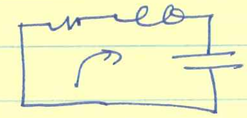
Initial Values

$i(0^+) = i(0^-)$ inductor
$V(0^+) = V(0^-)$ capacitor

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$



Solution has 2 unknown constants.

378 & 380

Eg. underdamped

$i(t) = e^{-\alpha t}\left[B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$ $\alpha, \omega_d$ defined in terms of RLC

$B_1, B_2$ constants

To find $B_1, B_2$ for a particular problem, need to know $i(t=0)$ & $\left.\frac{di}{dt}\right|_{t=0}$

Because: $i(t=0) = B_1$

$$\left.\frac{di}{dt}\right|_{t=0} = \omega_d B_2$$

~~Similar~~ Similar calc. can be done for underdamped.

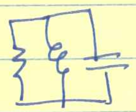
What if you are given $I_0 \equiv i(t=0)$ and $V_0 \equiv V_{cap}(t=0)$?

$KVL|_{t=0}$ $\quad i(t=0)R + L\left.\frac{di}{dt}\right|_{t=0} + V_{cap}(t=0) = 0$

$\Rightarrow \quad I_0 R + L\left.\frac{di}{dt}\right|_{t=0} + V_0 = 0$

$$\Rightarrow \left.\frac{di}{dt}\right|_{t=0} = \frac{-1}{L}(R I_0 + V_0)$$

|| RLC



$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0$$

Soln. has 2 unknown constants.
(True of any 2ND ODE)

Eg underdamped

$$V(t) = e^{-\alpha t}\left[B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$$

$$V(t=0) = B_1$$

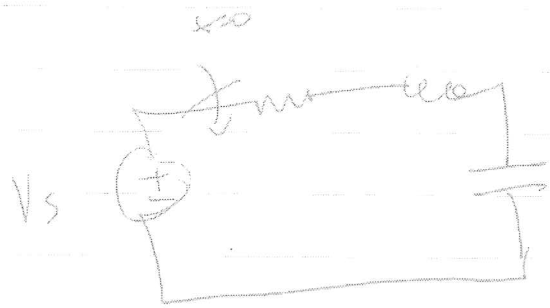
$$\left.\frac{dV}{dt}\right|_{t=0} = \omega_d B_2$$

What if you are given $V(t=0) \equiv V_0$ and $I_0 \equiv i_{ind}(t=0)$?

$$KCL\Big|_{t=0} \Rightarrow \quad \frac{V_0}{R} + I_0 + C\left.\frac{dV}{dt}\right|_{t=0} = 0$$

$$\Rightarrow \left.\frac{dV}{dt}\right|_{t=0} = -\frac{V_0 + I_0 R}{RC}$$

# RLC step response




KVL $L\frac{di}{dt} + Ri + v = V_s$ $\quad t>0$

$= 0 \quad t<0$

$q = Cv \Rightarrow i = C\,dv/dt$

$$\Rightarrow \frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC} \quad t>0$$

$$= 0 \quad t<0$$

$$\Rightarrow \frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = u(t)\,\frac{V_s}{LC}$$

later will be $\sin(\omega t)$

Soln $v(t) = v_t(t) + v_{ss}(t)$

transient steady state

↳ constant

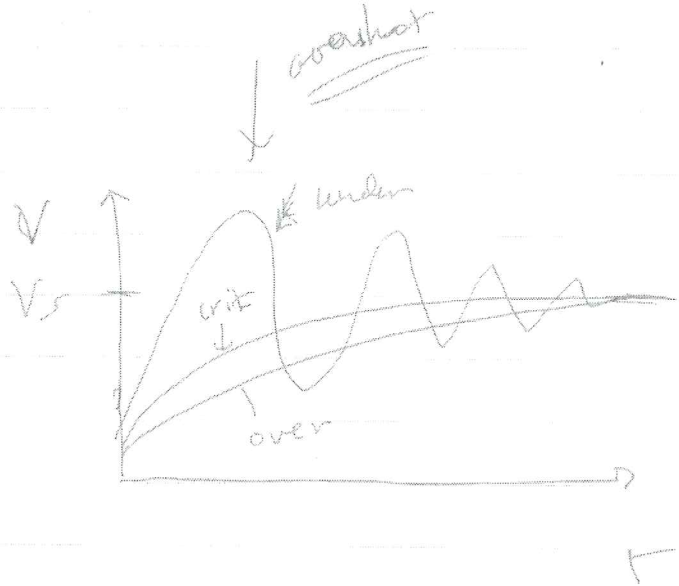
$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ over

$(A_1 + A_2 t)\,e^{-\alpha t}$ crit

$(A_1 \cos\omega_d t + A_2 \sin\omega_d t)\,e^{-\alpha t}$ under

$v_{ss}(t) = v(\infty) = V_s$

Prove.



