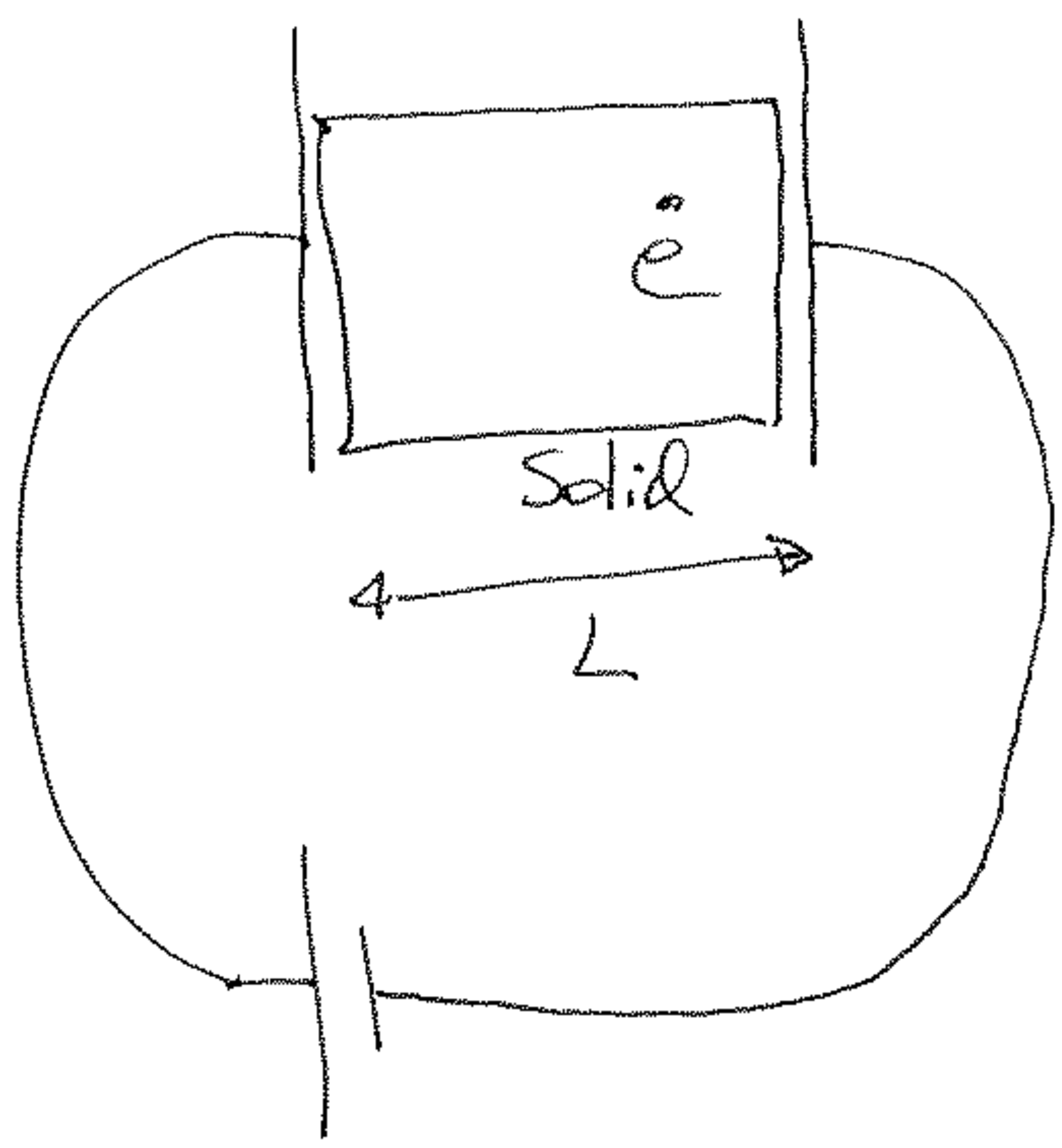
Ohm's "Law"

Not really a law, only true for certain materials.

What if we have a free electron inside a solid?



Can electron move?

Usually, No, most electrons are bound to the atoms in a solid.

If all electrons are bound and cannot move, we have an insulator: We apply a voltage, but electrons do not move.

