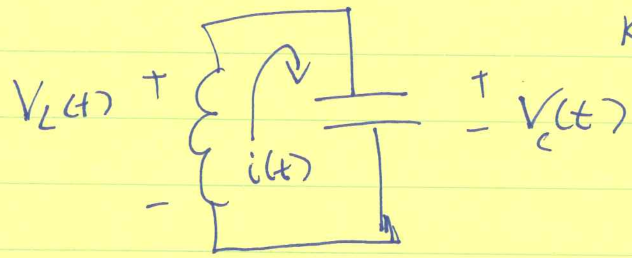
N.B. (-1) first ...



KVL ⇒ $V_L(t) = V_c(t)$ just call V

$V_L(t) = -L\frac{di}{dt}$ by def. of i

Energy

				$\frac{1}{2}LI^2$	$\frac{1}{2}CV^2$
Initially	$i=0$	$V \neq 0$ ($V>0$)		0	$\neq 0$

$V \neq 0 \Rightarrow i \downarrow$ b.c. $V = -L\frac{di}{dt}$ Slope of i vs t -ve

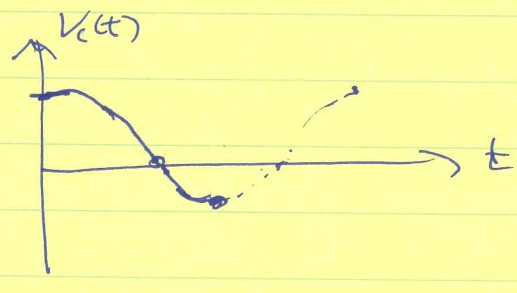
~~$i \uparrow \Rightarrow q \downarrow$~~

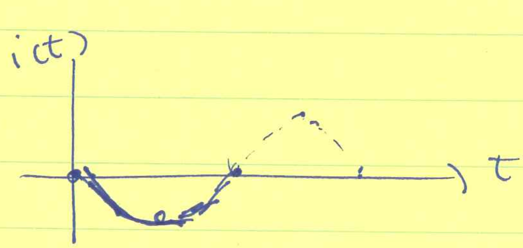
~~$i \uparrow \Rightarrow di/dt$~~ $\uparrow$

~~$\frac{di}{dt} \uparrow \Rightarrow V$~~ $\downarrow$

$\Rightarrow i < 0 \Rightarrow V_c(t) \downarrow$ b.c. $q \downarrow$ due to $i<0 \Rightarrow q = CV \downarrow$

$V_c(t) \downarrow \Rightarrow -L\frac{di}{dt} \uparrow$ slope becomes less -ve





		Energy	
		$\frac{1}{2}LI^2$	$\frac{1}{2}CV^2$
Then	$i \neq 0$ $V = 0$ ($i < 0$)	$\neq 0$	$= 0$

$i < 0 \Rightarrow V_C \downarrow$ as above

$\Rightarrow V_C < 0 \Rightarrow -L\frac{di}{dt} < 0 \Rightarrow \frac{di}{dt} > 0 \Rightarrow i \uparrow$

$i \uparrow \Rightarrow \frac{dv}{dt}$ becomes less negative

b.c. i less negative $\Rightarrow$ charge flows slower off cap

Energy goes back and forth between L, C.

Frequency is $\frac{1}{\sqrt{LC}}$ (will prove)

---

* later

If R in there, slowly ~~loss~~ lose energy.

B.c. $I \cdot V \neq 0$ in resistor → heat = lost energy to environment

$\Rightarrow$ eventually $i = 0$, $V = 0$

Rate is $\sim \frac{R}{L}$.

So compare $\frac{R}{L}$ to $\frac{1}{\sqrt{LC}}$

overdamped vs. underdamped.

Equations

KVL $\Rightarrow$ $V_L(t) = V_C(t)$

$$\frac{d}{dt}\left\{ -L\frac{di}{dt} = V \right\}$$

$$-L\frac{d^2 i}{dt^2} = \frac{dV}{dt} = \frac{1}{C} i$$

$q = CV$

$i = \frac{dq}{dt} = C\frac{dV}{dt}$

$$\Rightarrow \boxed{\frac{d^2 i}{dt^2} = -\frac{1}{LC} i}$$

2nd order D.E.

Note: Chs. 9, 14 deal with A, B, ω, math of sin, cos

Soln

$i(t) = A\sin(\omega t) + B\cos(\omega t)$ works. Assume $B=0$ for now.

