EECS 70A: Network Analysis

Homework #3

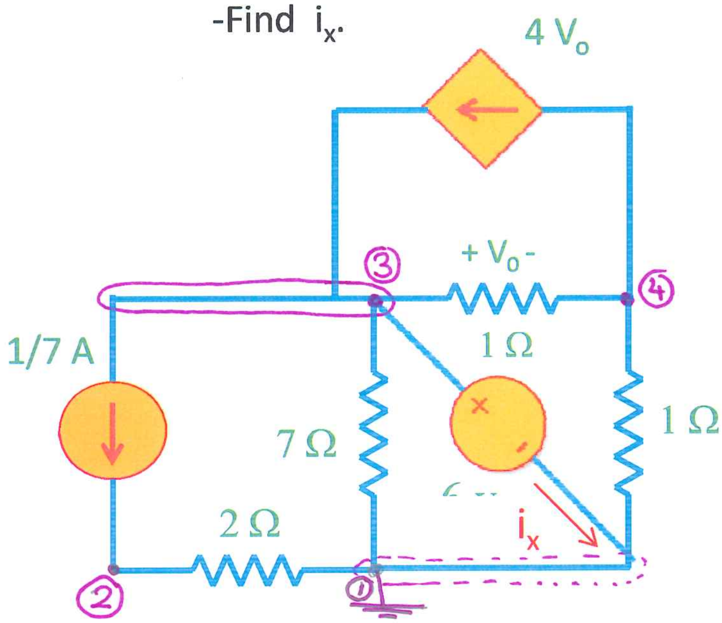
Due in discussion section,

Wednesday, May 5, 2010.

Problem 1:

-Use nodal analysis to find all node voltages.

-Find i_x.



↳ node 1 is the voltage ref.
So $V_1 = 0$ by defenition

Problem1)

① Define the reference node

You can choose any of the 4 nodes for the reference, if you choose node 1 or 3, one of the terminals of the voltage source is grounded and analysis will be easier. If you have chosen a reference other than node 1, your node voltage values might be different than this solution, but you should still get the same voltage drop across all elements and the same values for all currents.

There are 3 unknowns: V_2, V_3 and $V_4 \Rightarrow$ We need 3 independent equations.

② Apply KCL + Ohm's law to nodes:

@ node 4 :

$$\left.\begin{array}{l} \frac{V_4 - V_3}{1\Omega} + \frac{V_4 - \cancel{V_1}^{\,0}}{1\Omega} + 4V_0 = 0 \\ V_0 = V_3 - V_4 \end{array}\right\} \quad V_4 - V_3 + V_4 + 4(V_3 - V_4) = 0$$

$$V_3(-1+4) + V_4(1+1-4) = 0$$

$$3V_3 - 2V_4 = 0 \quad \text{(eq.1)}$$

@ node 2 :

$$\left.\begin{array}{l} \frac{V_2 - V_1}{2\Omega} - \frac{1}{7}A = 0 \\ V_1 = 0 \end{array}\right\} \Rightarrow V_2 = \frac{2}{7}\,v \quad \text{(eq.2)}$$

③ Apply KVL to voltage source: $V_3 = 6v$ (eq. 3)

④ Now that we have the 3 equations, solve them for V_2, V_3 and V_4:

Plug V_3 in equation1 : $3(6v) - 2V_4 = 0$

$$V_4 = \frac{18v}{2} = 9v$$

$$\left\{\begin{array}{l} V_2 = \frac{2}{7}\,v \\ V_3 = 6\,v \\ V_4 = 9\,v \end{array}\right.$$

Prob. 1

- Find i_x.

To find i_x, write a KCL at node 3:

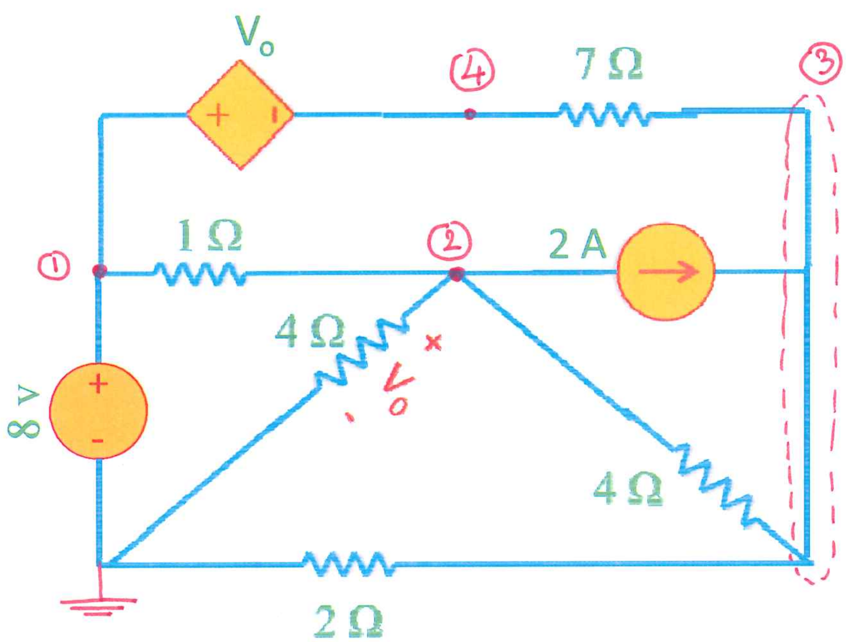
$$\frac{1}{7}A + \frac{V_3}{7\Omega} + i_x + \frac{V_3 - V_4}{1\Omega} - 4V_o = 0$$

$$\frac{1}{7}A + \frac{6v}{1\Omega} + i_x + \frac{6v - 9v}{1\Omega} - 4\left[\frac{A}{V}\right](6v - 9v) = 0$$

$$1A + i_x - 3A + 12A = 0$$

$i_x = -10\ A$

Problem 2: Solve for node voltages.



Problem 2)

V_1, V_2, V_3 and V_4 are the unknowns $\Longrightarrow$ We have to write 4 independent equations.

at node 1 : $V_1 = 8\,v$ (eq. 1)

KCL + Ohm @ node 2 : $\dfrac{V_2 - V_1}{1\,\Omega} + \dfrac{V_2}{4\,\Omega} + \dfrac{V_2 - V_3}{4\,\Omega} + 2\,A = 0$

$$-V_1 + V_2\left(1 + \frac{1}{4} + \frac{1}{4}\right) - \frac{V_3}{4} = -2$$

$\times 4$

$$-4V_1 + 6V_2 - V_3 = -8 \quad \text{(eq. 2)}$$

KCL + Ohm @ node 3 : $\dfrac{V_3 - V_4}{7\,\Omega} + \dfrac{V_3 - V_2}{4\,\Omega} + \dfrac{V_3}{2\,\Omega} - 2\,A = 0$

$$\frac{-V_2}{4\,\Omega} + V_3\left(\frac{1}{7\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{2\,\Omega}\right) - \frac{V_4}{7\,\Omega} = 2\,A$$

$\times 28$

$$-7V_2 + 25V_3 - 4V_4 = 56 \quad \text{(eq. 3)}$$

KVL for the VCVS : $\left.\begin{array}{l} V_1 - V_4 = V_o \\ V_o = V_2 \end{array}\right\} \Rightarrow V_1 - V_4 = V_2 \rightarrow V_1 - V_2 - V_4 = 0$ (eq. 4)

Replace $V_1 = 8\,v$ in eq. 2 & eq. 4 :

