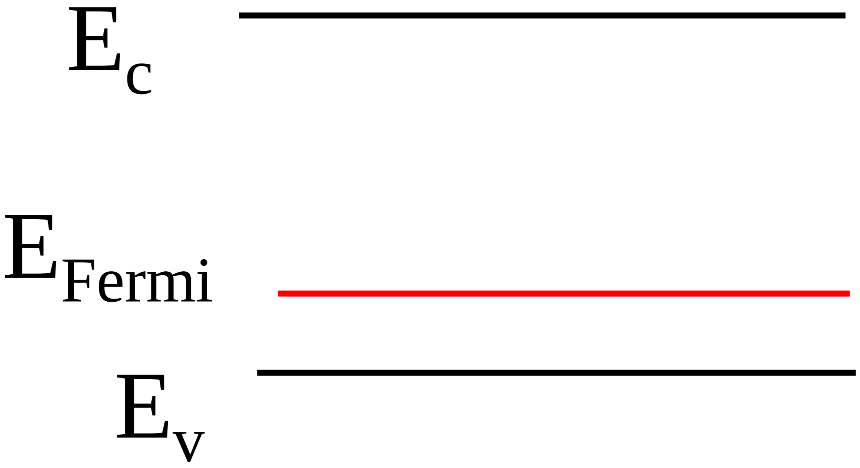
Lecture 3:
Two-terminal devices

Fermi levels



E_c

E_{Fermi}

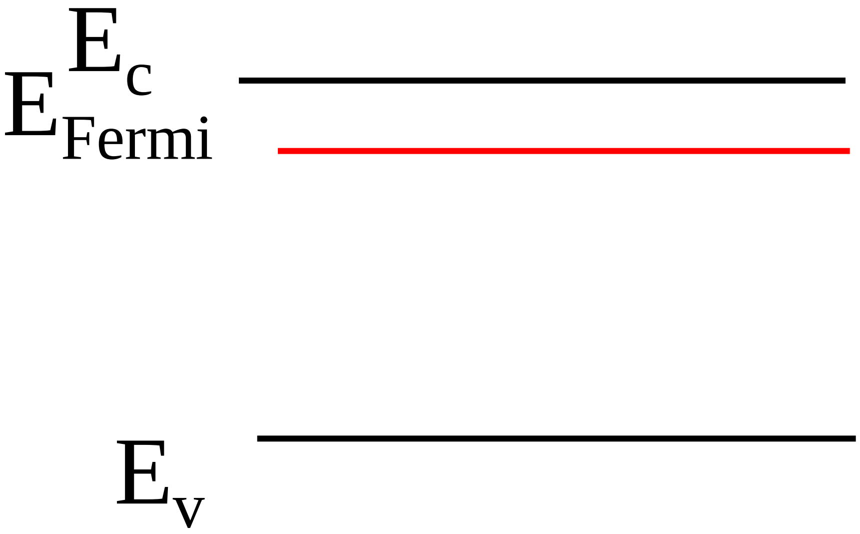
E_v

p-type

Fermi levels

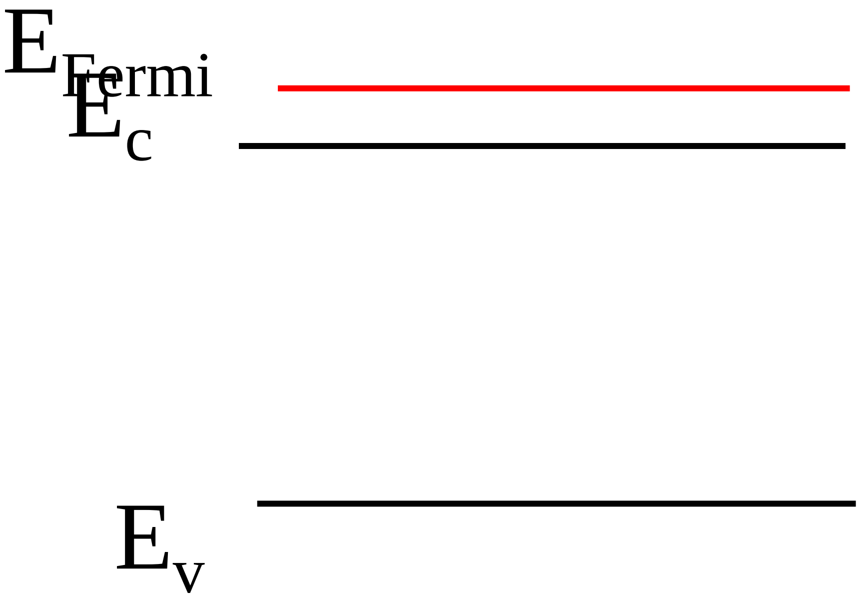
n-type

Fermi levels



n-type

Degenerate doping



E_{Fermi}

E_c

E_v

n-type

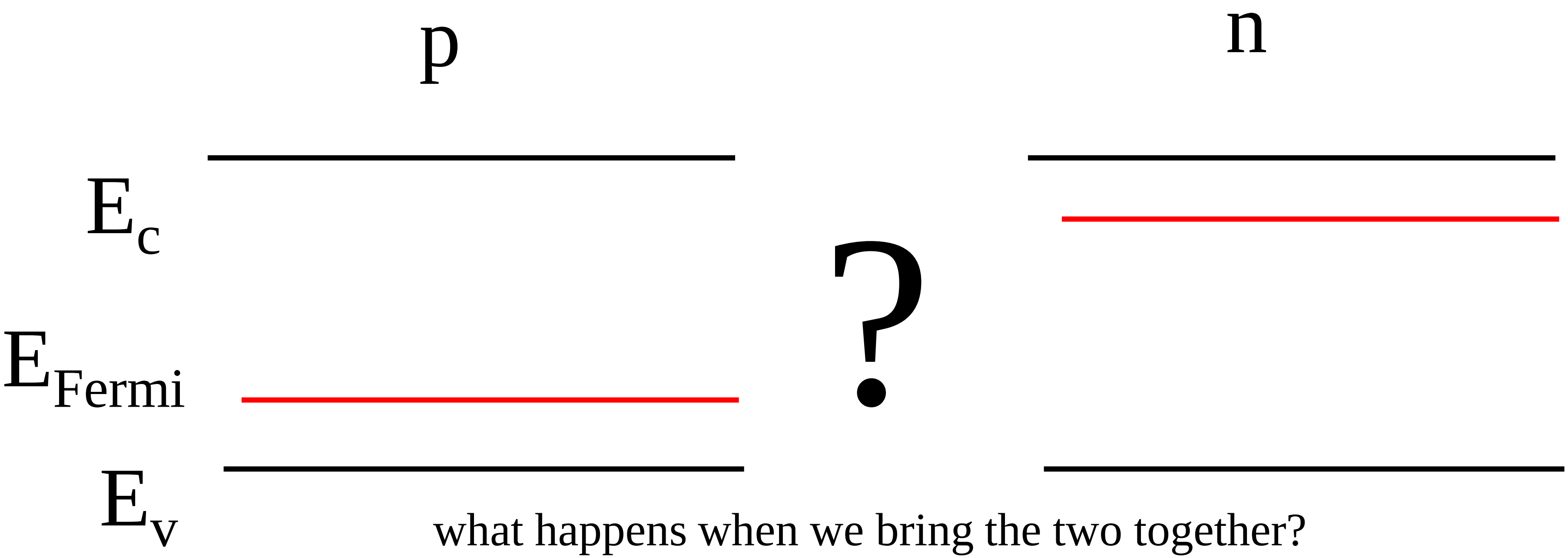
Invariance of Fermi level

- Fermi level must be constant if system is in equilibrium (no external voltage or force)
- Discuss on board

