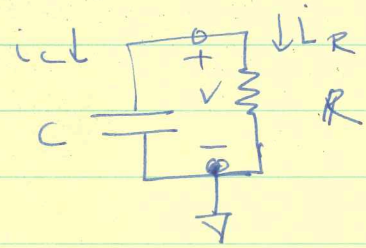
1st ORDER CIRCUITS

RC no source
with dc source

RL no source
with dc source

What if

Source free RC



As current flows thru R,
q on C ↓
⇒ V on C = q/C ↓
⇒ i ↓
Eventually q = 0 V = 0 i = 0
~~How~~ Capacitor is discharged.
How long does it take?
~~⇒ RC~~ Turns out RC.

$t = 0 \quad V(t=0) \equiv V_0$

KCL $\quad i_c + i_R = 0$

$$i_c = \frac{dq}{dt} = C\, dv/dt \qquad i_R = V/R$$

$$\Rightarrow \quad C\frac{dv}{dt} + \frac{V}{R} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} + \frac{V}{RC} = 0$$

1st ORDER DIFF. EQ.

Define $\tau = RC$

$$\frac{dv(t)}{dt} + \frac{1}{\tau} v(t) = 0$$

Diff eqns in the physical world:

IF you can find a solution
$V(t)$ that
1) Solves D.E.
2) Solves initial values
it is correct, unique.

There are many ~~meth~~ mathematical methods to solve them.

1) Guess
2) Know
3) Integrate

We will Focus on using easiest method.

$$\frac{dV}{dt} + \frac{1}{\tau} V = 0$$

$$\Rightarrow \int \frac{dV}{V} = -\int \frac{1}{\tau} dt$$

$$\ln V = -\frac{t}{\tau} + \ln A \qquad A = \text{constant}$$

$$\cancel{\ln \frac{V}{A}} \ \ln V - \ln A = \ln V/A = -t/\tau$$

$$\Rightarrow \cancel{V(t) = A e^{-t/\tau}}$$

$$e^{\ln V/A} = e^{-t/\tau}$$

$$= \frac{V}{A}$$

$$\Rightarrow V(t) = A e^{-t/\tau}$$

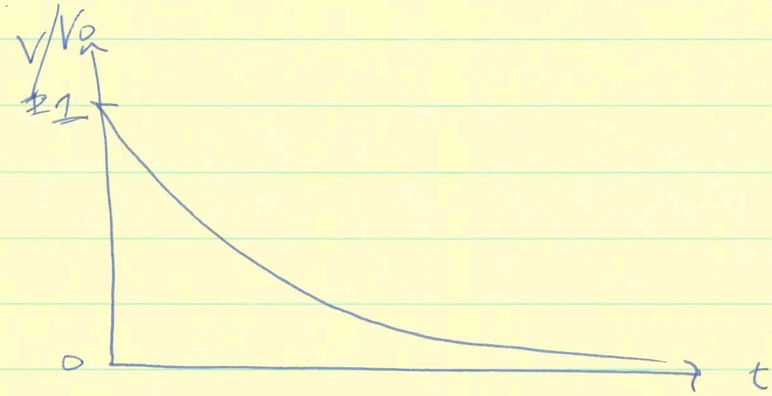
Check

$$\frac{dV}{dt} = \frac{d}{dt} A e^{-t/\tau} = -\frac{A}{\tau} e^{-t/\tau} = -\frac{1}{\tau} A e^{-t/\tau}$$

$$= -\frac{1}{\tau} V$$

$$V(t=0) = A e^{-0/\tau} = A \qquad \text{since } e^0 = 1$$

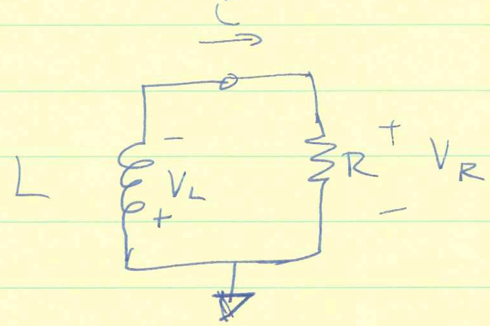
$$\Rightarrow V(t) = V_0 e^{-t/\tau}$$



After time τ V falls to $\frac{1}{e} V_0 = 0.37 V_0$
$e = 2.718$
2τ V falls to $\frac{1}{2e} V_0 = 0.135 V_0$