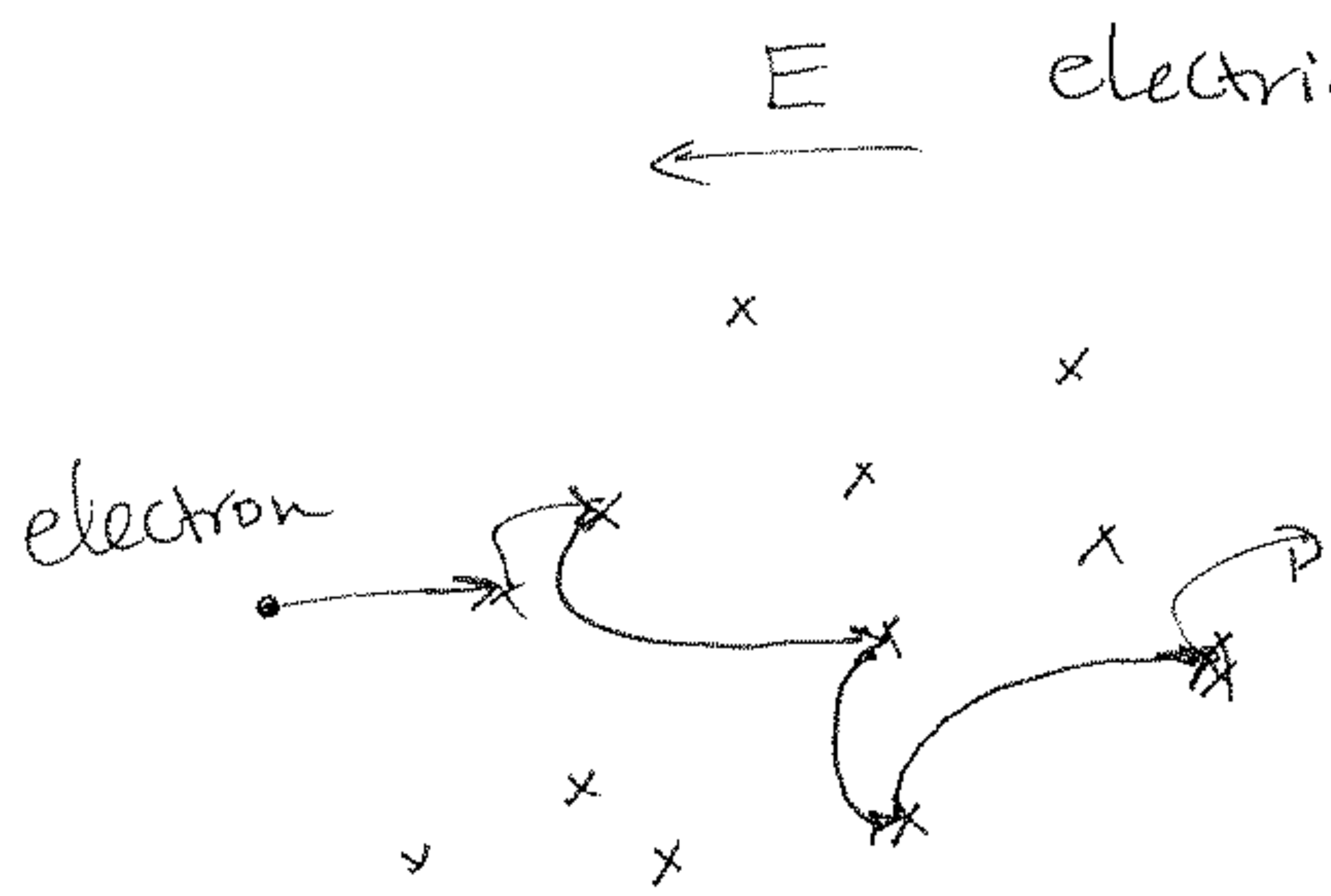
This means no current flows.

(why?)

In <u>metals</u> (gold, aluminum, copper, etc.)
electrons <u>can</u> move, but they scatter off of atoms.

Why are the consequences of this scattering?
<u>Resistance</u> <u>to</u> <u>Flow</u>.