(eq. 2) $\rightarrow$ $-4 \times 8 + 6V_2 - V_3 = -8 \Rightarrow 6V_2 - V_3 = 24$ (eq 2*)

(eq. 4) $\rightarrow$ $V_1 - V_2 - V_4 = 0 \rightarrow 8 - V_2 - V_4 = 0 \Rightarrow V_2 + V_4 = 8\,v$ (eq 4*)

(Prob. 2)

Solve equations 4*, 2* and 3 using Cramer's method:

$$\begin{array}{l} \text{(eq 4*)} \\ \text{(eq 2*)} \\ \text{(eq 3)} \end{array} \begin{bmatrix} 1 & 0 & 1 \\ 6 & -1 & 0 \\ -7 & 25 & -4 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 24 \\ 56 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 6 & -1 & 0 \\ -7 & 25 & -4 \end{vmatrix} = 1 \times \begin{vmatrix} -1 & 0 \\ 25 & -4 \end{vmatrix} + 0 \times \begin{vmatrix} 6 & 0 \\ -7 & -4 \end{vmatrix} + 1 \times \begin{vmatrix} 6 & -1 \\ -7 & 25 \end{vmatrix}$$

$$= 1 \times (4 - 0) + 0 + 1 \times (6 \times 25 - 7) = 147$$

$$\Delta_1 = \begin{vmatrix} 8 & 0 & 1 \\ 24 & -1 & 0 \\ 56 & 25 & -4 \end{vmatrix} = 688$$

$$\Delta_2 = \begin{vmatrix} 1 & 8 & 1 \\ 6 & 24 & 0 \\ -7 & 56 & -4 \end{vmatrix} = 600$$

$$\Delta_3 = \begin{vmatrix} 1 & 0 & 8 \\ 6 & -1 & 24 \\ -7 & 25 & 56 \end{vmatrix} = 488$$

$$V_2 = \frac{\Delta_1}{\Delta} = \frac{688}{147} = 4.68 \text{ v}$$

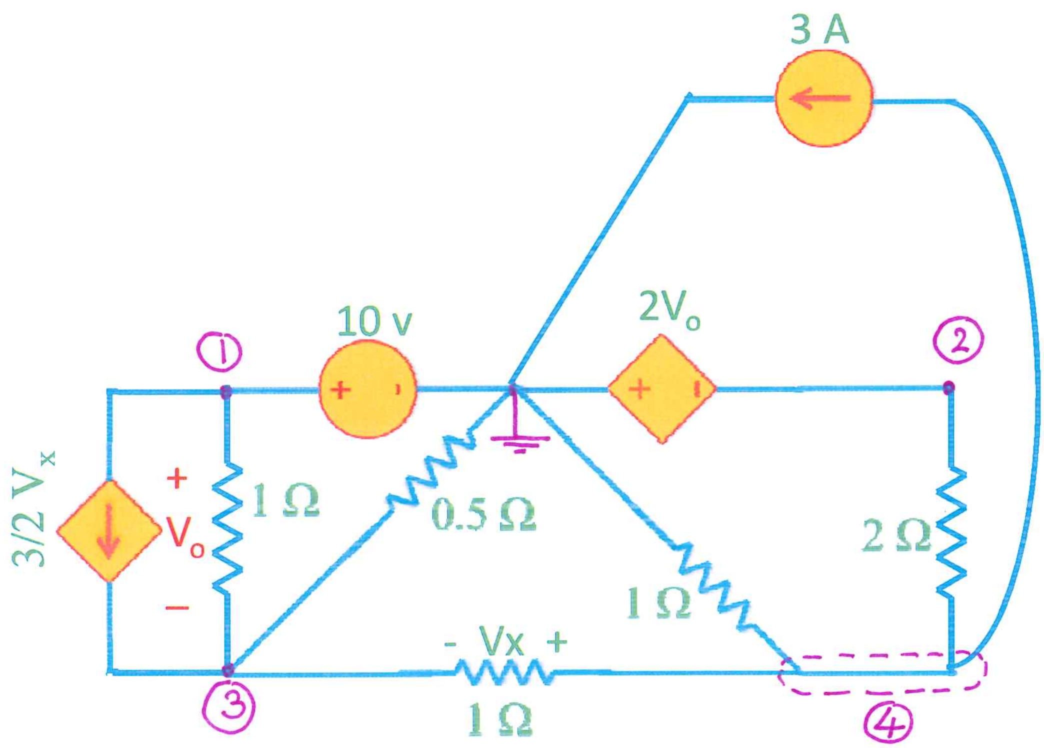
$$V_3 = \frac{\Delta_2}{\Delta} = \frac{600}{147} = 4.08 \text{ v}$$

$$V_4 = \frac{\Delta_3}{\Delta} = \frac{488}{147} = 3.319 \text{ v}$$

Problem 3:

-Use nodal analysis to find node voltages.

-Find the power supplied by the independent current source.



(Prob. 3)

Write equations 2*, 3* & 4 in Matrix form:

$$\begin{array}{c} 2* \\ 3* \\ 4 \end{array} \begin{bmatrix} 1 & -2 & 0 \\ 0 & 11 & -5 \\ 1 & 2 & -5 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -20 \\ 20 \\ 6 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 0 & 11 & -5 \\ 1 & 2 & -5 \end{vmatrix} = -35 \qquad \Delta_1 = \begin{vmatrix} -20 & -2 & 0 \\ 20 & 11 & -5 \\ 6 & 2 & -5 \end{vmatrix} = 760$$

$$\Delta_2 = \begin{vmatrix} 1 & -20 & 0 \\ 0 & 20 & -5 \\ 1 & 6 & -5 \end{vmatrix} = 30 \qquad \Delta_3 = \begin{vmatrix} 1 & -2 & -20 \\ 0 & 11 & 20 \\ 1 & 2 & 6 \end{vmatrix} = 206$$

$$V_2 = \frac{\Delta_1}{\Delta} = \frac{760}{-35} = -21.7 \text{ v}$$

$$V_3 = \frac{\Delta_2}{\Delta} = \frac{30}{-35} = -0.857 \text{ v}$$

$$V_4 = \frac{\Delta_3}{\Delta} = \frac{206}{-35} = -5.88 \text{ v}$$

- Find the current supplied by the VCCS:

$$P = V_x I = V_o(-3/2\, V_x)$$

$$V_o = V_1 - V_3 = 10 - (-0.857) = 10.857 \text{ v}$$

$$V_x = V_4 - V_3 = -5.88 - (-0.857) = -5.03 \text{ v}$$

$$P = (10.857)(-\tfrac{3}{2} \times -5.03) = 81.91 \text{ W}$$

Problem 3)

@ node 1: $\underline{V_1 = 10\,v}$ $\underline{(eq.1)}$

@ node 2: $\left.\begin{matrix} V_2 = -2V_o \\ V_o = V_1 - V_3 \end{matrix}\right\} \Rightarrow V_2 = -2(V_1 - V_3) \rightarrow \underline{-2V_1 - V_2 + 2V_3 = 0}$ $\underline{(eq.2)}$

KCL + Ohm @ node 3:

$$\left.\begin{matrix} -\frac{3}{2}V_x + \frac{V_3 - V_1}{1\Omega} + \frac{V_3 - V_4}{1\Omega} + \frac{V_3}{0.5\Omega} = 0 \\ V_x = V_4 - V_3 \end{matrix}\right\} \Rightarrow -V_1 + V_3\left(\frac{3}{2}+1+1+2\right) - V_4\left(\frac{3}{2}+1\right) = 0$$

$$\Rightarrow -V_1 + \frac{11}{2}V_3 - \frac{5}{2}V_4 = 0 \xrightarrow{\times 2} \underline{-2V_1 + 11V_3 - 5V_4 = 0 \quad (eq.3)}$$

