4.4. Under the generalized scaling rules, $W \to W/\kappa$, $L \to L/\kappa$, $t_{ox} \to t_{ox}/\kappa$, $V_g \to (\alpha/\kappa)V_g$, and $V_t \to (\alpha/\kappa)V_t$, Eq. (3.79) transforms as

$$I_{dsat} \to \frac{\alpha}{\kappa} C_{ox} W v_{sat} (V_g - V_t) \frac{\sqrt{1 + 2\alpha\mu_{eff}(V_g - V_t)/(mv_{sat}L)} - 1}{\sqrt{1 + 2\alpha\mu_{eff}(V_g - V_t)/(mv_{sat}L)} + 1}.$$

In the limit of $\mu_{eff}(V_g - V_t)/(mv_{sat}L) >> 1$ (short channel or velocity saturation), the fractional factor in the above equation equals 1 independent of α, so $I_{dsat} \to (\alpha/\kappa)I_{dsat}$. On the other hand, if $\alpha\mu_{eff}(V_g - V_t)/(mv_{sat}L) << 1$ (long channel), the fractional factor can be approximated by $\alpha\mu_{eff}(V_g - V_t)/(2mv_{sat}L)$, and one has $I_{dsat} \to (\alpha^2/\kappa)I_{dsat}$. In general, the scaling behavior is between the two limits.

4.5. (a)

$$V_t = V_{fb} + 2\psi_B + \frac{qN_a}{C_{ox}}(W_{dm} - x_s)$$

$$W_{dm} = \sqrt{\frac{4\varepsilon_{si}\psi_B}{qN_a} + x_s^2}$$

Eliminate N_a from the above eqs,

$$x_s = \frac{4\varepsilon_{si}\psi_B}{C_{ox}(V_t - V_{fb} - 2\psi_B)} - W_{dm}$$

$V_{fb} = -E_g/2q - \psi_B = -1.06$ V, so $x_s = 17$ nm.
Substituting in the 2nd eq., $N_a = 1.3\times10^{17}$ cm^{-3}.

(b) $m = 1 + 3t_{ox}/W_{dm} = 1.21$
Inv. Subth. Slope = $m\times$60mV/decade = 73 mV/decade

(c) $\lambda = W_{dm} + 3t_{ox} = 0.12$ µm
$L_{\min} \approx 2\lambda = 0.24$ µm

4.6. From Eq. (3.25) with V_t replaced by V_{on}, the linear region characteristics of the two parts are:

$$I_{ds1} = \mu_{eff} C_{ox} \frac{W_1}{L}\left(V_g - V_{on1} - \frac{m}{2}V_{ds}\right)V_{ds},$$

and

$$I_{ds2} = \mu_{eff} C_{ox} \frac{W_2}{L}\left(V_g - V_{on2} - \frac{m}{2}V_{ds}\right)V_{ds}.$$

Since they are connected in parallel, V_g and V_{ds} are the same and the total current is $I_{ds} = I_{ds1} + I_{ds2}$. It is straightforward to show that

$$I_{ds} = \mu_{eff} C_{ox} \frac{W_1 + W_2}{L}\left(V_g - V_{on} - \frac{m}{2}V_{ds}\right)V_{ds},$$

where $V_{on} = (W_1 V_{on1} + W_2 V_{on2})/(W_1 + W_2)$.

4.7. From Eq. (3. 23) with V_t replaced by V_{on}, the linear region characteristics of the two parts are:

$$I_{ds} = \mu_{eff} C_{ox} \frac{W}{L_1}\left(V_g - V_{on1}\right)V_{ds1},$$

and

$$I_{ds} = \mu_{eff} C_{ox} \frac{W}{L_2}\left(V_g - V_{on2}\right)V_{ds2}.$$

Here the second order terms in V_{ds1} and V_{ds2} have been neglected (low-drain bias). Since they are connected in series, I_{ds} are the same and the total voltage is $V_{ds} = V_{ds1} + V_{ds2}$. Therefore,

$$V_{ds} = \frac{I_{ds}}{\mu_{eff} C_{ox} W}\left(\frac{L_1}{V_g - V_{on1}} + \frac{L_2}{V_g - V_{on2}}\right).$$

Since $V_{on1} \approx V_{on2}$, one considers only the first-order terms of $(V_{on1} - V_{on2})$ by letting $V_{on2} = V_{on1} + \delta$ and expanding the second term in a power series of δ:

$$V_{ds} \approx \frac{I_{ds}}{\mu_{eff} C_{ox} W}\left(\frac{L_1 + L_2}{V_g - V_{on1}} + \frac{L_2}{(V_g - V_{on1})^2}\delta\right) = \frac{I_{ds}}{\mu_{eff} C_{ox} W}\frac{L_1 + L_2}{V_g - V_{on1}}\left(1 + \frac{L_2/(L_1 + L_2)}{V_g - V_{on1}}\delta\right).$$