τ is a characteristic time for capacitor to discharge

RL circuit



KVL $\quad V_L = V_R$

$$L\frac{di}{dt} = iR \quad \Rightarrow \quad \frac{di}{dt} + \frac{1}{\tau} i = 0 \qquad \tau = L/R$$

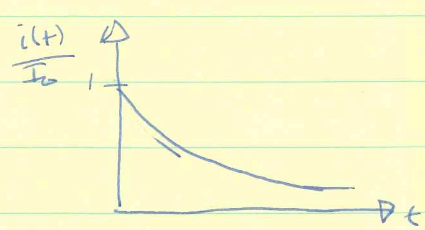
Same as before: D.E.

~~$F(t) + \frac{1}{\tau}$.~~

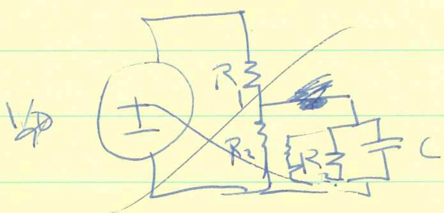
$$\frac{d\,F(t)}{dt} + \frac{1}{\tau} F(t) = 0$$

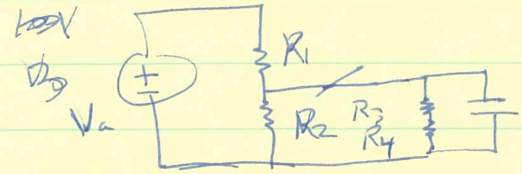
Always $\quad F(t) = A\,e^{-t/\tau}$

$$i(t) = I_0\,e^{-t/\tau}$$



Eq RC

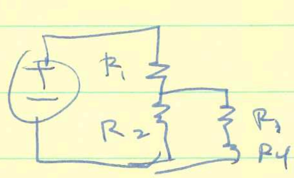


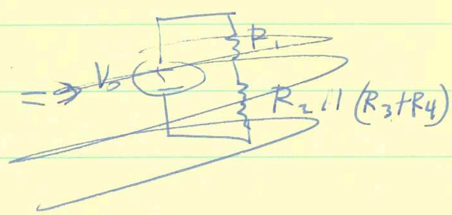


t = 0 open ~~switch~~ switch

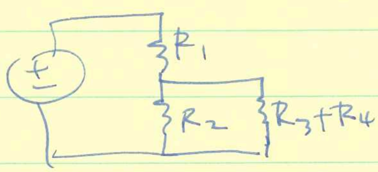
Find voltage across cap vs time.

Before opening switch,





=>



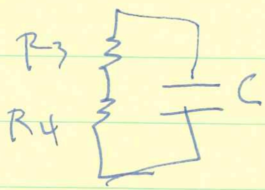
=

$R_2 \parallel (R_3 + R_4) = \dfrac{R_2(R_3+R_4)}{R_2+(R_3+R_4)}$

$\cancel{V_0} \Rightarrow V_{C(0)} = V_a \dfrac{R}{R_1 + \dfrac{R_2(R_3+R_4)}{R_3+R_4}}$

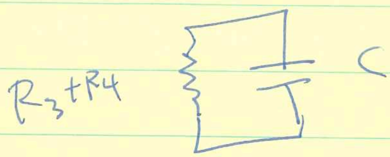
$= \underline{\underline{V_0}}$

After switch open have:



V_0 as given

=



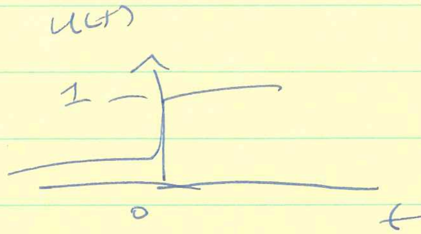
$$V(t) = V_0 e^{-t/\tau} \qquad \tau = (R_3 + R_4) C$$