KCL + Ohm @ node 4:

$$\frac{V_4 - V_3}{1\Omega} + \frac{V_4}{1\Omega} + \frac{V_4 - V_2}{2\Omega} + 3A = 0$$

$$-\frac{V_2}{2\Omega} - \frac{V_3}{1\Omega} + V_4\left(\frac{1}{1\Omega} + \frac{1}{1\Omega} + \frac{1}{2\Omega}\right) = -3A \Rightarrow -\frac{V_2}{2} - V_3 + \frac{5}{2}V_4 = -3$$

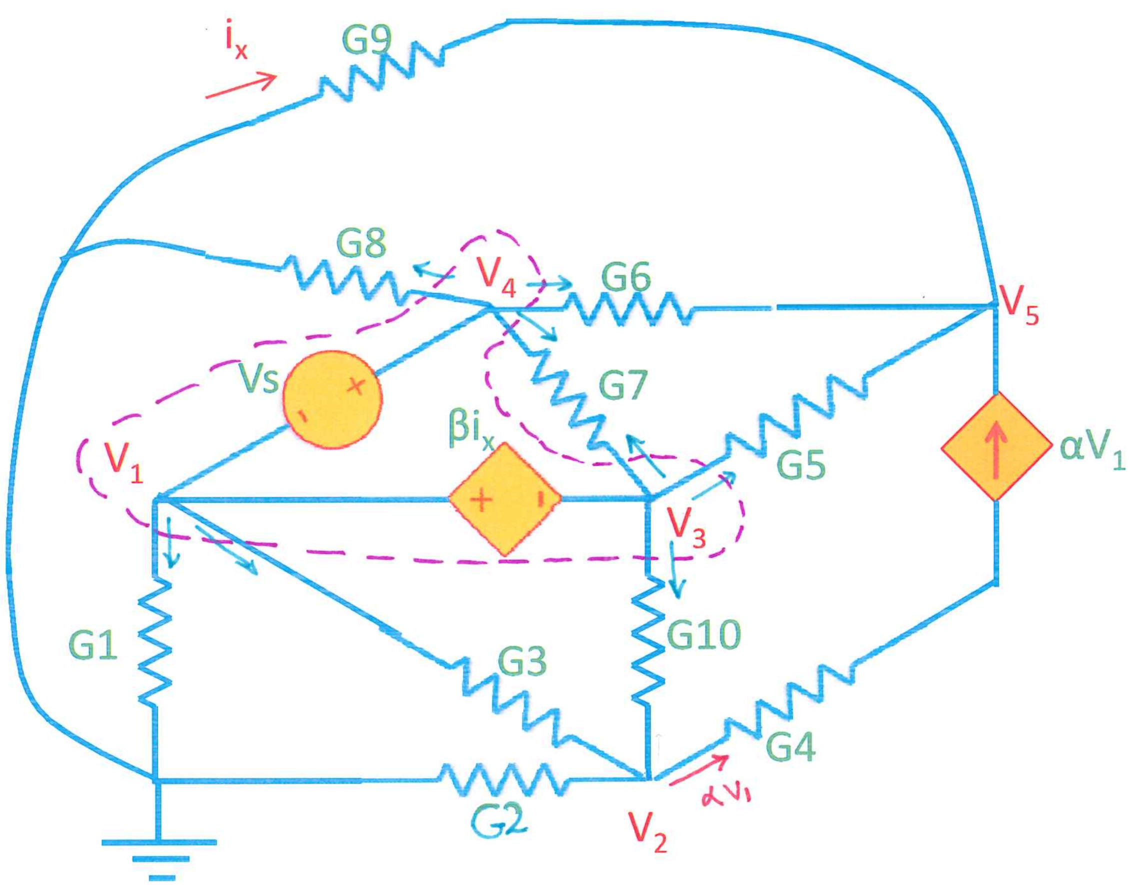
$$\xrightarrow{\times(-2)} \underline{V_2 + 2V_3 - 5V_4 = 6 \quad (eq.4)}$$

Replace equation 1 ($V_1 = 10v$) in eq.3 and eq.2:

eq.2: $-2\times 10 - V_2 + 2V_3 = 0 \rightarrow V_2 - 2V_3 = -20$ (eq 2*)

eq.3: $-2\times 10 + 11V_3 - 5V_4 = 0 \rightarrow 11V_3 - 5V_4 = 20$ (eq 3*)

Problem 4: Write the node-voltage equations and put them in matrix form.



Problem 4) 5 unknowns $\Rightarrow$ need 5 equations

KCL + Ohm @ node 2:

$$G_2V_2 + G_3(V_2-V_1) + G_{10}(V_2-V_3) + \alpha V_1 = 0$$

$$(\alpha - G_3)V_1 + (G_2+G_3+G_{10})V_2 - G_{10}V_3 = 0 \qquad (eq.\,1)$$

KCL + Ohm @ node 5:

$$G_9V_5 + G_6(V_5-V_4) + G_5(V_5-V_3) - \alpha V_1 = 0$$

$$-\alpha V_1 - G_5V_3 - G_6V_4 + (G_9+G_6+G_5)V_5 = 0 \qquad (eq.\,2)$$

These two terms cancel out $\rightarrow G_7$ could have been part of the supernode.

KCL + Ohm @ supernode:

$$\overbrace{G_1V_1 + G_3(V_1-V_2)}^{\text{node 1}} + \overbrace{G_{10}(V_3-V_2) + G_5(V_3-V_5) + \cancel{G_7(V_3-V_4)}}^{\text{node 3}} + \underbrace{\cancel{G_7(V_4-V_3)} + G_6(V_4-V_5) + G_8V_4}_{\text{node 4}} = 0$$

$$(G_1+G_3)V_1 + V_2(-G_3-G_{10}) + V_3(G_{10}+G_5) + V_4(G_6+G_8) + V_5(-G_5-G_6) = 0 \qquad (eq.\,3)$$

We still need 2 more equations $\Rightarrow$ write 2 KVLs at the supernode:

$$V_4 - V_1 = V_s \qquad (eq.\,4)$$

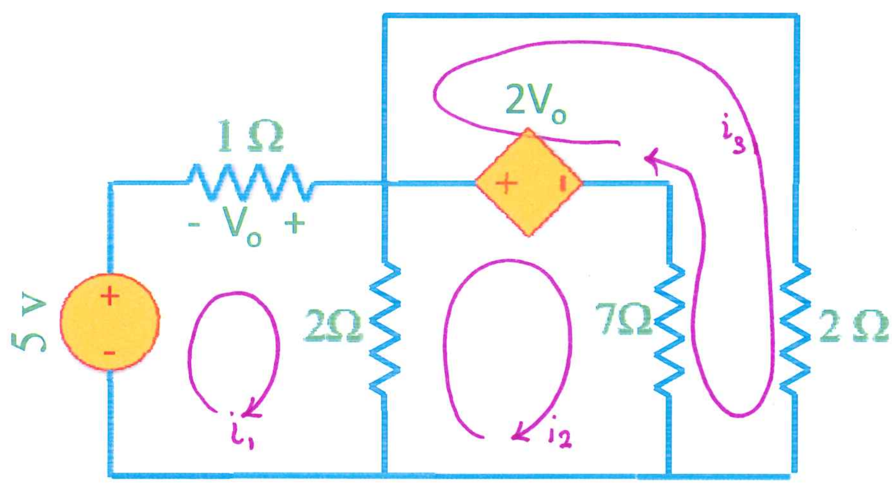
$$\left.\begin{array}{l} V_1 - V_3 = \beta i_x \\ i_x = -G_9V_5 \end{array}\right\} \Rightarrow V_1 - V_3 = -G_9\beta V_5 \Rightarrow V_1 - V_3 + G_9\beta V_5 = 0 \qquad (eq.\,5)$$