Junction types

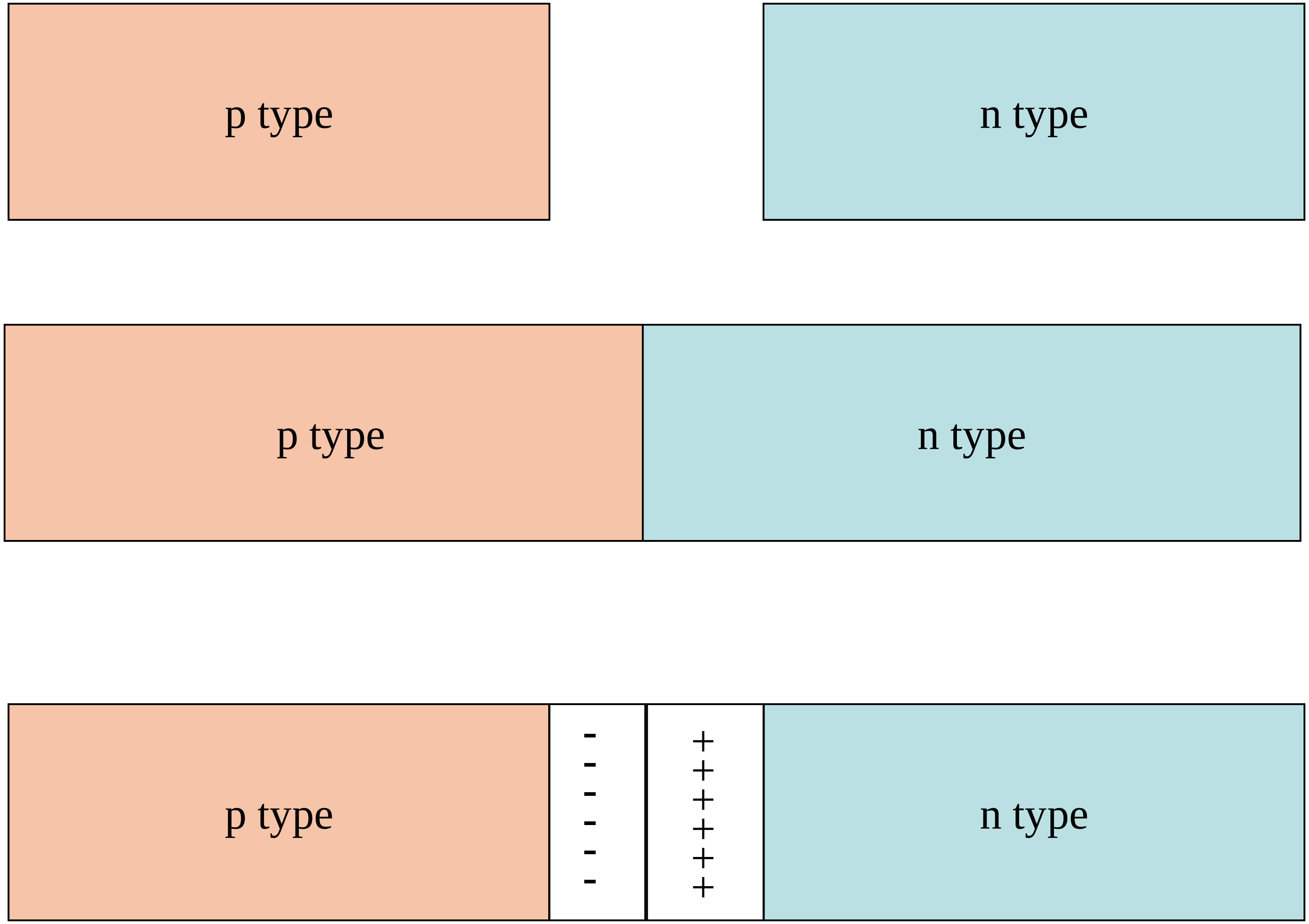
- Homojunction
- Heterojunction

p-n junction



what happens when we bring the two together?
Electrons move from n to p
Holes move from p to n