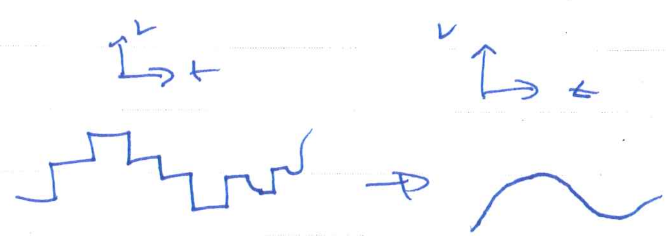

## Applications:

Car ignition system

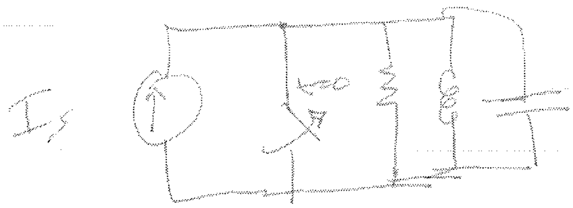
Battery 12V
Need ~ 10,000V momentarily across spark plug

Use RLC overshoot and switch to generate the large voltage.

## Smoothing

∥ RLC step



KCL $\frac{v}{R} + i + C\frac{dv}{dt} = I_s$

Using $v = L\,di/dt$ $\Rightarrow$

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

$$i(t) = i_t(t) + i_{ss}(t)$$

$$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{over}$$

$$i(t) = I_{ss} + (A_1 + A_2 t)e^{-\alpha t} \qquad \text{crit}$$

$$i(t) = I_{ss} + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \qquad \text{under}$$

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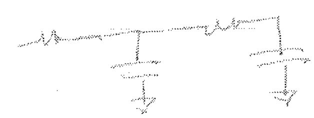
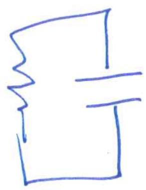
TA will cover 2nd order RC circuits
+ RL circuits

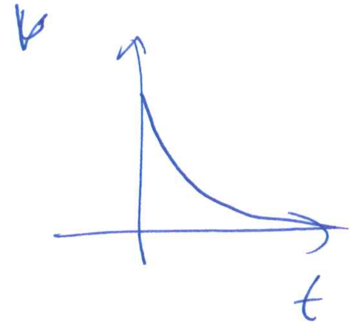
e.g.



To date

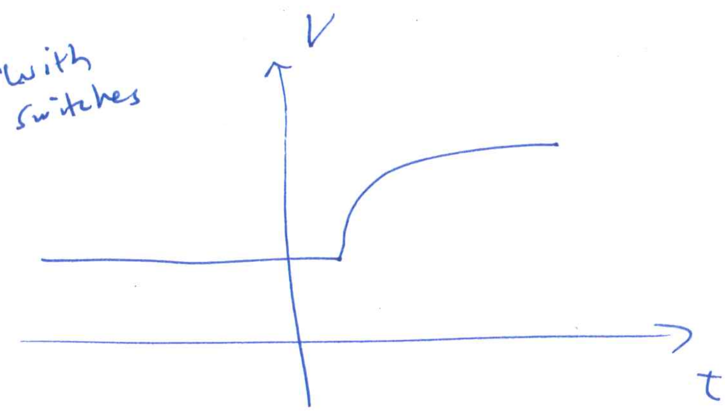


RC

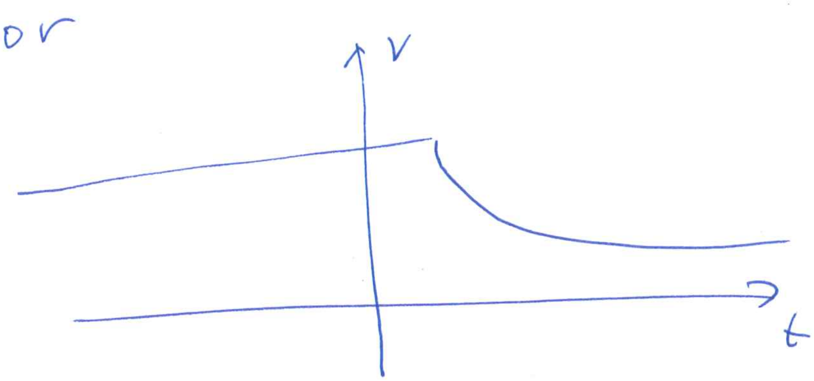



RL similar

In general:
RC, RL with switches




or




$v = iR$

$q = Cv \quad i = C\frac{dV}{dt}$

$v = L\frac{di}{dt}$

No overshoot to final destination.

RLC has overshoot. Why?