(Prob. 4)

Now write the equations in matrix form :

$$\begin{bmatrix} \alpha - G_3 & G_2+G_3+G_{10} & -G_{10} & 0 & 0 \\ -\alpha & 0 & -G_5 & -G_6 & G_9+G_6+G_5 \\ G_1+G_3 & -G_3-G_{10} & G_{10}+G_5 & G_6+G_8 & -G_5-G_6 \\ 1 & 0 & -1 & 0 & \beta G_9 \\ -1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_S \end{bmatrix}$$

Problem 5: Solve for mesh currents.



Problem 5)

KVL @ Mesh 1: $-5+(1\Omega)(i_1)+(2\Omega)(i_1-i_2)=0$

$i_1(1+2)-2i_2=5$

$3i_1-2i_2=5$ (eq1)

KVL @ mesh2: $\left.\begin{array}{l} 2(i_2-i_1)+2V_o+7(i_2-i_3) \\ V_o=(1\Omega)(-i_1)=-i_1 \end{array}\right\} \Rightarrow -4i_1+9i_2-7i_3=0$ (eq.2)

KVL @ mesh 3: $\left.\begin{array}{l} -2V_o+2i_3+7(i_3-i_2)=0 \\ V_o=-i_1 \end{array}\right\} \Rightarrow 2i_1-7i_2+9i_3=0$ (eq. 3)

$$\begin{array}{c} eq1 \\ eq2 \\ eq3 \end{array} \begin{bmatrix} 3 & -2 & 0 \\ -4 & 9 & -7 \\ 2 & -7 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$\Delta = 52$

$\Delta_1 = 160$

$\Delta_2 = 110$

$\Delta_3 = 50$

$i_1 = \frac{\Delta_1}{\Delta} = \frac{160}{52} = 3.077$ A

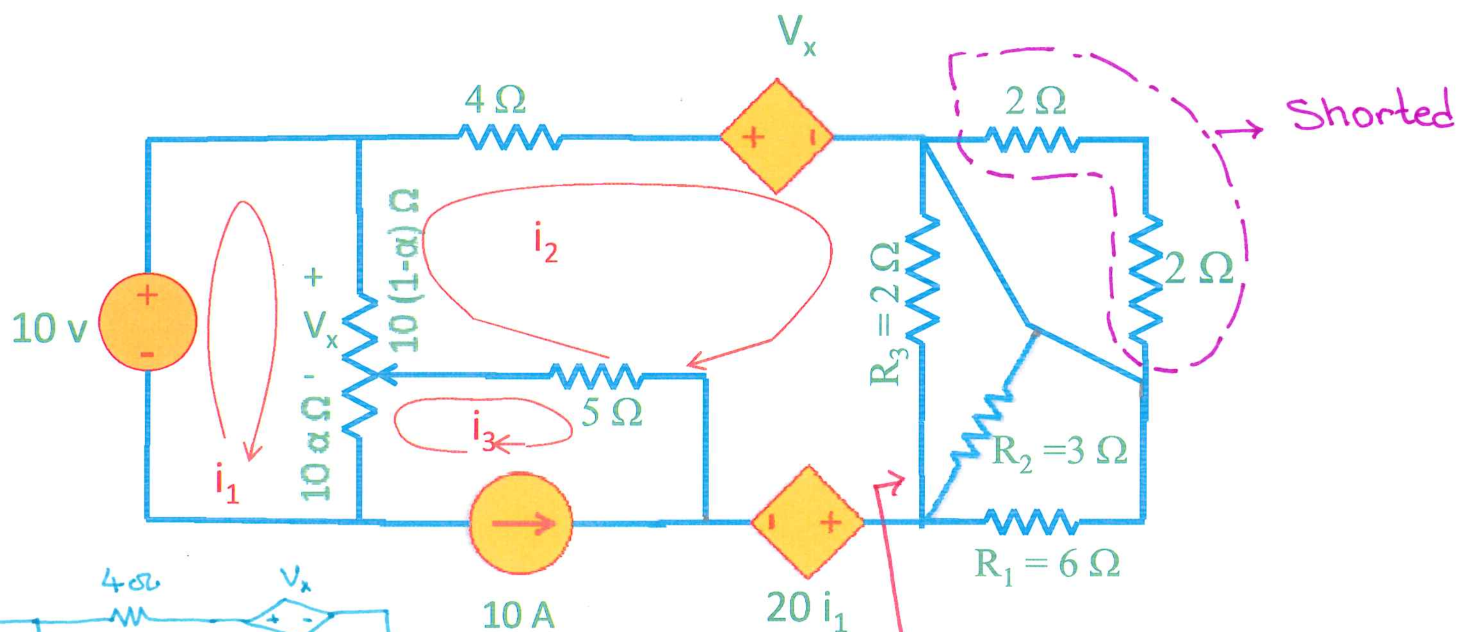
$i_2 = \frac{\Delta_2}{\Delta} = \frac{110}{52} = 2.12$ A

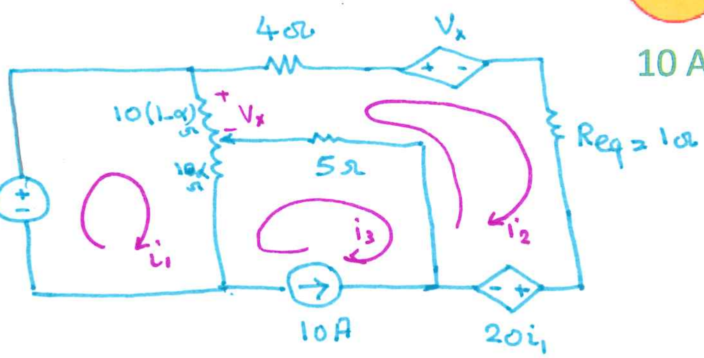
$i_3 = \frac{\Delta_3}{\Delta} = \frac{50}{52} = 0.96$ A

Problem 6:

-Obtain mesh currents i_1 through i_3 (in terms of α).