p-n junction



p type

n type

p type

n type

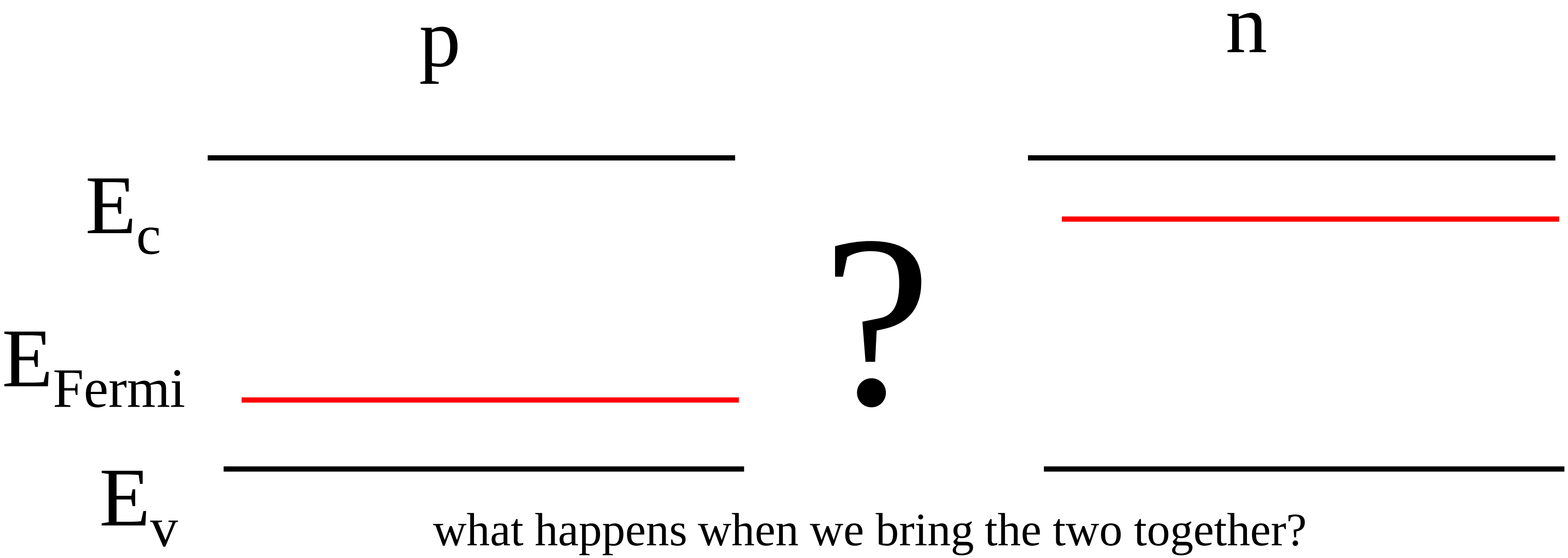
p type

- - - - - -

\+ + + + + +

n type

p-n junction



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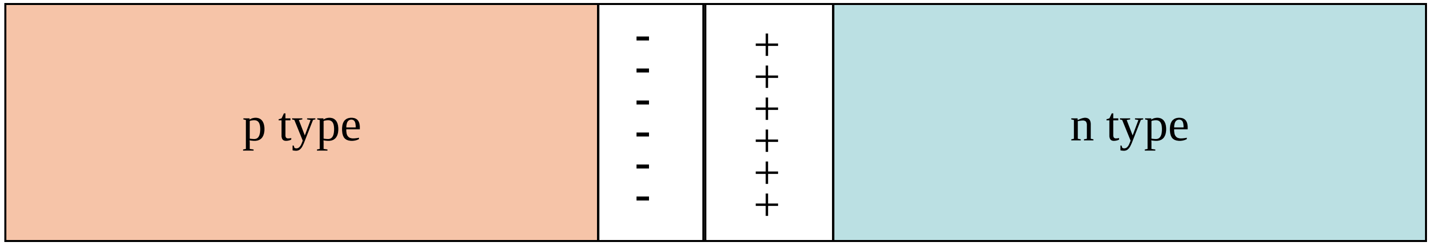
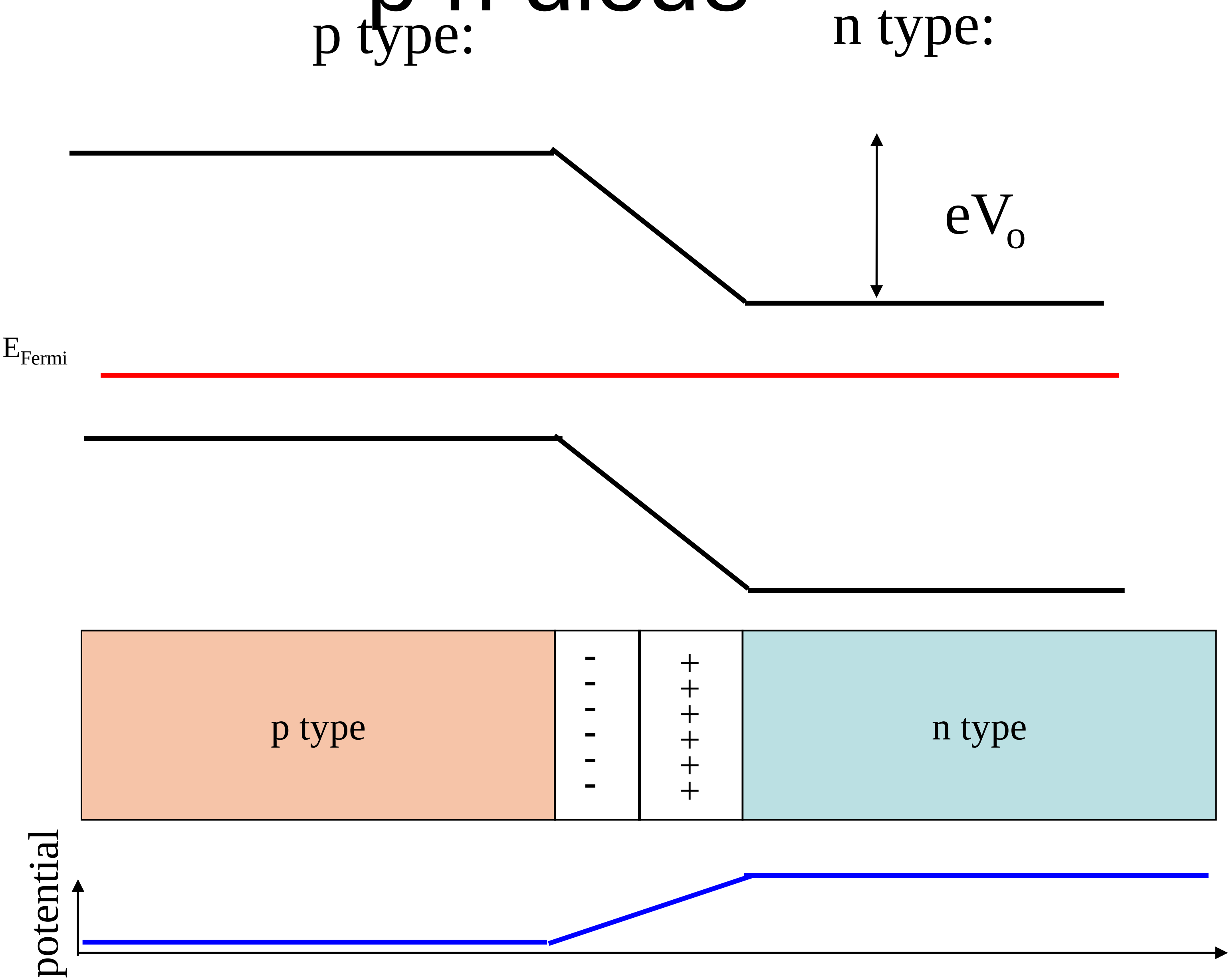