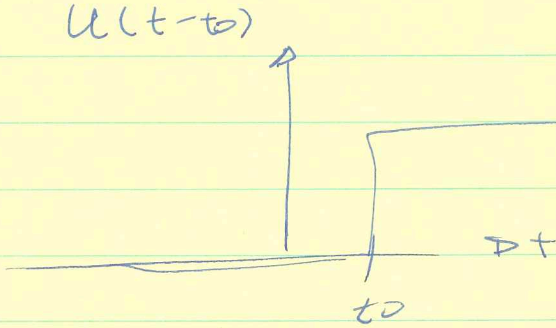
$V_{R_3} =$

RL can be done similarly

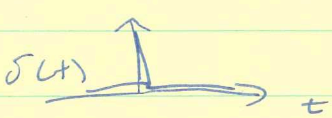
~~Eg RL~~

$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$





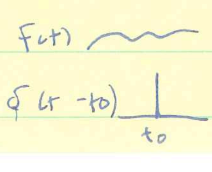
$$\delta(t) \equiv \frac{d}{dt} u(t) = \begin{cases} 0 & t<0 \\ \text{Undefined} & t=0 \\ 0 & t>0 \end{cases}$$



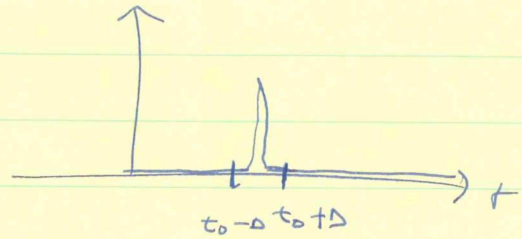
$$\int_{0^-}^{0^+} \delta(t)\, dt = 1$$

Properties $\int_a^b f(t)\, \delta(t-t_0)\, dt = f(t_0)$

Why?,



$f(t)\,\delta(t-t_0)$

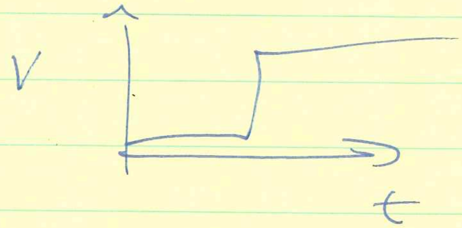


$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = \int_{t_0-\Delta}^{t_0+\Delta} f(t)\,\delta(t-t_0)\,dt$$

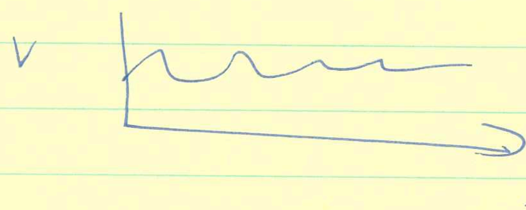
$$\approx f(t_0) \underbrace{\int_{t_0-\Delta}^{t_0+\Delta} \delta(t-t_0)\,dt}_{\substack{\shortparallel \\ 1 \text{ by defn of } \delta(t)}}$$