- Find the total power dissipated in R_1, R_2 and R_3($P1+P2+P3$ in terms of α).





$$Req = R_1 \| R_2 \| R_3$$

$$= 6\Omega \| 3\Omega \| 2\Omega$$

$$= \frac{6 \times 3}{6+3} \Omega \| 2\Omega$$

$$= 2\Omega \| 2\Omega = 1\Omega$$

Problem 6)

KVL @ mesh1

$$-10 + 10(1-\alpha)(i_1 - i_2) + 10\alpha(i_1 - i_3) = 0$$

$$10i_1 - 10(1-\alpha)i_2 - 10\alpha i_3 = 10 \xrightarrow{\div 10} \underline{i_1 + (\alpha-1)i_2 - \alpha i_3 = 1} \quad (eq. 1)$$

KVL @ mesh2:

$$-\cancel{V_x} + 4i_2 + \cancel{V_x} + \overset{1\Omega}{R_{eq}}\, i_2 + 20i_1 + 5(i_2 - i_3) = 0$$

$$20i_1 + (4+1+5)i_2 - 5i_3 = 0$$

$$20i_1 + 10i_2 - 5i_3 = 0 \xrightarrow{\div 5} \underline{4i_1 + 2i_2 - i_3 = 0} \quad (eq. 2)$$

@ mesh 3: $i_3 = -10\,A$

replace i_3 by (-10) in eq. 1 & eq. 2 :

eq. 1 $\rightarrow$ $i_1 + (\alpha-1)i_2 - 10\alpha = 1 \Rightarrow i_1 + (\alpha-1)i_2 = 1 + 10\alpha$ (eq 1*)

eq. 2 $\rightarrow$ $4i_1 + 2i_2 - 10 = 0 \xRightarrow{\div 2} 2i_1 + i_2 = 5$ (eq. 2*)

$$\begin{cases} i_1 + (\alpha-1)i_2 = 1 + 10\alpha \\ 2i_1 + i_2 = 5 \end{cases} \xrightarrow{\times 2} \begin{cases} 2i_1 + 2(\alpha-1)i_2 = 2 + 20\alpha \\ 2i_1 + i_2 = 5 \end{cases}$$

$$2(\alpha-1)i_2 - i_2 = 2 + 20\alpha - 5$$

$$(2\alpha - 3)i_2 = 20\alpha - 3 \Rightarrow i_2 = \frac{20\alpha - 3}{2\alpha - 3}\ A$$

(Prob. 6)

$$2i_1 + i_2 = 5 \longrightarrow i_1 = \frac{5 - i_2}{2}$$

$$i_1 = \frac{1}{2}\left(5 - \frac{20\alpha - 3}{2\alpha - 3}\right) = \frac{1}{2}\left(\frac{10\alpha - 15 - 20\alpha + 3}{2\alpha - 3}\right)$$

$$i_1 = \frac{-10\alpha - 12}{2(2\alpha - 3)} = \frac{-(5\alpha + 6)}{(2\alpha - 3)} \quad A$$

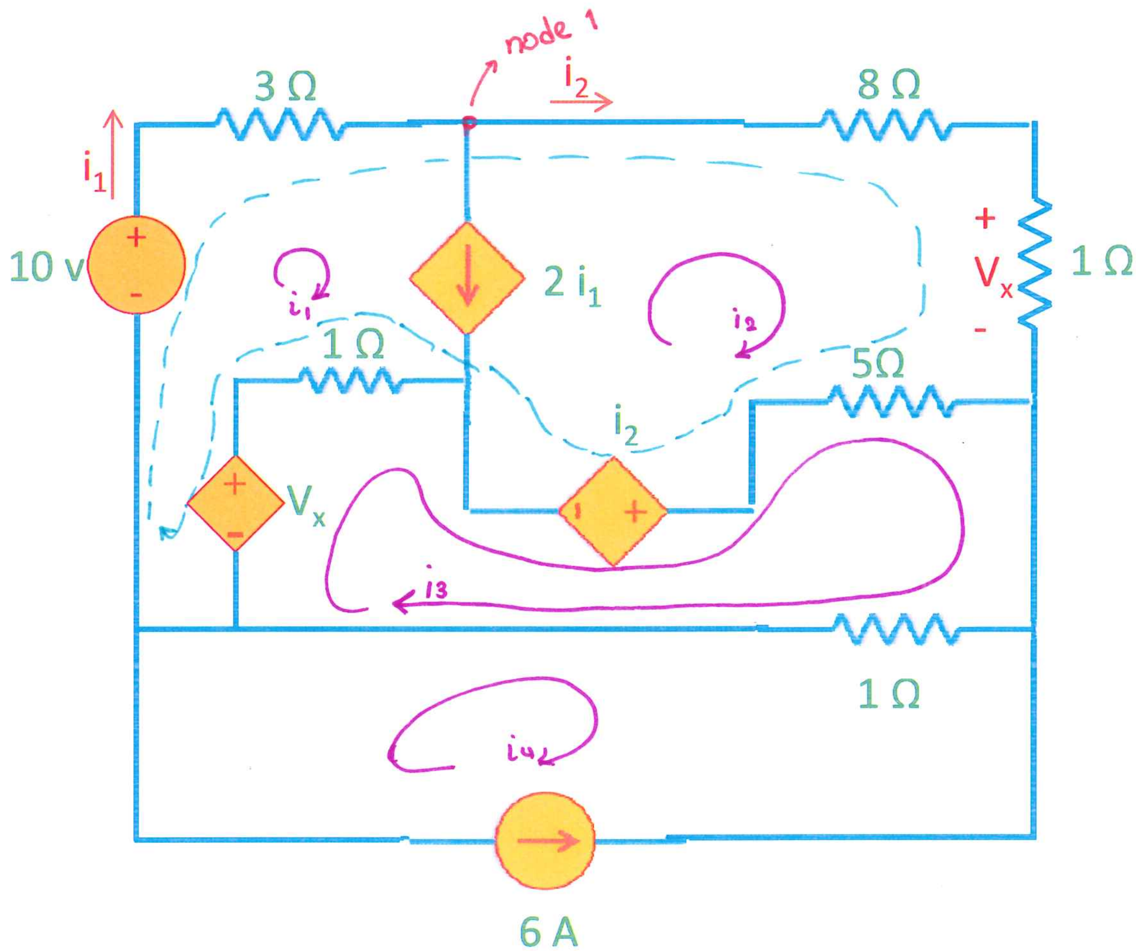
- Find the total power absorbed in R_1, R_2 & R_3 :

$$P_{R_1} + P_{R_2} + P_{R_3} = P_{Req}$$

$$P = V \cdot I = (I \cdot R) \cdot I = I^2 R$$

$$P_{Req} = i_2^2 \cdot Req = \left(\frac{-(5\alpha + 6)}{2\alpha - 3}\right)^2 \cdot 1\Omega = \left(\frac{5\alpha + 6}{2\alpha - 3}\right)^2 \quad W$$

Problem 7: Solve for mesh currents.



Problem7)

@mesh4: $i_4 = -6A$ (eq. 1)

KVL @ mesh3: $-V_x + 1\Omega(i_3 - i_1) - i_2 + 5\Omega(i_3 - i_2) + 1\Omega(i_3 - i_4) = 0$

$V_x = 1\Omega \times i_2 = i_2$

$-i_2 + i_3 - i_1 - i_2 + 5i_3 - 5i_2 + i_3 - i_4 = 0$

$-i_1 - i_2(1+1+5) + i_3(1+5+1) - i_4 = 0$

$-i_1 - 7i_2 + 7i_3 - i_4 = 0$ (eq. 2)

KVL @ supermesh:

$-10 + 3i_1 + 8i_2 + 1i_2 + 5(i_2 - i_3) + i_2 + 1(i_1 - i_3) + V_x = 0$

$V_x = 1\Omega \times i_2 = i_2$

$i_1(3+1) + i_2(8+1+5+1+1) + i_3(-5-1) = 10$

$4i_1 + 16i_2 - 6i_3 = 10 \xrightarrow{\div 2} 2i_1 + 8i_2 - 3i_3 = 5$ (eq. 3)

KCL @ supermesh (node 1): $i_1 - i_2 = 2i_1 \Rightarrow i_1 + i_2 = 0$ (eq. 4)

Replace equation 1 in equation 2:

$-i_1 - 7i_2 + 7i_3 - (-6) = 0 \rightarrow -i_1 - 7i_2 + 7i_3 = -6$ (eq 2*)

Solving with Cramer's rule:

$$\begin{array}{r} \text{eq 4} \\ \text{eq 3} \\ \text{eq 2*} \end{array} \begin{bmatrix} 1 & 1 & 0 \\ 2 & 8 & -3 \\ -1 & -7 & 7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -6 \end{bmatrix}$$

$\Delta = 24$

$\Delta_1 = -17$

$\Delta_2 = 17$

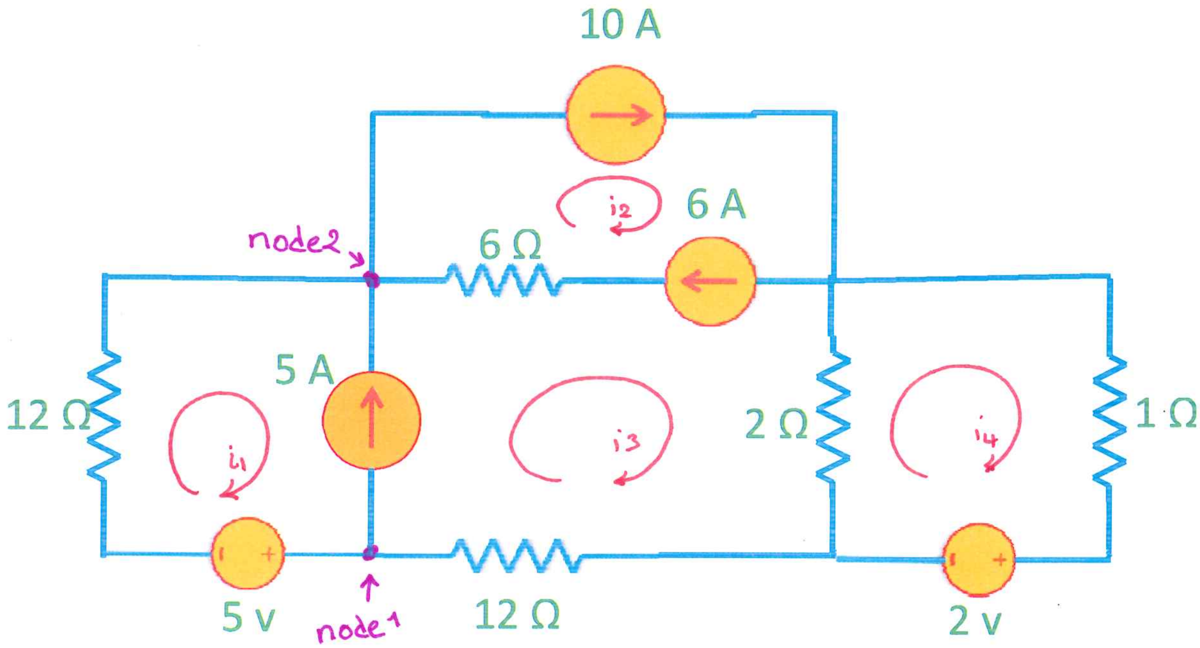
$\Delta_3 = -6$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{-17}{24} = -0.7 \text{ A}$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{17}{24} = 0.7 \text{ A}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{-6}{24} = -0.25 \text{ A}$$

Problem 8: Solve for mesh currents.



Problem 8)

@ mesh2 : $i_2 = 10A$ (eq. 1)

KVL @ mesh 4: $2(i_4 - i_3) + i_4 + 2 = 0$

$3i_4 - 2i_3 = -2$ (eq. 2)

There are 4 unknowns; i_1, i_2, i_3 and i_4 so we need 4 eq.s, We have got two equations so far, and must write two more. We can not write any KVL equations for mesh 1, mesh2, mesh3 or any of the supermeshes, so we have to write two KCL equations.

KCL @ node 1 : $i_3 - i_1 = 5A$ (eq. 3)

KCL @ node 2 : $i_1 + 6A + 5A = 10A \Rightarrow i_1 = -1\,A$ (eq. 4)

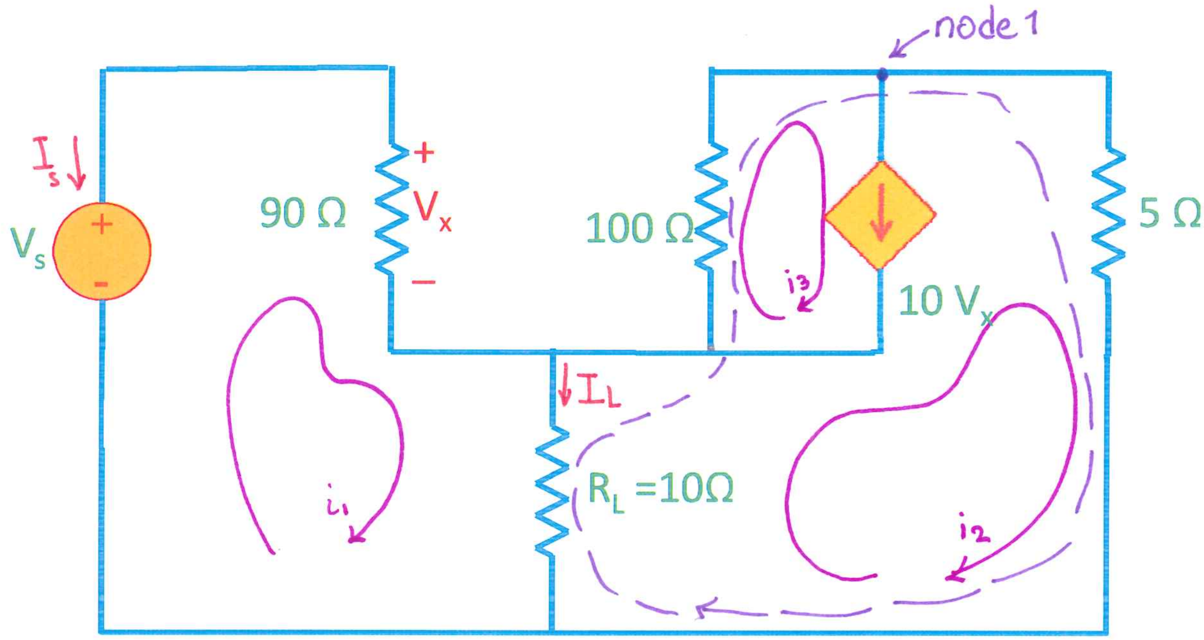
Replace i_1 by $-1A$ in eq. 3 $\Rightarrow i_3 - (-1A) = 5A \rightarrow i_3 = 4A$

Replace i_3 by $4A$ in eq. 2 $\Rightarrow 3i_4 - 2(4A) = -2$

$$i_4 = \frac{8-2}{3} = \frac{6}{3}$$

$$i_4 = 2A$$

Problem 9: Use your circuit analysis skills to find the ratio of the power absorbed by R_L to the power supplied by Vs.