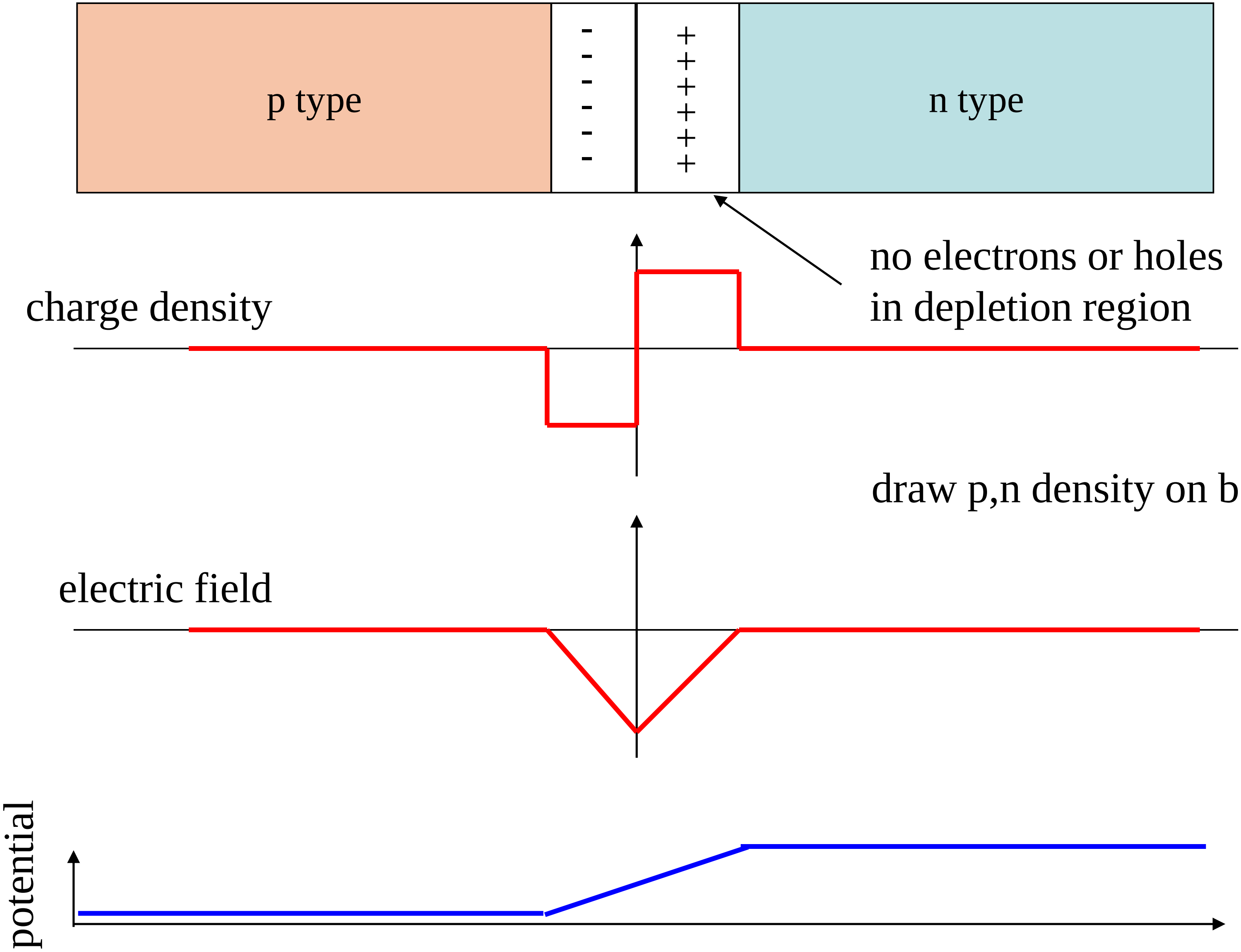
How to deal with?

- Away from junction, same as bulk
- In “depletion region” or “space charge region” no carriers
- But, there is an electric field which causes potential energy difference
- “Band bending” method: shift all levels of p side relative to n side
- Rule: align Fermi levels

p n diode



p n diode



no electrons or holes
in depletion region

draw p,n density on board

Poisson equation

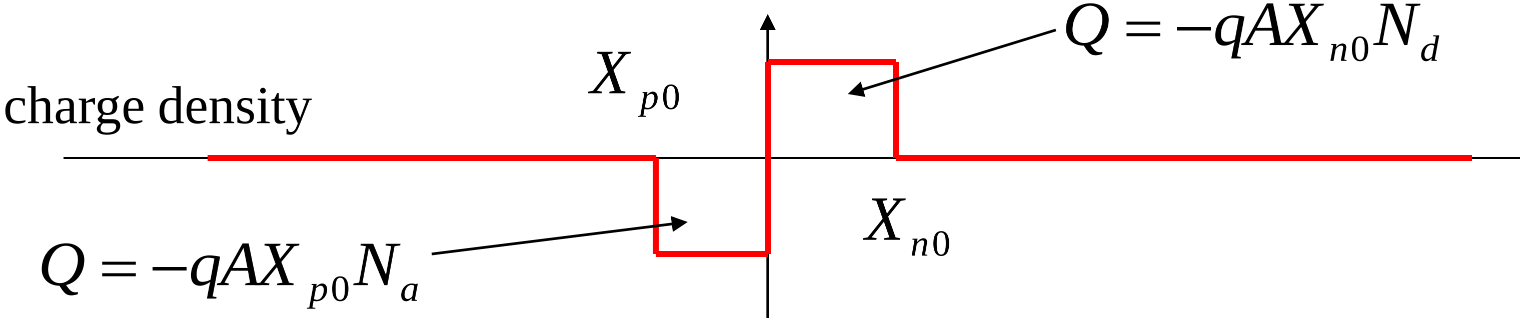
Actually first of Maxwell's four equations:

$$\vec{\nabla} \cdot \vec{E} = -\frac{\rho}{\varepsilon}$$

In the x-direction only:

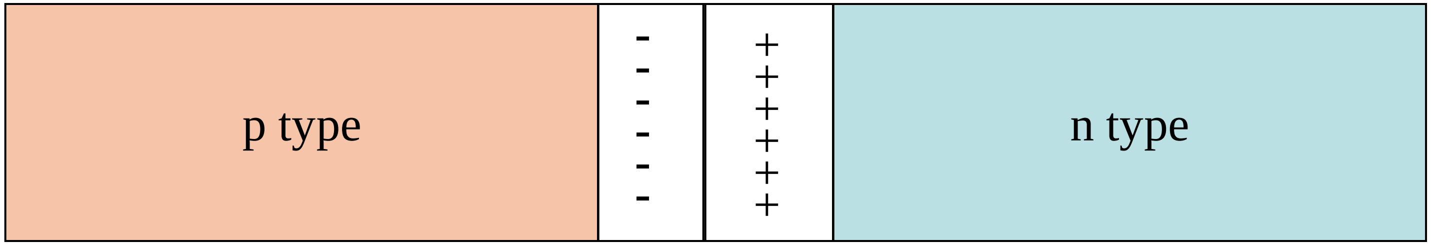
$$\frac{dE}{dx} = \frac{q}{\varepsilon_p}\left(p - n + N_d^+ - N_a^-\right)$$