$i(t) = A\sin(\omega t)$ $\quad\omega = ??$

$$\frac{d^2}{dt^2} i = -A\omega^2 \sin(\omega t) = -\frac{1}{LC} A \sin(\omega t)$$

$$\Rightarrow \omega = \sqrt{\frac{1}{LC}}$$

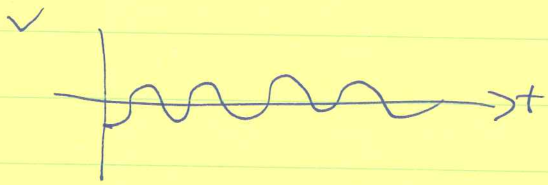
$$\Rightarrow i(t) = A\sin\left(\frac{1}{\sqrt{LC}} t\right)$$

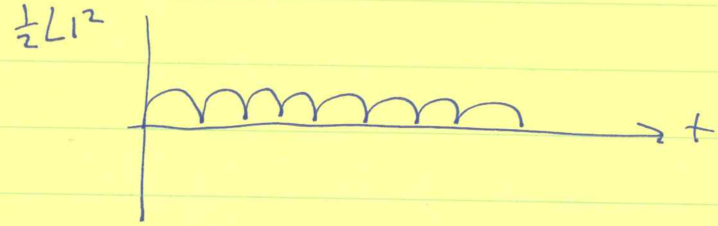
$$V = -L\frac{di}{dt} \Rightarrow$$

$$V(t) = -LA\sqrt{\frac{1}{LC}}\cos\left(\frac{1}{\sqrt{LC}} t\right)$$
$$= -A\sqrt{\frac{L}{C}}\cos\left(\frac{1}{\sqrt{LC}} t\right)$$

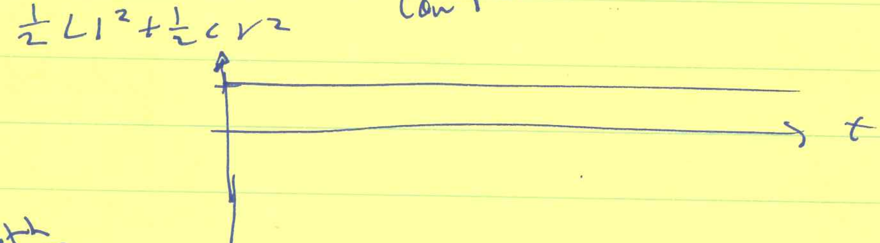
$i(t)$, $v(t)$ said to be out of phase.

$\Rightarrow$ $\emptyset$ Damping

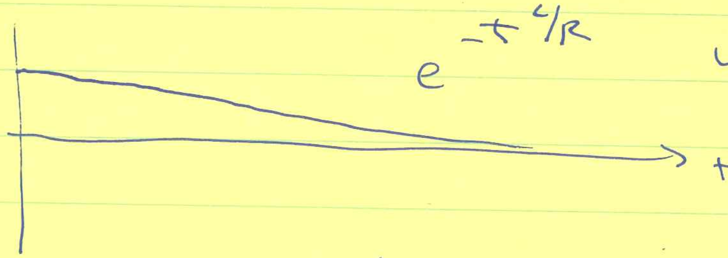




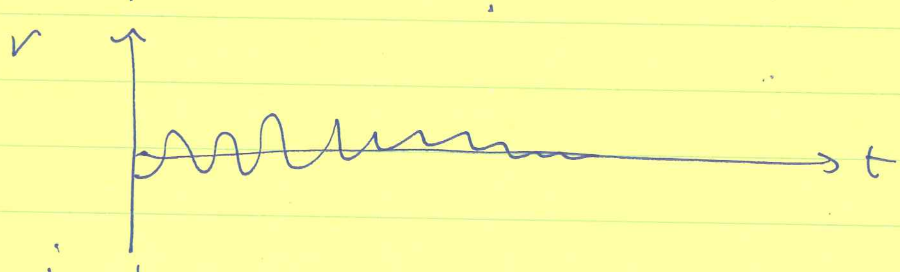
Can prove. How? Show...

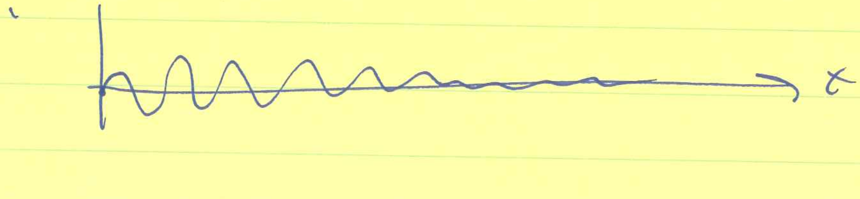


With Damping



will prove later





II

Sometimes damping severe

⇒

or

Damping severe if

$$\left(\frac{L}{R}\right)^{-1} > \sqrt{\frac{1}{LC}}$$

Found $i(t) = A\sin\left(\sqrt{\frac{1}{LC}}\,t\right)$ no damping

Want to find $i(t)$ with damping.

Will be $i(t) \approx A e^{-t/\tau} \sin\left(\sqrt{\frac{1}{LC}}\,t\right)$

Want exact soln..

Need math