Problem 9)

To find the power ratio, we must first find I_s and I_L. You can use nodal or mesh analysis. Here mesh analysis has been used:

KVL @ mesh 1: $-V_s + 90i_1 + 10(i_1 - i_2) = 0$

$$100i_1 - 10i_2 = V_s \quad \text{(eq. 1)}$$

KVL @ supermesh: $10(i_2 - i_1) + 100i_3 + 5i_2 = 0$

$$-10i_1 + 15i_2 + 100i_3 = 0$$

$$\xrightarrow{\div 5} -2i_1 + 3i_2 + 20i_3 = 0 \quad \text{(eq. 2)}$$

KCL @ supermesh, node 1: $\left.\begin{array}{l} i_3 - i_2 = 10V_x \\ V_x = 90i_1 \end{array}\right\} \Rightarrow i_3 - i_2 = 10(90i_1)$

$$\Rightarrow 900i_1 + i_2 - i_3 = 0 \quad \text{(eq. 3)}$$

Solving the equations:

$$\begin{bmatrix} 100 & -10 & 0 \\ -2 & 3 & 20 \\ 900 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

(Prob. 9)

$$\Delta = -182280$$

$$\Delta_1 = \begin{vmatrix} V_s & -10 & 0 \\ 0 & 3 & 20 \\ 0 & 1 & -1 \end{vmatrix} = V_s \begin{vmatrix} 3 & 20 \\ 1 & -1 \end{vmatrix} + 10 \begin{vmatrix} 0 & 20 \\ 0 & -1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 3 \\ 0 & 1 \end{vmatrix}$$

$$= V_s(-3-20) = -23 V_s$$

$$\Delta_2 = \begin{vmatrix} 100 & V_s & 0 \\ -2 & 0 & 20 \\ 900 & 0 & -1 \end{vmatrix} = 100 \begin{vmatrix} 0 & 20 \\ 0 & -1 \end{vmatrix} - V_s \begin{vmatrix} -2 & 20 \\ 900 & -1 \end{vmatrix} + 0$$

$$= -V_s(2 - 20 \times 900) = 17998 V_s$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{-23 V_s}{-182280} = 1.26 \times 10^{-4} V_s \Rightarrow P_{source} = V_s \cdot I_s = V_s \cdot (-i_1) = V_s \cdot (-1.26 \times 10^{-4} V_s)$$

$$\Rightarrow P_{source} = -1.26 \times 10^{-4} V_s^2 \ [W]$$

$$= -0.126 V_s^2 \ [mW]$$

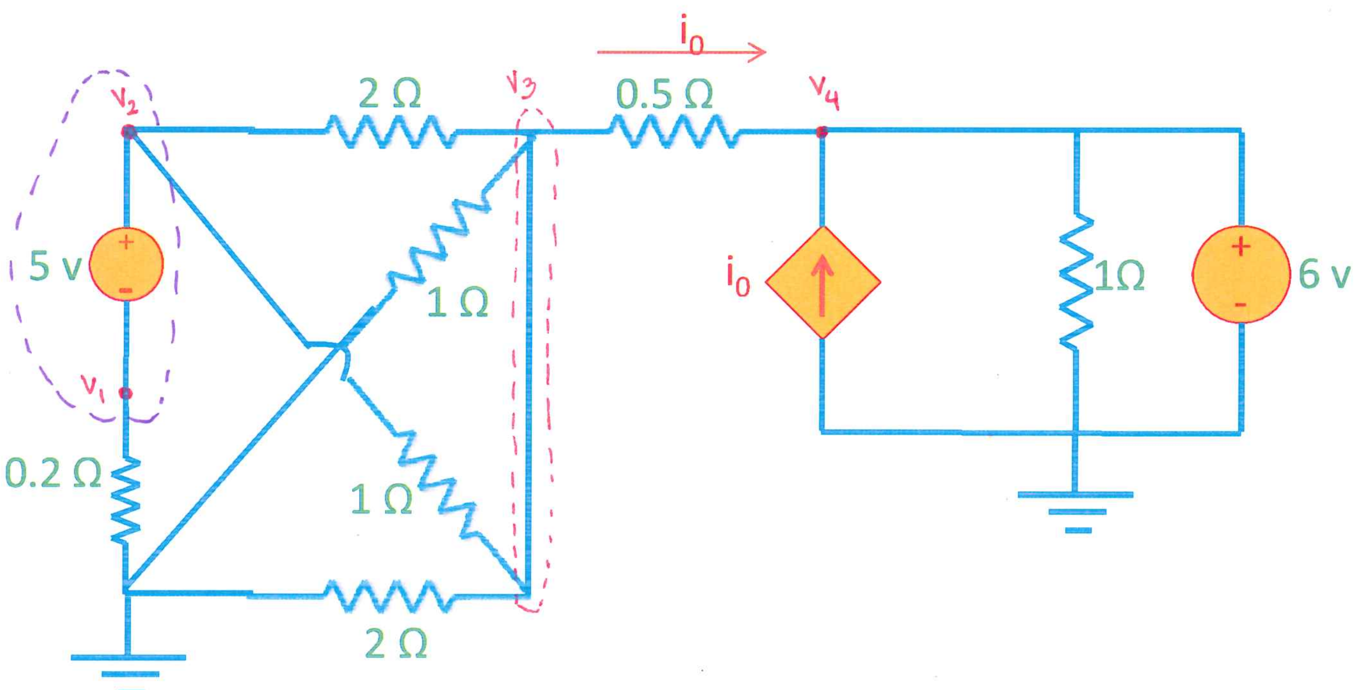
$$i_2 = \frac{\Delta_2}{\Delta} = \frac{17998 V_s}{-182280} \simeq -0.1 V_s \ ; \ P_{R_L} = (I_L)^2 \cdot R_L = (i_1 - i_2)^2 \cdot R_L$$

$$P_{R_L} = \underbrace{(1.26 \times 10^{-4} + 0.1)^2}_{\approx 0.1} V_s^2 \cdot 10\Omega \simeq 0.1 V_s^2 \ [W]$$

$$P_{R_L} \approx 100 V_s^2 \ [mW]$$

$$\Rightarrow \frac{P_{R_L}}{P_{source}} = \frac{100 V_s^2}{-0.126 V_s^2} = -793.6$$

Problem 10: Find i_0.



Problem 10)

Because of the way this circuit looks, it is easier to do nodal analysis. To find i_o, first we have to obtain V_3 and V_4

KCL + Ohm @ node 3:

$$\frac{V_3 - V_2}{2\Omega} + \frac{V_3}{1\Omega} + \frac{V_3 - V_2}{1\Omega} + \frac{V_3}{2\Omega} + \frac{V_3 - V_4}{0.5\Omega} = 0$$

$$-V_2\left(\frac{1}{2} + 1\right) + V_3\left(\frac{1}{2} + 1 + 1 + \frac{1}{2} + 2\right) - 2V_4 = 0$$

$$-\frac{3}{2}V_2 + 5V_3 - 2V_4 = 0 \xrightarrow{\times 2} -3V_2 + 10V_3 - 4V_4 = 0 \qquad \text{(eq. 1)}$$

KCL + Ohm @ the supernode:

$$\frac{V_1}{0.2\Omega} + \frac{V_2 - V_3}{2\Omega} + \frac{V_2 - V_3}{1\Omega} = 0$$

$$5V_1 + \frac{3}{2}V_2 - \frac{3}{2}V_3 = 0 \xrightarrow{\times 2} 10V_1 + 3V_2 - 3V_3 = 0 \qquad \text{(eq. 2)}$$

@ node 4: $V_4 = 6v$ (eq. 3)

KVL @ supernode: $V_2 - V_1 = 5v \rightarrow -V_1 + V_2 = 5v$ (eq. 4)