Charge density

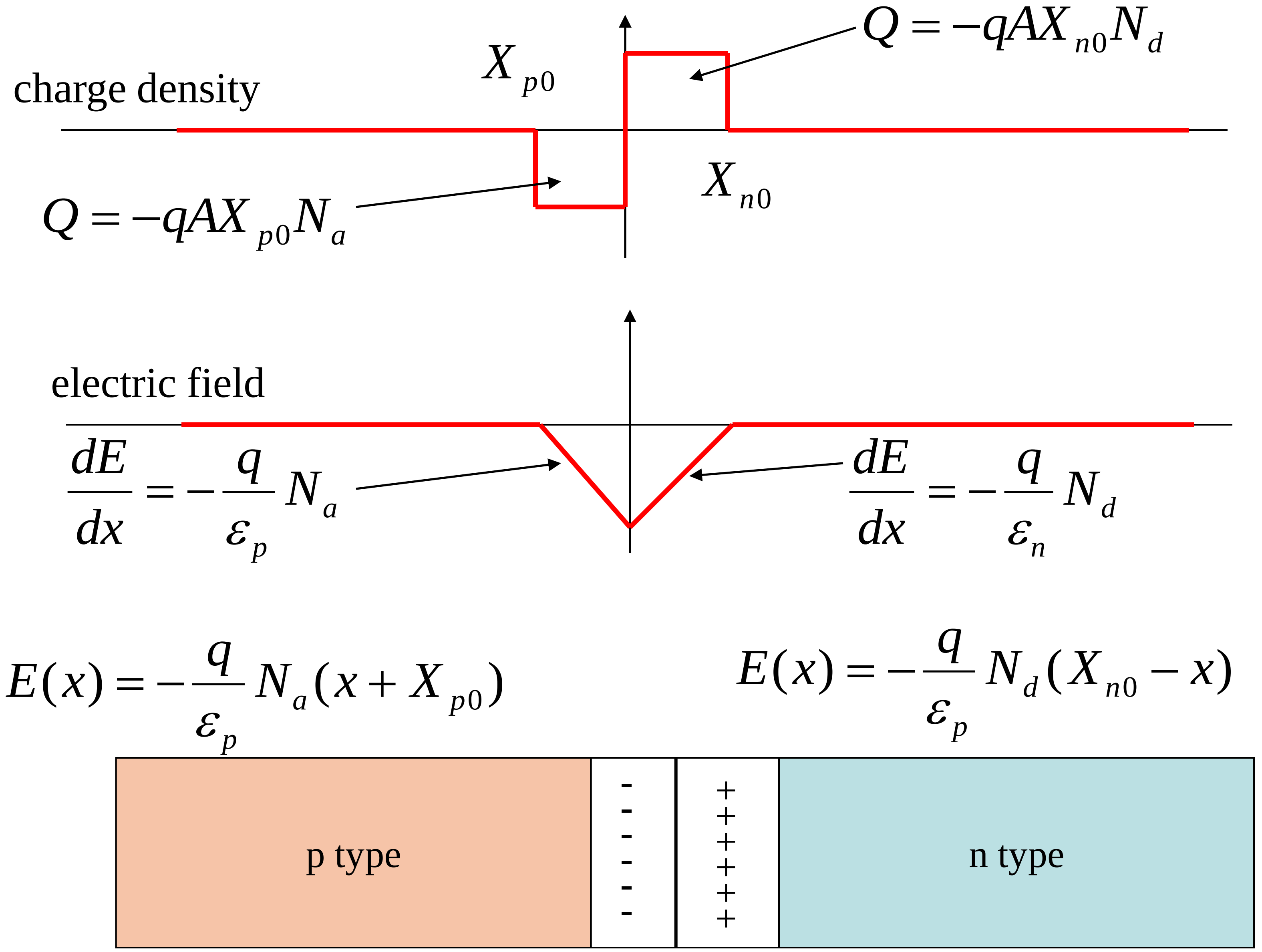


$$Q = -qAX_{n0}N_d$$

$$Q = -qAX_{p0}N_a$$



Electric field



charge density

X_{p0}

X_{n0}

$$Q = -qAX_{n0}N_d$$

$$Q = -qAX_{p0}N_a$$

electric field

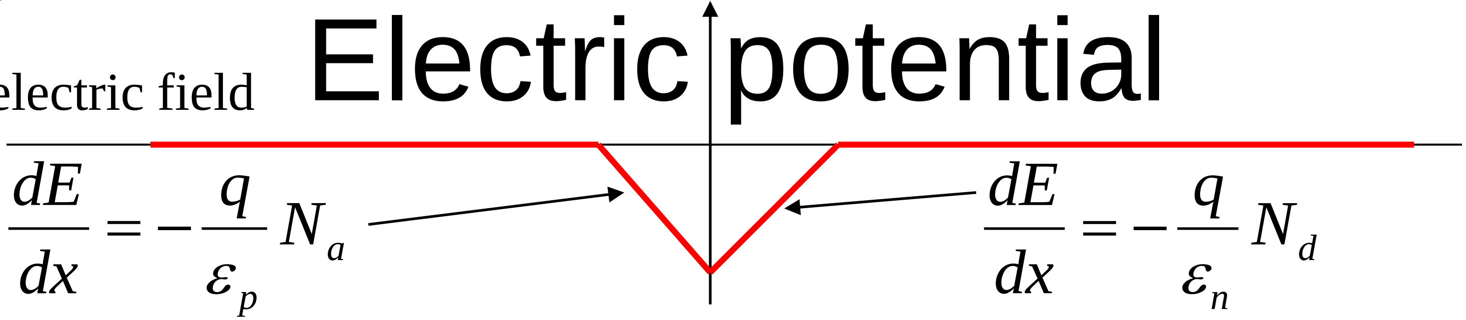
$$\frac{dE}{dx} = -\frac{q}{\varepsilon_p} N_a$$

$$\frac{dE}{dx} = -\frac{q}{\varepsilon_n} N_d$$

$$E(x) = -\frac{q}{\varepsilon_p} N_a (x + X_{p0})$$

$$E(x) = -\frac{q}{\varepsilon_p} N_d (X_{n0} - x)$$

Electric potential



$$\frac{dE}{dx} = -\frac{q}{\varepsilon_p} N_a$$

$$\frac{dE}{dx} = -\frac{q}{\varepsilon_n} N_d$$

$$E(x) = -\frac{q}{\varepsilon_p} N_a (x + X_{p0})$$

$$E(x) = -\frac{q}{\varepsilon_p} N_d (X_{n0} - x)$$

$$V(x) = -\int E(x)dx = \frac{q}{\varepsilon_p} N_a (\frac{x^2}{2} + X_{p0}x) + C$$

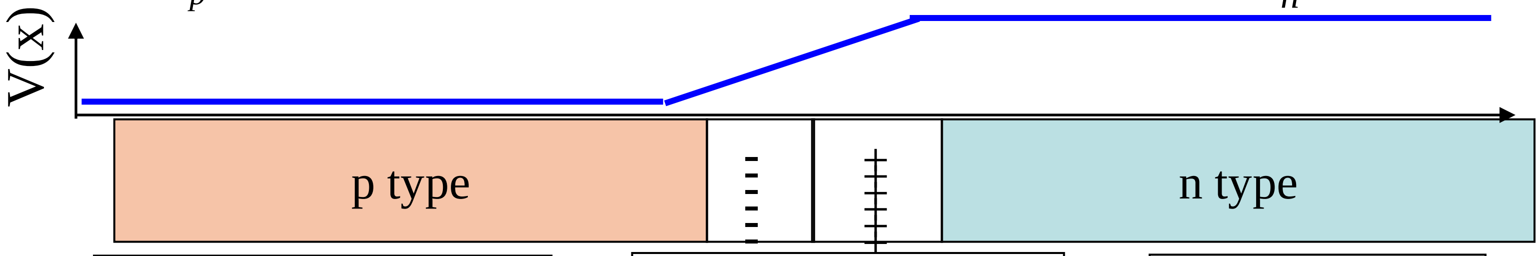
Define zero of V(x) at p side

$$V(x) = \frac{q}{\varepsilon_p} N_a \left(\frac{x^2}{2} + X_{p0}x + \frac{X_{p0}{}^2}{2} \right)$$

$$V(x) = \frac{q}{\varepsilon_n} N_d \left(X_{n0}x - \frac{x^2}{2} \right) + \phi_{p0}$$

