VOUT =5V
BIT =1


+5V
WRITE
READ


READ
Q≠0
V=+5V

$$q = CV$$

$$i = \frac{dq}{dt} = C\frac{dV}{dt}$$

The voltage on a capacitor cannot change abruptly.

Because the $\frac{dV}{dt} \to \infty \Rightarrow i \to \infty$ not physical.

$$= \frac{\varepsilon A}{d}$$

C is time independent.

q, V, i can depend on time i.e.

$q(t) \quad V(t) \quad i(t)$

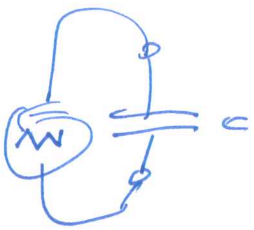
$$q(t) = C\,V(t)$$

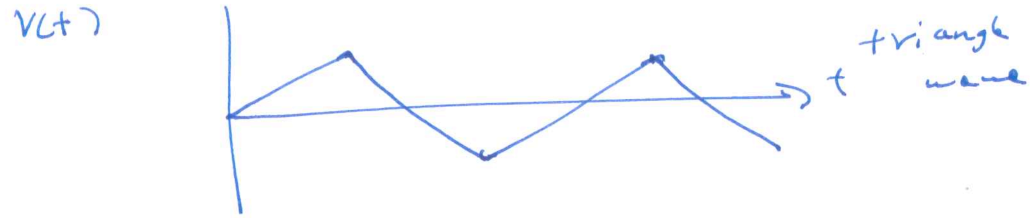
$$i(t) = C\frac{dV(t)}{dt}$$

$$\Rightarrow V(t) = \frac{1}{C}\int i(t)\,dt$$

$$\Rightarrow q(t) = \int i(t)\,dt$$

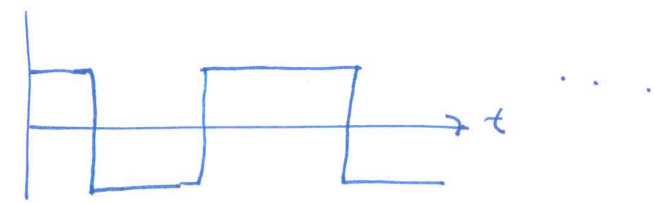
Example



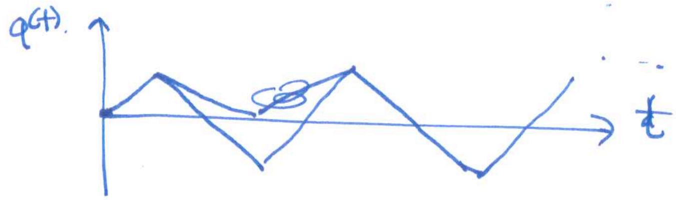


What is $i(t)$, $q(t)$?

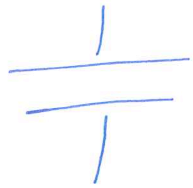
$i(t) = C\frac{dv(t)}{dt}$



$q(t) = \int i(t)\,dt$



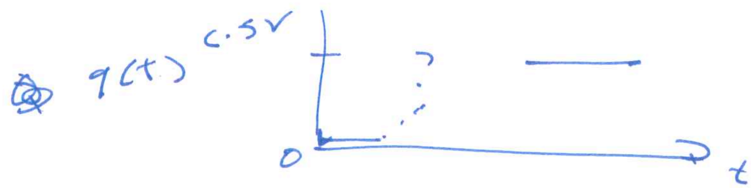
one bit memory <u>write</u>:

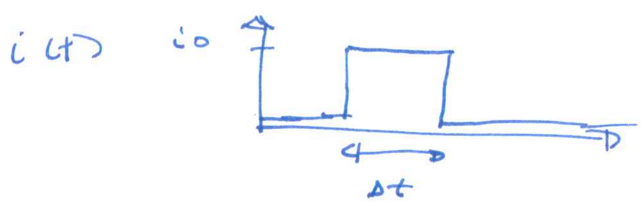


$q = CV$

$V = \frac{q}{C} = 5V$

$\Rightarrow q = C \cdot 5V$





$i_0 \Delta t = C \cdot 5V$

Tells you how long, how much current to apply.