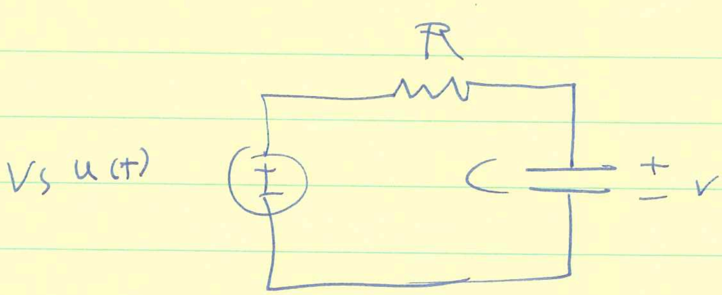
$$= f(t_0)$$

Step response



Later will assume sin waves





Before closing switch $v = V_0$

$v(0^-) = v(0^+) = V_0$

KCL $\quad C\frac{dv}{dt} + \frac{V - V_s u(t)}{R} = 0$

$$\Rightarrow \quad \frac{dv}{dt} + \frac{V}{RC} = \frac{V_s}{RC} u(t)$$

$t > 0 \Rightarrow$

$$\frac{dV}{dt} + \frac{V}{RC} = \frac{V_s}{RC}$$

$$\frac{dv}{dt} = -\frac{v - V_s}{RC}$$

$$\int_{v_0}^{v(t)} \frac{dv}{V - V_s} = -\int_0^t \frac{dt}{RC}$$

$$\ln (V - V_s) \Big|_{V_0}^{V(t)} = -\frac{t}{RC} \Big|_0^t$$

$$\ln (V(t) - V_s) - \ln (V_0 - V_s) = -\frac{t}{RC}$$

$$\ln \frac{V(t) - V_s}{V_0 - V_s} = -\frac{t}{RC}$$

$$e^{LHS} = e^{RHS}$$

$$\forall t \quad \frac{V - V_s}{V_0 - V_s} = e^{-t/\tau} \qquad \tau = RC$$

$$V(t) = \underbrace{V_s}_{\text{steady state (always there)}} + \underbrace{(V_0 - V_s)\, e^{-t/\tau}}_{\text{(TRANSIENT DIES AS } t \to \infty)}$$

$$\Rightarrow V(t) = \begin{cases} V_0 & t < 0 \\ V_s + (V_0 - V_s)\, e^{-t/\tau} & t > 0 \end{cases}$$

Could also write as

$$V(t) = V(t=\infty) + \left(V(t_0) - V(t=\infty)\right) e^{-\frac{t-t_0}{\tau}}$$

OR

$$V(t) = \underbrace{V_0\, e^{-t/\tau}}_{\text{natural response}} + \underbrace{V_s\left(1 - e^{-t/\tau}\right)}_{\text{Forced response}}$$

natural response

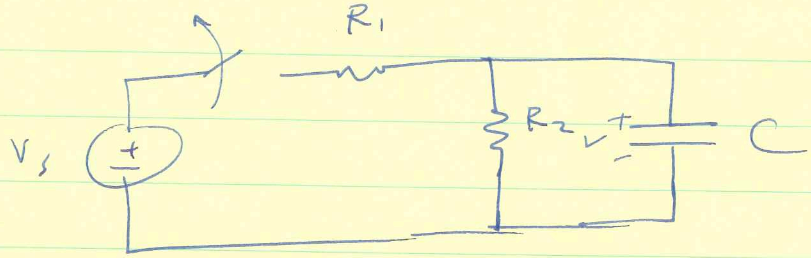
stored energy

what happens if there is no source

Forced response

what happens if there is a source.

Example RC w/ source



~~Initial voltage V_o accross capacitor.~~

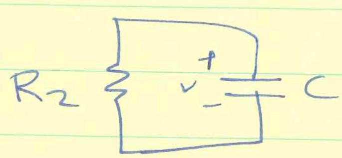
~~Find $V(t)$ after switch closes.~~

Switch opens at $t=0$.

Was closed a long time.