$$V(0) = \frac{q}{\varepsilon_p} N_a \frac{X_{p0}{}^2}{2} \equiv \phi_{p0}$$

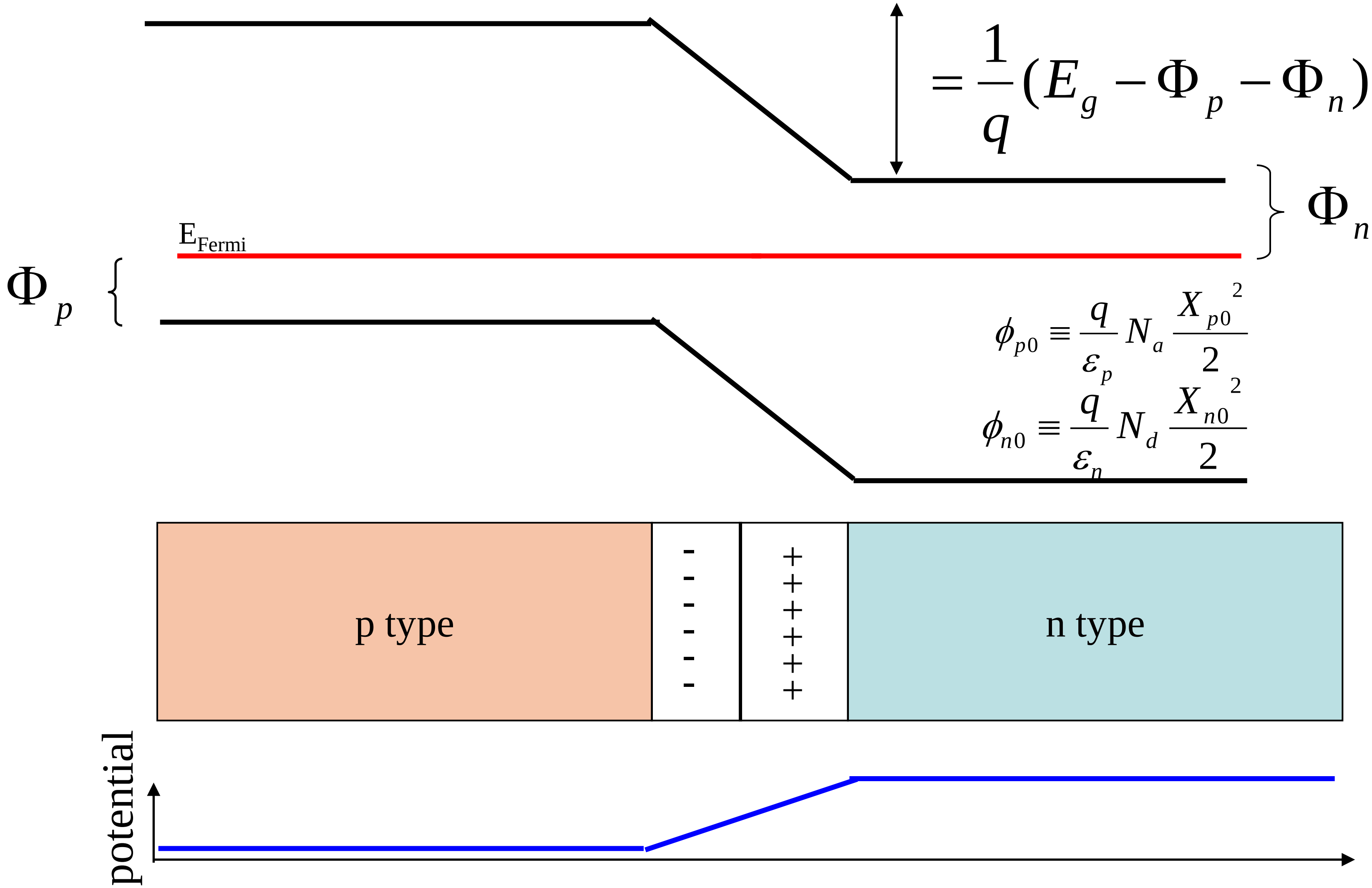
$$\phi_{n0} \equiv V(x) = \frac{q}{2\varepsilon_n} N_d X_{n0}{}^2$$



built in potential

$$\phi_{bi} \equiv \phi_{p0} + \phi_{n0}$$

$$= \frac{1}{q}(E_g - \Phi_p - \Phi_n)$$



$$\phi_{p0} \equiv \frac{q}{\varepsilon_p} N_a \frac{X_{p0}^{\;2}}{2}$$

$$\phi_{n0} \equiv \frac{q}{\varepsilon_n} N_d \frac{X_{n0}^{\;2}}{2}$$

Depletion region width

$$\phi_{bi} \equiv \phi_{p0} + \phi_{n0} = \frac{1}{q}(E_g - \Phi_p - \Phi_n)$$

$$\phi_{p0} \equiv \frac{q}{\varepsilon_p} N_a \frac{X_{p0}^2}{2} \qquad \phi_{n0} \equiv \frac{q}{\varepsilon_n} N_d \frac{X_{n0}^2}{2}$$

$$Q_p = -qAX_{p0}N_a \qquad Q_n = -qAX_{n0}N_d$$

$$Q_p = Q_n \Rightarrow qAX_{p0}N_a = -qAX_{n0}N_d$$

$$\Rightarrow$$

$$W = \sqrt{\frac{2\varepsilon\phi_{bi}}{q}\left(\frac{1}{N_d} + \frac{1}{N_a}\right)}$$