How much current flows?



v = velocity

$$F = ma = m\, dv/dt$$

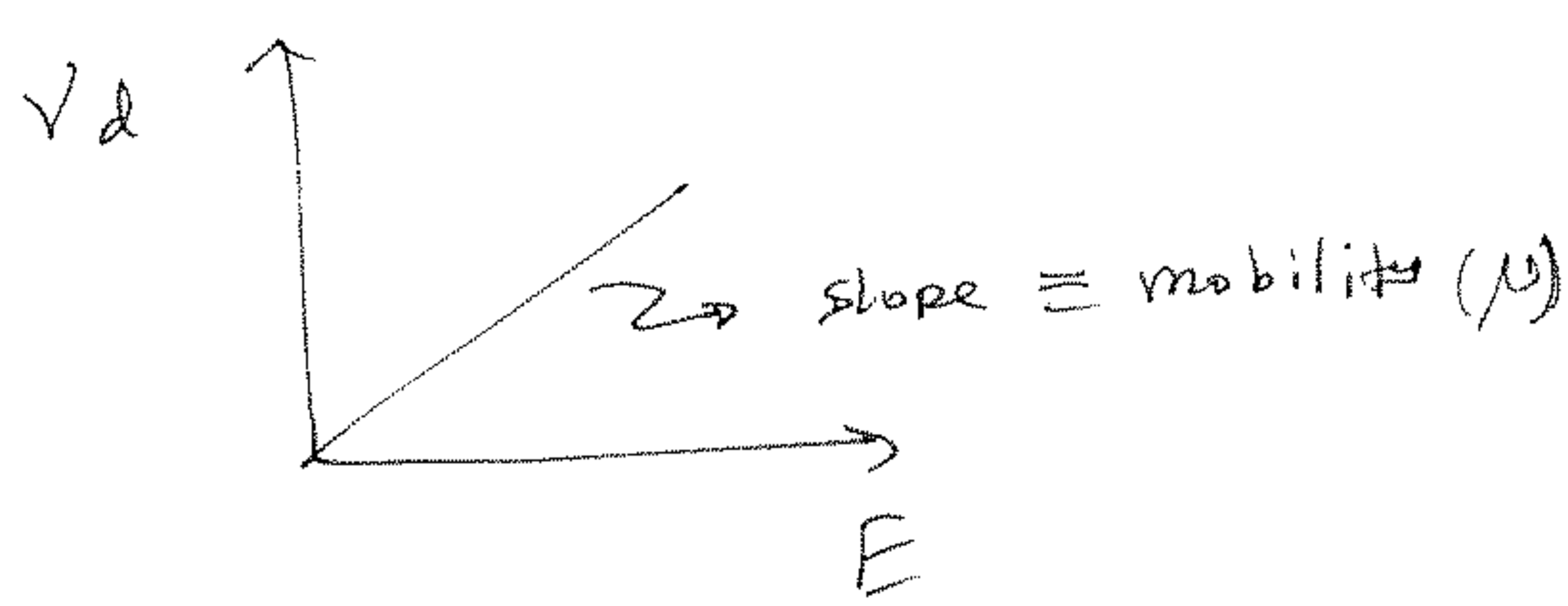
$$eE = m\frac{dv}{dt} = m\frac{\Delta v}{\Delta t}$$

$$\Rightarrow \Delta v = \frac{e\,\Delta t}{m} E$$

$\tau \equiv$ average ~~scattering~~ time between scattering events

$$\Rightarrow v_{average} = v_{drift} = \frac{e\tau}{m} E$$

Like a swarm of mosquitos in a gentle breeze...



Want to know how I, V related.

$E = V/L$

How v_d, I related? $\uparrow v_d \Rightarrow \uparrow I$

$\uparrow n \Rightarrow \uparrow I$ $\quad n \equiv \dfrac{\#\text{ electrons}}{m^3}$

$\uparrow A \Rightarrow \uparrow I$ $\quad$ density

"It can be shown that" (Phys. 7D)

$$I = e n v_d A$$

So: $I = e n v_d A = e n \left(\frac{e\tau}{m}\right) E A$

$$= \frac{n e^2 \tau}{m} \frac{V}{L} A$$

$$\Rightarrow V = \left(\frac{L}{A} \frac{m}{n e^2 \tau}\right) I = IR \quad \text{"Ohm's law"}$$

$$R = \frac{L}{A} \frac{m}{n e^2 \tau}$$

R = resistance $[\Omega]$ = [ohms]

Larger R means lower current for a given applied voltage.

$$R = \frac{m}{ne^2\tau} \frac{L}{A}$$

($\frac{m}{ne^2\tau}$) ↑ only depends on material, not geometry.