In this time I to v to => power ~~dissipa~~ dissipated.

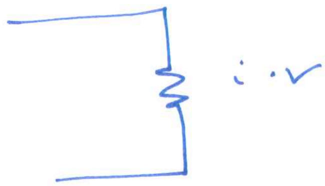
<u>Stored</u> in capacitor

$$\Rightarrow \quad W = \int p\,dt = \int i v\,dt = \int C \frac{dv}{dt} v\,dt$$

$$= C \int v\,dv = \boxed{\frac{1}{2} C V^2 = W}$$

Energy / Power

For a resistor

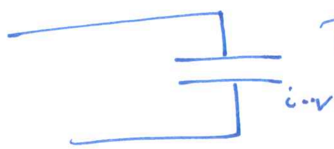


$P = i \cdot v$

$\Rightarrow$ heat

(cannot recover)

capacitor

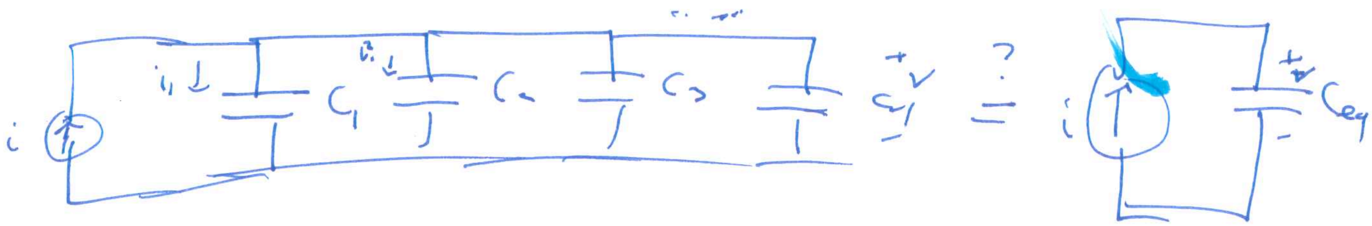


$P = i \cdot v$ electrical energy

can recover.

(will see how later.)

|| (Parallel) Capacitors



$i = i_1 + i_2 \cdots i_N$

~~$i = C_{eq} \frac{dv}{dt}$~~

$i_1 = C_1 \frac{dv}{dt}$

$i_2 = C_2 \frac{dv}{dt}$ (same V)

$i = C_1 \frac{dV}{dt} + C_2 \frac{dv}{dt} + \cdots C_N \frac{dV}{dt}$

$= \sum_{k=1}^{N} C_k \frac{dV}{dt} = C_{eq} \frac{dV}{dt}$

$C_{eq} = C_1 + C_2 + C_3 + \cdots C_N$

Series Capacitors

$$V = v_1 + v_2 + v_3 + \dots V_N$$

$$V_1 = \frac{1}{C_1}\int i(t)\,dt$$

$$v_2 = \frac{1}{C_2}\int i(t)\,dt \quad \text{same } i$$

$$V = \frac{1}{C_1}\int i(t)\,dt + \frac{1}{C_2}\int i(t)\,dt + \dots \frac{1}{C_N}\int i(t)\,dt$$

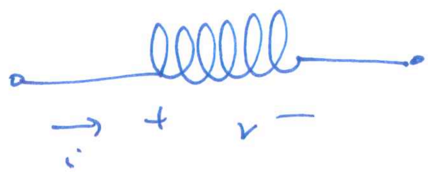
$$= \frac{1}{C_{eq}}\int i(t)\,dt = \cancel{\frac{\;}{C_{eq}}}$$

$\Rightarrow$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \frac{1}{C_N}$$

Inductors

An inductor consists of a coil of conducting wire.



$$V = L \frac{di}{dt}$$

$$L = \frac{N^2 \mu A}{l}$$

$\mu = 4\pi\, 10^{-6}$ H/m
vacuum.
larger for magnetic materials.