Numerical example

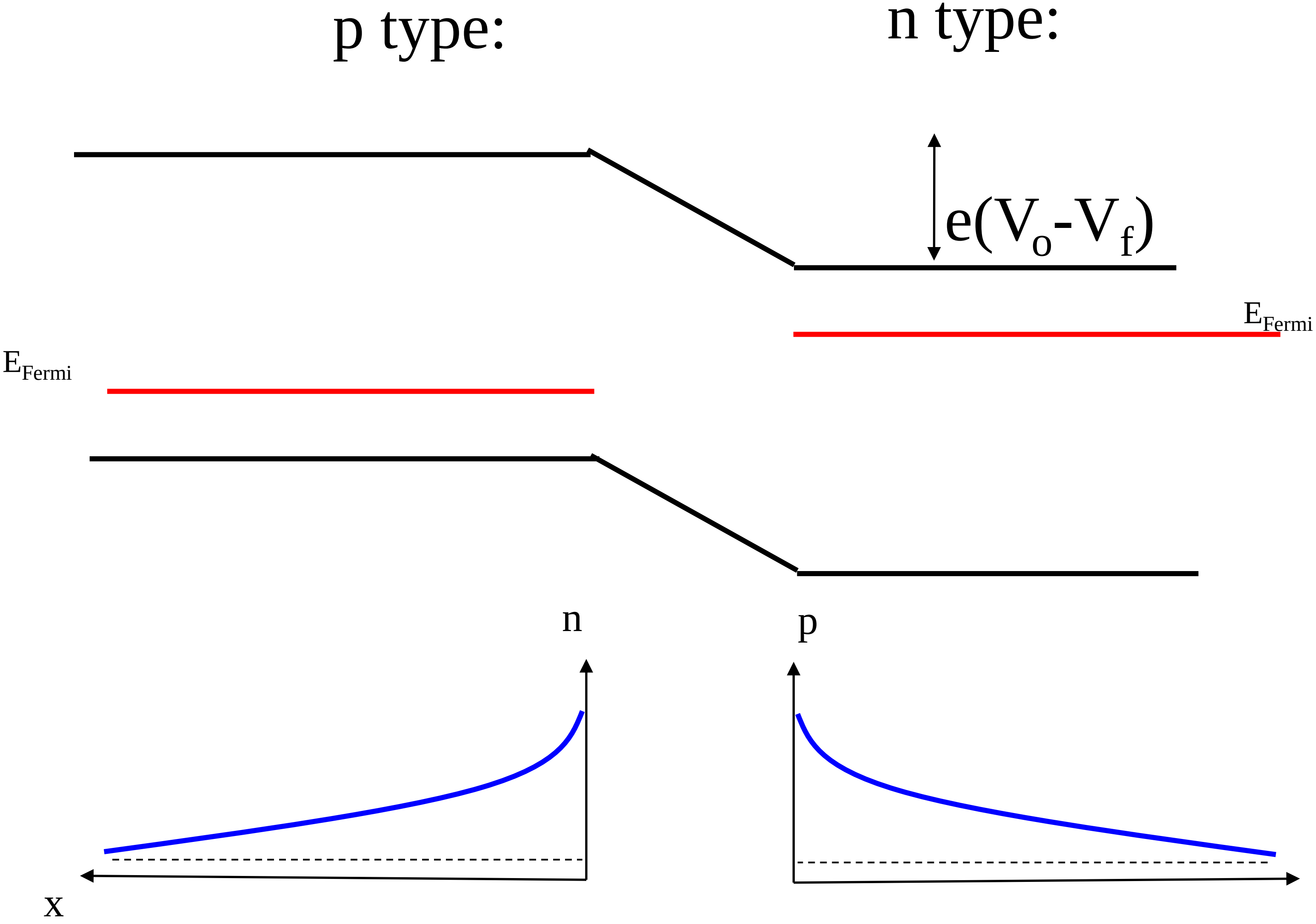
$$N_d = 10^{16} cm^{-3}$$

$$N_a = 10^{18} cm^{-3}$$

$$\Rightarrow W = X_{p0} + X_{n0} = 0.33 \mu m$$

$$\Rightarrow X_{p0} = 8.3 A$$

Forward bias

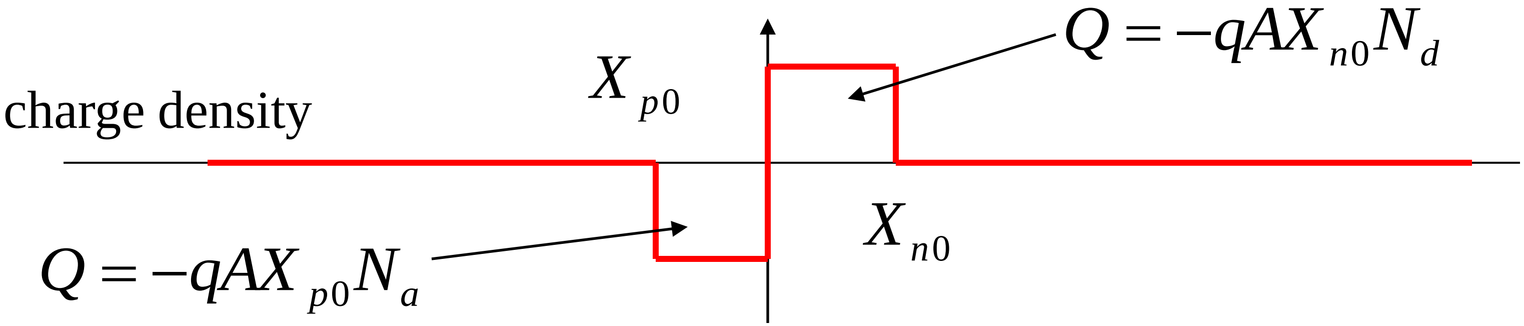


Discuss current vs. x

$$j_n=dn/dx$$

$$j=j_n+j_p$$

Capacitance



$$X_{p0} = \sqrt{\frac{2\varepsilon\phi_{bi}}{q}\frac{1}{N_a(1+N_a/N_d)}} \rightarrow \sqrt{\frac{2\varepsilon(\phi_{bi}-V)}{q}\frac{1}{N_a(1+N_a/N_d)}}$$

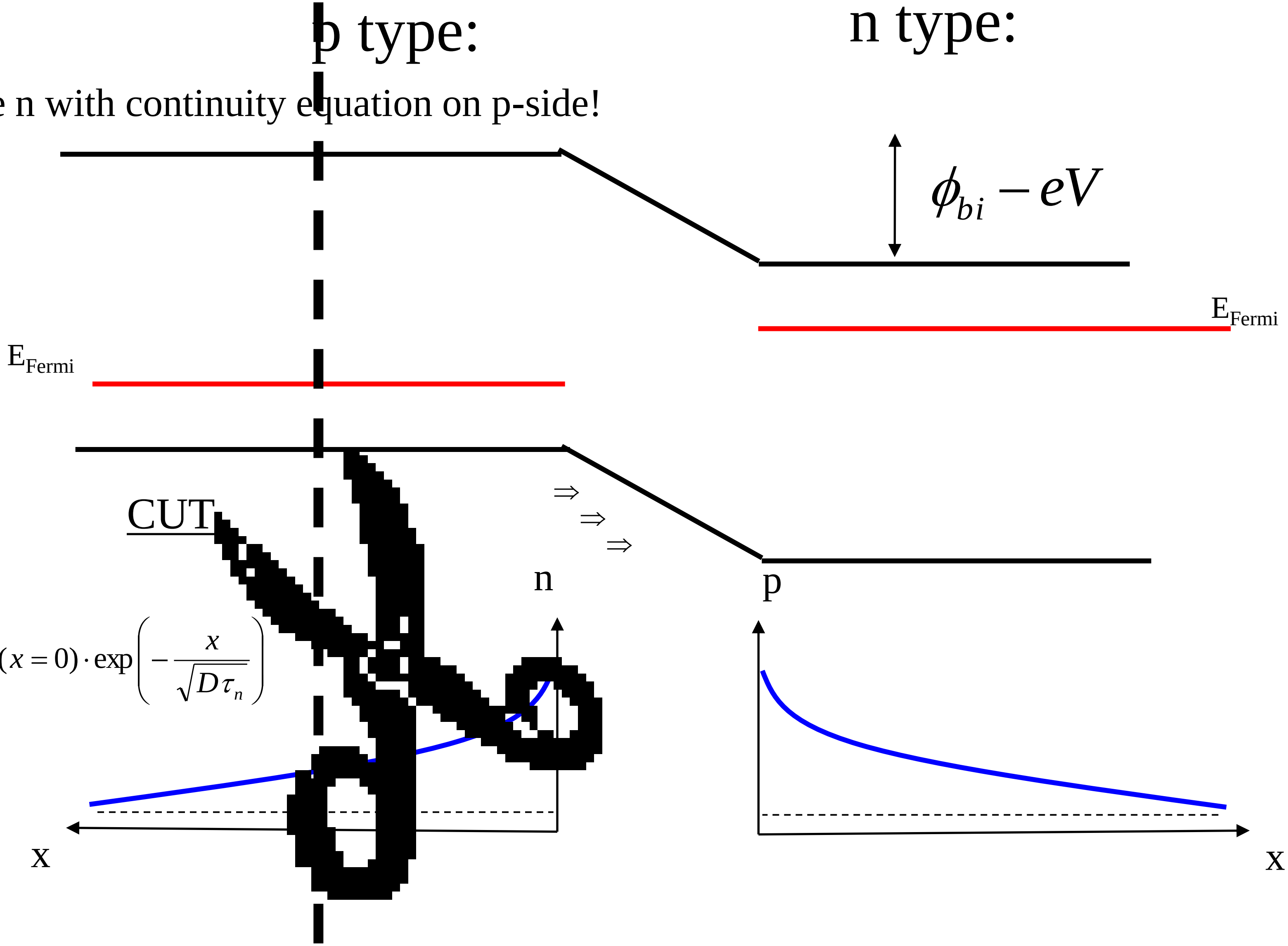
$$Q = -qAX_{p0}N_a = -qAN_a\sqrt{\frac{2\varepsilon(\phi_{bi}-V)}{q}\frac{1}{N_a(1+N_a/N_d)}}$$