This, in turn, can be approximated by

$$V_{ds} \approx \frac{I_{ds}}{\mu_{eff} C_{ox} W}\frac{(L_1 + L_2)/(V_g - V_{on1})}{1 - \frac{L_2/(L_1 + L_2)}{V_g - V_{on1}}\delta} = \frac{I_{ds}}{\mu_{eff} C_{ox} W}\frac{L_1 + L_2}{V_g - V_{on1} - (V_{on2} - V_{on1})L_2/(L_1 + L_2)},$$

which can be written as

$$V_{ds} = \frac{I_{ds}}{\mu_{eff} C_{ox} W}\frac{L_1 + L_2}{V_g - V_{on}},$$

where the equivalent linear threshold voltage is $V_{on} = (L_1 V_{on1} + L_2 V_{on2})/(L_1 + L_2)$.

The energy dissipated in R is

$$E = \int_0^\infty RI^2 dt = \frac{V_{dd}{}^2}{R}\int_0^\infty e^{-2t/RC} dt = \frac{1}{2}CV_{dd}{}^2,$$

which is independent of R. The same amount of energy is stored in C.

If the voltage source is now switched to 0, one has

$$V(t) + RC\frac{dV}{dt} = 0.$$

With the initial condition $V(t = 0) = V_{dd}$, the solution for $V(t)$ is

$$V(t) = V_{dd}e^{-t/RC}.$$

The energy stored in C is now all dissipated in R:

$$E = \int_0^\infty RI^2 dt = \frac{V_{dd}{}^2}{R}\int_0^\infty e^{-2t/RC} dt = \frac{1}{2}CV_{dd}{}^2.$$

5.3. From Eq. (3.25) and the inversion charge expression above Eq. (3.58), the transit time is

$$\tau_{tr} = \frac{WLC_{ox}(V_g - V_t - mV_{ds}/2)}{\mu_{eff}C_{ox}(W/L)(V_g - V_t - mV_{ds}/2)V_{ds}} = \frac{L^2}{\mu_{eff}V_{ds}}$$

for a MOSFET device biased in the linear region.

From Eq. (3.28) and the inversion charge expression above Eq. (3.60), the transit time is

$$\tau_{tr} = \frac{(2/3)WLC_{ox}(V_g - V_t)}{\mu_{eff}C_{ox}(W/L)(V_g - V_t)^2/2m} = \frac{4mL^2}{3\mu_{eff}(V_g - V_t)}$$

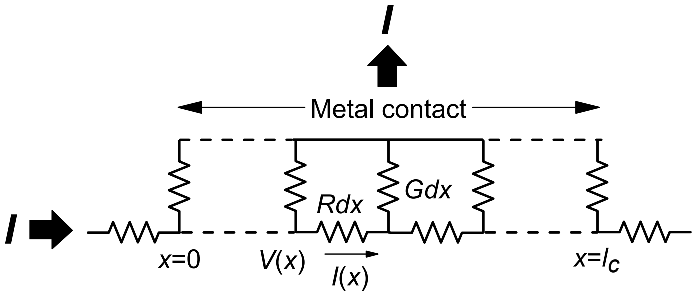
for a long-channel MOSFET biased in saturation.

5.4. From Eq. (3.79) and the inversion charge expression in Exercise 3.10, the transit time is

$$\tau_{tr} \equiv \frac{Q_i}{I_{dsat}} = \frac{L}{v_{sat}} \frac{\sqrt{1+2\mu_{eff}(V_g - V_t)/(mv_{sat}L)} + 1/3}{\sqrt{1+2\mu_{eff}(V_g - V_t)/(mv_{sat}L)} - 1}$$

for a short-channel MOSFET biased in saturation. The limiting value of τ_{tr} is L/v_{sat} when the device becomes fully velocity saturated as $L \to 0$.

5.5. The *transmission line model* of contact resistance in a planar geometry is represented by the distributed network below. The current flows from a thin resistive film (diffusion with a sheet resistivity ρ_{sd}) into a ground plane (metal) with an interfacial contact resistivity ρ_c between them (Fig. 5.16).



Following a similar approach as in Eqs. (5.23)-(5.25), one can write

$$V(x+dx) - V(x) = \frac{dV}{dx}dx = -I(x)Rdx,$$

and

$$I(x+dx) - I(x) = \frac{dI}{dx}dx = -V(x)Gdx.$$

Here $R = \rho_{sd}/W$ and $G = W/\rho_c$. From the above two equations, one obtains

$$\frac{d^2 f}{dx^2} = RGf = \frac{\rho_{sd}}{\rho_c} f,$$

where $f(x) = V(x)$ or $I(x)$.