L time independent

v, $\frac{di}{dt}$ time dependent i.e.

$$v(t) \qquad \frac{d\,i(t)}{dt}$$

$$v = L\frac{di}{dt} \Rightarrow i(t) = \frac{1}{L}\int v(t)\,dt$$

$v \neq 0 \quad i \neq 0 \Rightarrow p = iv \neq 0$

$$W = \int p\,dt = \int iv\,dt$$

$$= \int i\frac{di}{dt}L\,dt = \frac{1}{2}LI^2$$

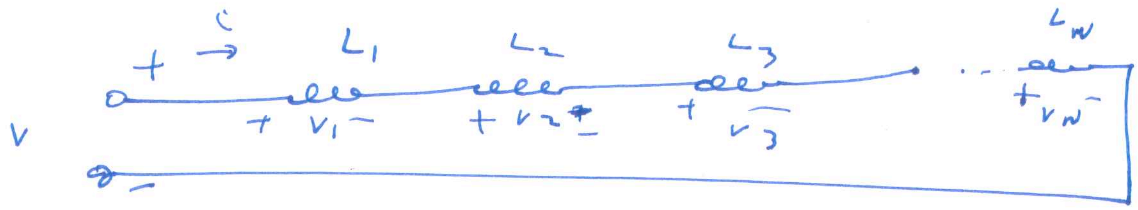
stored energy.

An inductor acts like a short circuit DC.

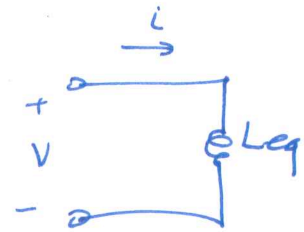
The current through an inductor cannot change instantaneously.

B.c. then $\frac{di}{dt} \to \infty \Rightarrow v \Rightarrow \infty$ not physical.

Series Inductors



$\stackrel{?}{=}$



$$V = V_1 + V_2 + V_3 + \cdots V_N$$

$$\cancel{V} \quad V_1 = L_1 \frac{di}{dt}$$

$$v_2 = v_2 \frac{di}{dt} \quad \underline{\text{same}}:$$

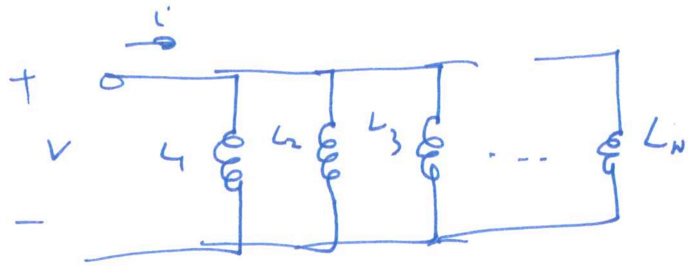
$$\vdots$$

$$V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots L_N \frac{di}{dt}$$

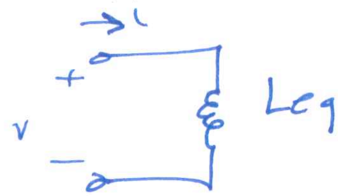
$$= (L_1 + L_2 + \cdots L_N) \frac{di}{dt}$$

$$\Rightarrow L_{eq} = L_1 + L_2 + L_3 + \cdots L_N$$

|| Ind.



$\stackrel{?}{=}$



$i = i_1 + i_2 \cdots i_N$

$i_1 = \frac{1}{L_1} \int v(dt)$

$i_2 = \frac{1}{L_2} \int v\,dt$ Same $\underline{v}$

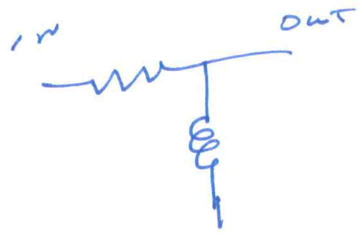
$\vdots$

$i = \frac{1}{L_1} \int v\,dt + \frac{1}{L_2} \int v\,dt + \cdots \frac{1}{L_N} \int v\,dt$