$$C = \frac{dQ}{dV} = \frac{A}{2}\sqrt{\frac{2q\varepsilon}{(\phi_{bi}-V)}\frac{N_dN_a}{(N_d+N_a)}}$$

This will be important in high-speed bipolar transistors.

“Short” diodes

Calculate n with continuity equation on p-side!



$$\delta n(x) = \delta n(x=0) \cdot \exp\left(-\frac{x}{\sqrt{D\tau_n}}\right)$$

Exactly what we will do with bipolar transistors!

"Short" diodes

$$D\frac{\partial^2 \delta n}{\partial x^2} = \frac{\delta n}{\tau} \qquad \Rightarrow \delta n(x) = \delta n(x=0)\cdot \exp\left(-\frac{x}{\sqrt{D\tau_n}}\right)$$

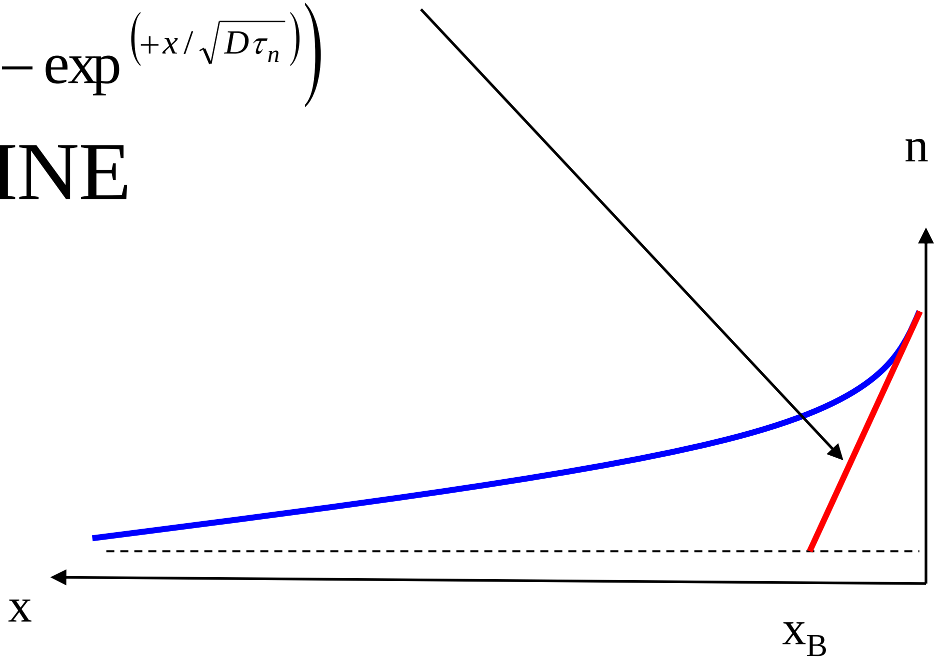
New boundary condition: $\delta n(x = x_B) = 0$

$$\delta n(x) = A\exp\left(-\frac{x}{\sqrt{D\tau_n}}\right) + B\exp\left(+\frac{x}{\sqrt{D\tau_n}}\right)$$

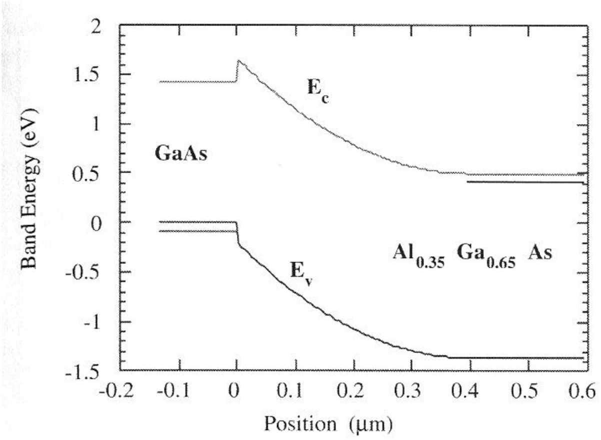
$$\rightarrow (prefactor)\left(\exp^{\left(-x/\sqrt{D\tau_n}\right)} - \exp^{\left(+x/\sqrt{D\tau_n}\right)}\right)$$

$\approx$ A STRAIGHT LINE

$$If\ \mathrm{x_N} < \sqrt{D\tau_n}$$



p^+ N I-V curve



From Liu

As in p-n homojunction:
Diffusion current: Electrons with energy > eV_0
Exponentially increased. (Explain).
Drift current :not really effected.

$$I = I_{diffusion} - |I_{drift}|$$

exponentially larger

about the same

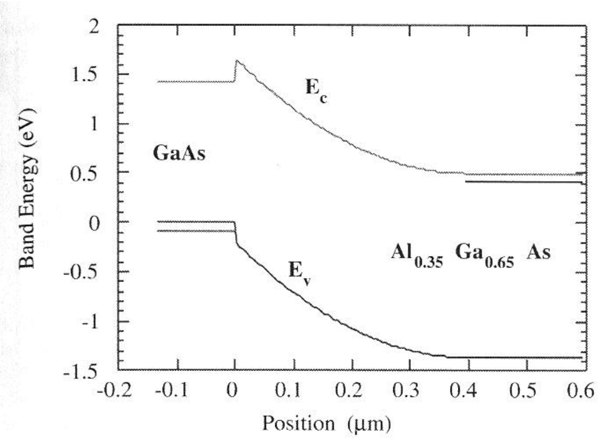
$$\boxed{I = I_0\left(e^{qV/kT} - 1\right) \rightarrow I_0\left(e^{qV_n/kT} - 1\right)}$$

But only voltage drop across N region matters

$$\boxed{I = I_0\left(\exp\left[\frac{qV}{kT}\frac{1}{1+K}\right] - 1\right)}$$

$$K = \frac{\varepsilon_N N_d}{\varepsilon_p N_a}$$

Grated p^+ N I-V curve



From Liu

As in p-n homojunction
but MORE CURRENT

$$I = I_{diffusion} - |I_{drift}|$$

exponentially larger

about the same

$$\boxed{I = I_0\left(e^{qV/kT} - 1\right)}$$

I_0 larger in grated p^+N than in abrupt p^+N.