Resistivity $\rho = \frac{m}{ne^2\tau}$

units $[\Omega\text{-m}] = [\text{ohm- meter}]$

Conductance $G \equiv 1/R$

Units siemen $[S]$ $[\mho]$

Conductivity $\sigma \equiv 1/\rho$

Units $[S/m]$

Example Cu $\rho = 1.72 \mu\Omega\text{-cm}$
(Pure Cu)

Note: Cu that contains impurities has higher ρ.

What is the resistance of a copper wire with diameter 1mm, length 10 cm?

$$R = \rho \frac{L}{A} = \ldots$$

MKSA way

$$\rho = 1.72 \mu\Omega\text{-cm} = 1.72 \ 10^{-6} \Omega \ 10^{-2} m$$
$$= 1.72 \times 10^{-8} \Omega\text{-m}$$

$$L = 10 cm = 10 \ 10^{-2} m = 10^{-1} m$$

$$A = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi}{4} (1mm)^2 = \frac{\pi}{4} \left(10^{-3} m\right)^2$$
$$= \frac{\pi}{4} 10^{-6} m^2$$

$$\Rightarrow R = 1.72 \times 10^{-8} \Omega\text{-}\cancel{m} \frac{10^{-1} \cancel{m}}{\frac{\pi}{4} 10^{-6} \cancel{m^2}} =$$

$$\underbrace{\frac{4}{\pi} \times 1.72}_{2.191} \ 10^{-8-1+6} \Omega \cong 2 \ 10^{-3} \Omega = 2 m\Omega$$

(small)

Other way

$$R = \rho \frac{L}{A} = 1.72\ \mu\Omega\text{-cm}\ \frac{10\ cm}{\pi \left(\frac{1mm}{2}\right)^2}$$

$$= 1.72 \times 10 \times \frac{4}{\pi}\ \frac{\mu\Omega\ cm\ cm}{(mm)^2}$$

$$\approx 2 \times 10\ \frac{\mu\Omega\ cm\ cm}{(mm)^2}$$

$$= 2 \times 10\ \frac{10^{-6}\Omega\ 10^{-2}m\ 10^{-2}m}{(10^{-3}m)(10^{-3}m)}$$

$$= 2 \times 10 \times 10^{-6-2-2+3+3}\ \Omega = 2 \times 10^{-3}\ \Omega$$

$$= 2\ m\Omega$$

Au Ag Cu Al etc about same $\rho \approx 1-10\ \mu\Omega\text{-cm}$
$= 10^{-8}\ \Omega\text{-m}$

NiCr
Plastic, glass, paper, etc.
$\rho \approx 10^{10} - 10^{12}\ \Omega\text{-m}$

"Insulators"

Special materials

Carbon → diamond : insulator
Carbon → black carbon $\rho \sim 10^{-5}\ \Omega\text{-m}$

Silicon, Germanium → "semiconductors"

Pure $10^{2}\ \Omega\text{-m}$
⇒ almost insulator

"doped" ⇒ down to $10^{-3}\ \Omega\text{-cm}$
$= 10^{-5}\ \Omega\text{-m}$

doped p or n type ⇒ p-n diode

Since Cu has R small, approximate as $\underline{R=0}$

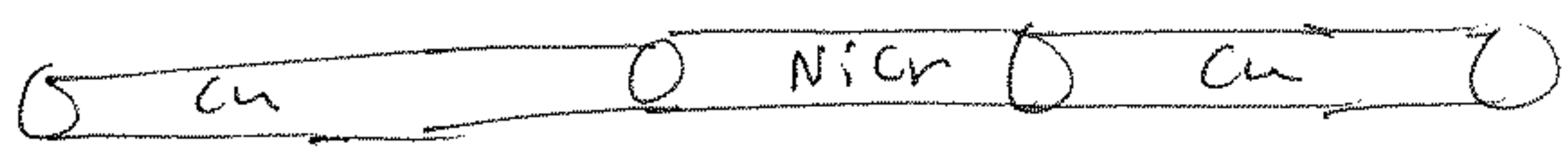
This is an "ideal metal"