Find $V(t)$

After opens looks like

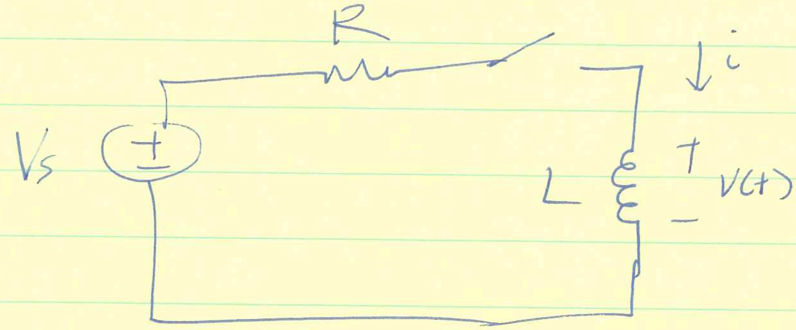


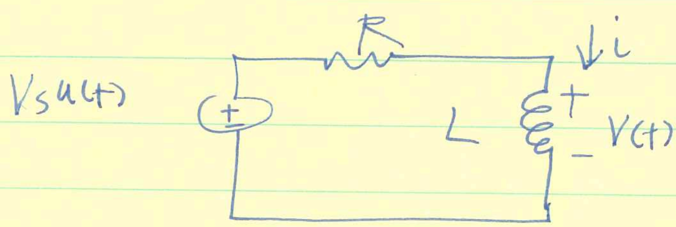
$$V(t) = V(t=0)\, e^{-t/R_2 C}$$

$$V(t=0) = ? \qquad V_s \frac{R_2}{R_1+R_2}$$

$$\Rightarrow V(t) = V_s \frac{R_2}{R_1+R_2} e^{-t/R_2 C}$$

RL





$i = i_t + i_{ss}$

i_{ss} constant

$i_t = A e^{-t/\tau}$

$i_{ss} = V_s/R \Rightarrow i(t) = A e^{-t/\tau} + \frac{V_s}{R}$

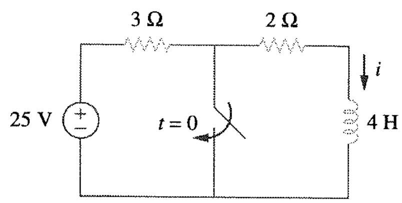
$i(0^+) = i(0^-) = I_0$

$i(0) = A + \frac{V_s}{R} = I_0 \Rightarrow A = I_0 - V_s/R$

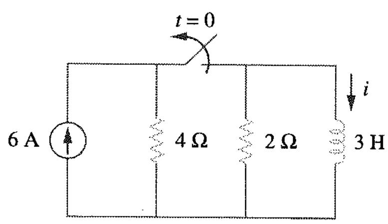
$\Rightarrow \boxed{i(t) = \left(I_0 - V_s/R\right) e^{-t/\tau} + V_s/R}$

Chapter 7, Problem 53.

Determine the inductor current $i(t)$ for both $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.119.



(a)



(b)

Figure 7.119
For Prob. 7.53.

Chapter 7, Solution 53.

(a) Before t = 0, $i = \dfrac{25}{3+2} = \underline{\mathbf{5\ A}}$

After t = 0, $i(t) = i(0)e^{-t/\tau}$

$$\tau = \frac{L}{R} = \frac{4}{2} = 2, \qquad i(0) = 5$$

$$i(t) = \underline{5e^{-t/2}\ u(t)A}$$

(b) Before t = 0, the inductor acts as a short circuit so that the 2 Ω and 4 Ω resistors are short-circuited.

$$i(t) = \underline{\mathbf{6\ A}}$$

After t = 0, we have an RL circuit.

$$i(t) = i(0)e^{-t/\tau}, \qquad \tau = \frac{L}{R} = \frac{3}{2}$$

$$i(t) = \underline{6e^{-2t/3}\ u(t)A}$$

Chapter 7, Problem 63.

Obtain $v(t)$ and $i(t)$ in the circuit of Fig. 7.128.

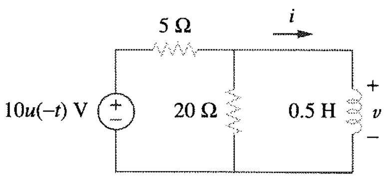


Figure 7.128
For Prob. 7.63.

Chapter 7, Solution 63.

For $t < 0$, $\quad u(-t) = 1, \quad i(0) = \dfrac{10}{5} = 2$

For $t > 0$, $\quad u(-t) = 0, \quad i(\infty) = 0$

$$R_{th} = 5 \| 20 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{4} = \frac{1}{8}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = \underline{2e^{-8t}\, u(t) A}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-8)(2)e^{-8t}$$

$$v(t) = \underline{-8e^{-8t} u(t)\ V}$$