$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}\right) \int v\,dt$

$\Rightarrow \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}}$

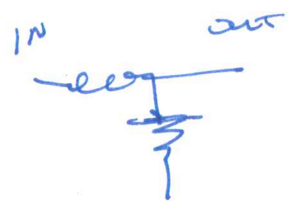
HPF



$\omega > \frac{R}{L} \Rightarrow$ pass

$\omega < \frac{R}{L} \Rightarrow$ NO PASS

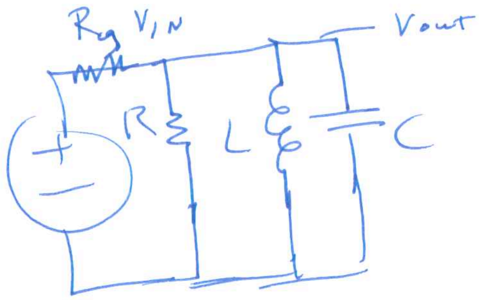
LPF



$\omega < \frac{L}{R} \Rightarrow$ PASS

$\omega > \frac{L}{R}$ NO PASS

RLC Filter

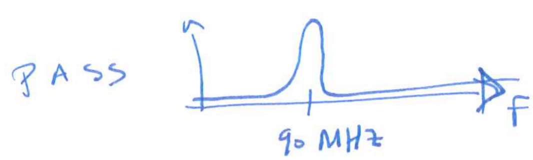


$R \parallel i\omega L \parallel \frac{1}{i\omega C}$

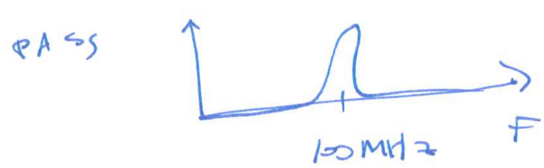
DC L short circuit $\Rightarrow V_{out} = 0$

$F \uparrow \Rightarrow$ C short circuit $\Rightarrow V_{out} = 0$

$f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$ PASS



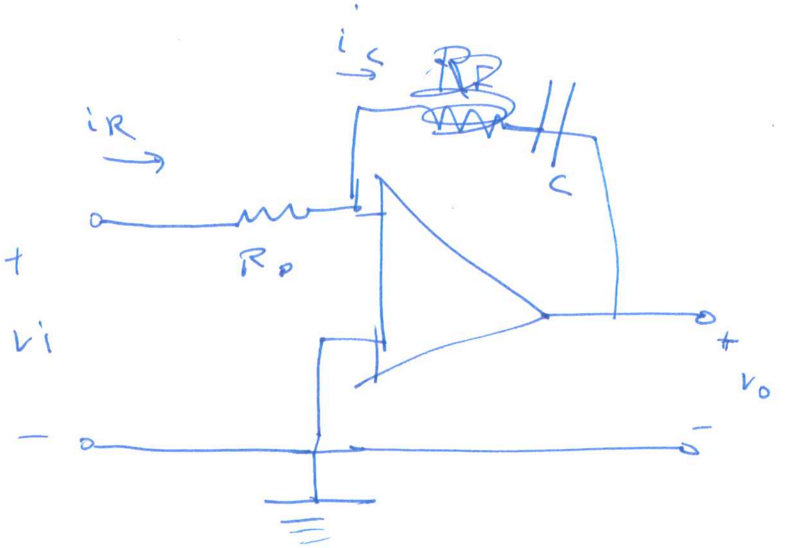
"Tune" knob



SIMPLE RADIO USE.

SHOW SPECTRUM

Integrator



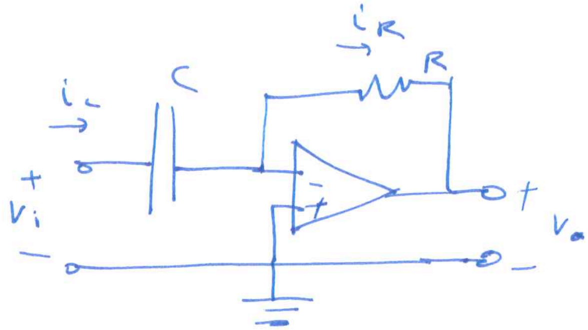
$$i_R = i_c$$

$$i_R = \frac{v_i}{R} \qquad i_c = -C\frac{dv_o}{dt}$$

$$\frac{v_i}{R} = -C\frac{dv_o}{dt}$$

$$\Rightarrow v_o = -\frac{1}{RC}\int v_i(t)\,dt$$

Differentiator



$$i_c = C\frac{dv_i}{dt}$$

$$i_R = -\frac{v_o}{R}$$

$$\Rightarrow \boxed{v_o = -RC\frac{dv_i}{dt}}$$

Have done:

+

−

integrat

differentiatee

What about $\times$, $\div$??? Not Linear.

R, L, C, op amp all linear.

cannot do.