1) $\rho \to 0$

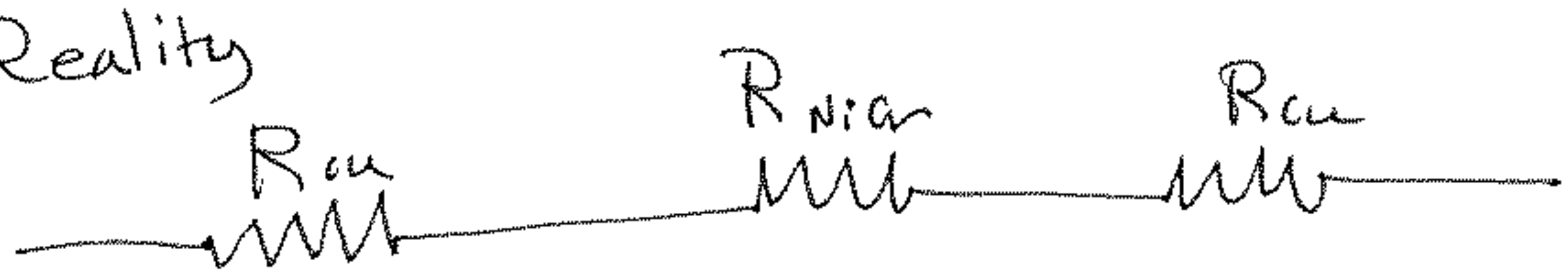
2) $E = 0$ inside everywhere

$1 \iff 2$ (Shown in physics class)

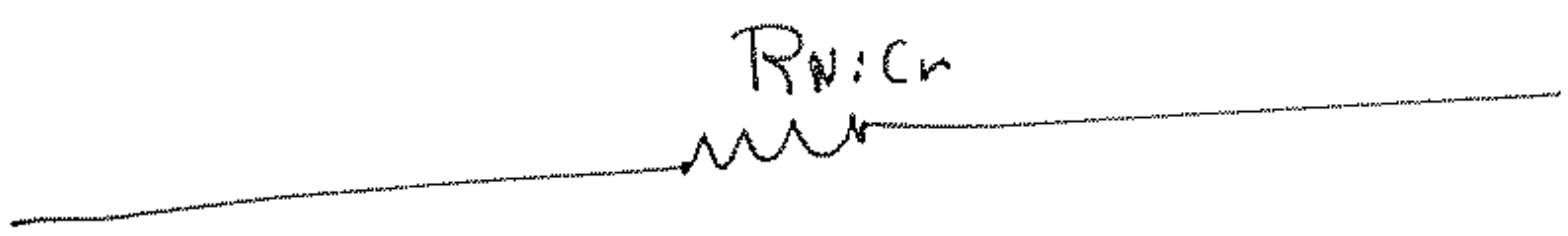
E.g.



Reality



But ignore R_{Cu}:



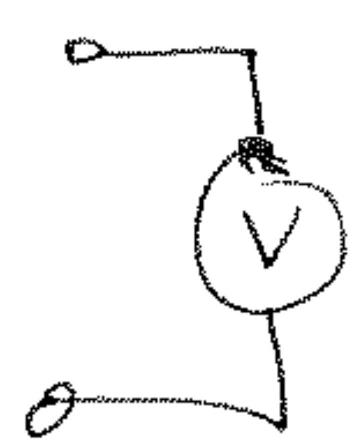
Additional circuit elements

Variable resistor

Potentiometer
"e.g. trimpot"

meters

ammeter — in a circuit, behaves as a short



Voltmeter — in a circuit, behaves as an open

Fuse

Small resistor. If current goes above a specific value becomes open.

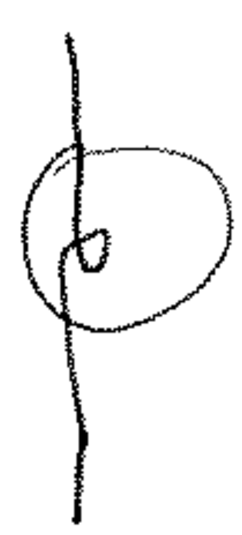
Switch

open close

Speaker

Usually 4Ω or 8Ω

Light bulb



Incandescent like a resistor.

But LED like a diode.

Transformer