5.6. The solution to the second-order differential equation in the above exercise is of the $\sinh[(\rho_{sd}/\rho_c)^{1/2}x]$ and $\cosh[(\rho_{sd}/\rho_c)^{1/2}x]$ form. With the boundary condition $I(x=l_c) = 0$ where $x = 0$ is the leading edge and $x = l_c$ is the far end of the contact window (see figure in Exercise 5.5), the solution is

$$I(x) = I_0 \sinh\left[\sqrt{\frac{\rho_{sd}}{\rho_c}}(l_c - x)\right],$$

where I_0 is a constant multiplying factor. From the second equation in Exercise 5.5, the voltage is

$$V(x) = -\frac{\rho_c}{W}\frac{dI}{dx} = \frac{\sqrt{\rho_c \rho_{sd}}}{W} I_0 \cosh\left[\sqrt{\frac{\rho_{sd}}{\rho_c}}(l_c - x)\right].$$

The contact resistance is then

$$R_{co} = \frac{V(x=0)}{I(x=0)} = \frac{\sqrt{\rho_c \rho_{sd}}}{W} \coth\left[l_c \sqrt{\frac{\rho_{sd}}{\rho_c}}\right].$$

5.7. From Eqs. (5.43) and (5.45), $\tau_{b\min} < \tau$ if

$$C_{out} + C_L > 2C_{out} + 2\sqrt{C_{in} C_L}\,.$$

This inequality is quadratic in $C_L^{1/2}$, which can be solved to yield

$$C_L > \left(\sqrt{C_{in}} + \sqrt{C_{in} + C_{out}}\right)^2.$$

Under this condition, the insertion of one or a few properly designed buffer stage(s) will help reduce the overall delay.

5.11.

Power = $CV_{dd}^2 f$ where $f = 1/(2\tau)$ and $C = C_{in}+C_{out}+C_L$

So Power = $V_{dd}^2/2R_{sw}$

If $(W_n, W_p) \rightarrow (kW_n, kW_p)$,

$R_{sw} \rightarrow R_{sw}/k$

$C_{in}+C_{out} \rightarrow k(C_{in}+C_{out})$

Therefore,

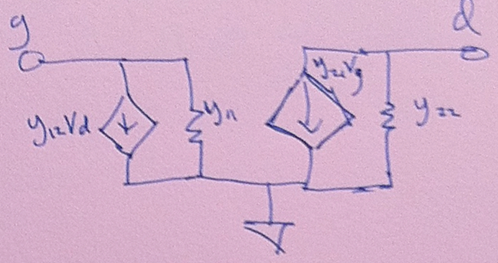
Delay $\tau \rightarrow R_{sw}(C_{in}+C_{out}+C_L/k)$

Power $\rightarrow$ k times original value.

Problem #3

$$\begin{pmatrix} i_g \\ i_d \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} v_g \\ v_d \end{pmatrix}$$

equiv. to



Compare to HW prob. #3 circuit diagram $\Rightarrow$

$y_{12} = 0 \qquad y_{21} = g_m$

$y_{22} = \frac{1}{r_o} \qquad y_{11} = (i\omega C_{gs}) + \frac{1}{r_{leak}}$

From table $\quad h_{21} = \frac{y_{21}}{y_{11}}$

$$\Rightarrow h_{21} = \frac{g_m}{i\omega C_{gs} + \frac{1}{r_{leak}}}$$

If r_{leak} large $\Rightarrow i\omega C_{gs} \gg \frac{1}{r_{leak}}$

$$\Rightarrow h_{21} \simeq \frac{g_m}{i\omega C_{gs}} \Rightarrow \boxed{F_T \simeq \frac{g_m}{2\pi C_{gs}}}$$

$\boxed{F_{max} = \infty.}$ Why? At any frequency we can design an impedance matching network at the input and output to achieve power gain. Therefore there is no upper limit on frequency for power gain.

Another way to see this is to calculate $U = \frac{|y_{21} - y_{12}|^2}{4(\mathrm{Re}Y_{11}\,\mathrm{Re}Y_{22} - \mathrm{R}Y_{12}\,\mathrm{R}Y_{21})}$

$U = \frac{g_m^2}{4 \frac{1}{r_{leak}} \frac{1}{r_o}}$ <u>independent of ω</u> $\Rightarrow U$ does not decrease with ω.

$\Rightarrow F_{max} = \infty$