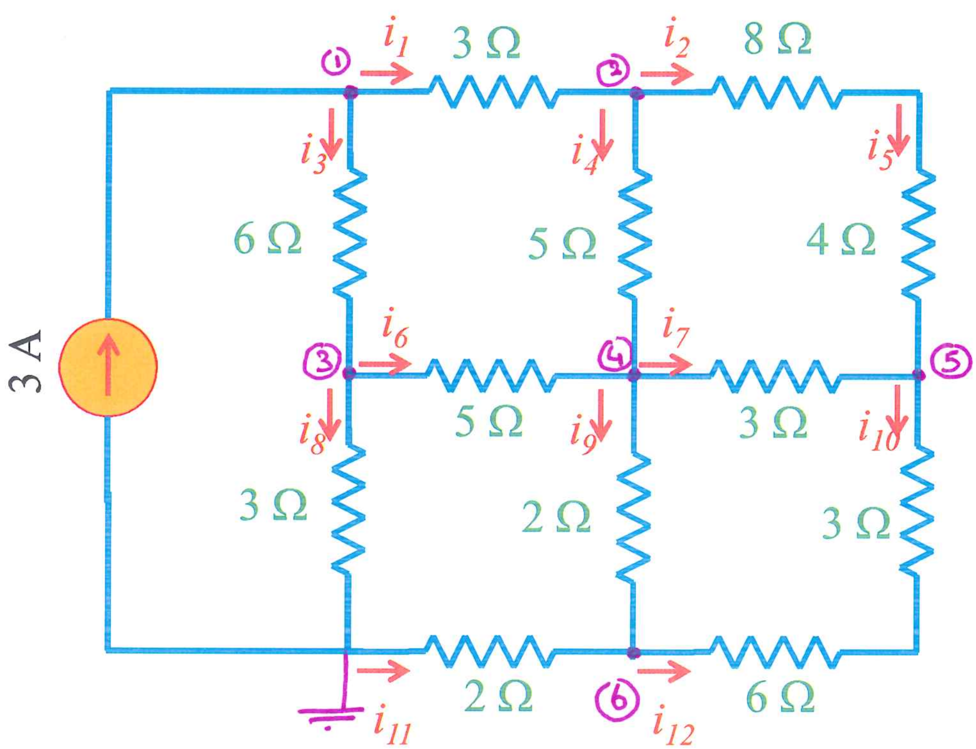
Replace V_4 from eq. 2 in eq. 1:

$$-3V_2 + 10V_3 = 24 \qquad \text{(eq 1*)}$$

$$\begin{bmatrix} 0 & -3 & 10 \\ 10 & 3 & -3 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 5 \end{bmatrix} \rightarrow V_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 0 & -3 & 24 \\ 10 & 3 & 0 \\ -1 & 1 & 5 \end{vmatrix}}{\begin{vmatrix} 0 & -3 & 10 \\ 10 & 3 & -3 \\ -1 & 1 & 0 \end{vmatrix}} = \frac{462}{121} = 3.8v$$

$$i_o = \frac{V_3 - V_4}{0.5\Omega} = \frac{3.8v - 6v}{0.5\Omega} = -4.36\ A$$

Problem 11: Find node voltages and currents.



Problem 11)

KCL + Ohm's law @

node 1: $\frac{V_1 - V_3}{6} + \frac{V_1 - V_2}{3} = 3\,A \Rightarrow V_1\left(\frac{1}{6}+\frac{1}{3}\right) - \frac{V_3}{6} - \frac{V_2}{3} = 3$

$\xrightarrow{\times 6} 3V_1 - V_3 - 2V_2 = 18$ (eq. 1)

$20+12+5$

node 2: $\frac{V_2 - V_1}{3\Omega} + \frac{V_2 - V_4}{5\Omega} + \frac{V_2 - V_5}{8\Omega + 4\Omega} = 0 \Rightarrow \frac{-V_1}{3} + V_2\left(\frac{1}{3}+\frac{1}{5}+\frac{1}{12}\right) - \frac{V_4}{5} - \frac{V_5}{12} = 0$

$\xrightarrow{\times 60} -20V_1 + 37V_2 - 12V_4 - 5V_5 = 0$ (eq. 2)

$5/30 + 6/30 + 10/30$

node 3: $\frac{V_3 - V_1}{6\Omega} + \frac{V_3 - V_4}{5\Omega} + \frac{V_3}{3\Omega} = 0 \Rightarrow \frac{-V_1}{6} + V_3\left(\frac{1}{6}+\frac{1}{5}+\frac{1}{3}\right) - \frac{V_4}{5} = 0$

$\xrightarrow{\times 30} -5V_1 + 21V_3 - 6V_4 = 0$ (eq. 3)

node 4: $\frac{V_4 - V_3}{5\Omega} + \frac{V_4 - V_2}{5\Omega} + \frac{V_4 - V_5}{3\Omega} + \frac{V_4 - V_6}{2\Omega} = 0$

$-\frac{V_2}{5} - \frac{V_3}{5} + V_4\left(\frac{1}{5}+\frac{1}{5}+\frac{1}{3}+\frac{1}{2}\right) - \frac{V_5}{3} - \frac{V_6}{2} = 0$

$\xrightarrow{\times 30} -6V_2 - 6V_3 + V_4(6+6+10+15) - 10V_5 - 15V_6 = 0$

$-6V_2 - 6V_3 + 37V_4 - 10V_5 - 15V_6 = 0$ (eq. 4)

node 5: $\frac{V_5 - V_2}{8\Omega + 4\Omega} + \frac{V_5 - V_4}{3\Omega} + \frac{V_5 - V_6}{3\Omega + 6\Omega} = 0$

$3+12+4$

$-\frac{V_2}{12} - \frac{V_4}{3} + V_5\left(\frac{1}{12}+\frac{1}{3}+\frac{1}{9}\right) - \frac{V_6}{9} = 0 \xrightarrow{\times 36} -3V_2 - 12V_4 + 19V_5 - 4V_6 = 0$ (eq. 5)

$2/18 + 9/18 + 9/18$

node 6: $\frac{V_6 - V_5}{6\Omega + 3\Omega} + \frac{V_6 - V_4}{2\Omega} + \frac{V_6}{2\Omega} = 0 \Rightarrow \frac{-V_4}{2} - \frac{V_5}{9} + V_6\left(\frac{1}{9}+\frac{1}{2}+\frac{1}{2}\right) = 0$

$\xrightarrow{\times 18} -9V_4 - 2V_5 + 20V_6 = 0$ (eq. 6)

(Prob. 11)

$$\begin{bmatrix} 3 & -2 & -1 & 0 & 0 & 0 \\ -20 & 37 & 0 & -12 & -5 & 0 \\ -5 & 0 & 21 & -6 & 0 & 0 \\ 0 & -6 & -6 & 37 & -10 & -15 \\ 0 & -3 & 0 & -12 & 19 & -4 \\ 0 & 0 & 0 & -9 & -2 & 20 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 18 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 14.33\ v \\ 10.08\ v \\ 4.83\ v \\ 4.98\ v \\ 5.32\ v \\ 2.77\ v \end{bmatrix}$$

$$i_1 = \frac{V_1 - V_2}{3\Omega} = \frac{14.33 - 10.08^V}{3\Omega} = 1.416 \text{ A}$$

$$i_2 = \frac{V_2 - V_5}{8\Omega + 4\Omega} = \frac{(10.08 - 5.32)^V}{12\Omega} = 0.37 \text{ A}$$

$$i_3 = \frac{V_1 - V_3}{6\Omega} = \frac{(14.33 - 4.83)^V}{6\Omega} = 1.58 \text{ A}$$

$$i_4 = \frac{V_2 - V_4}{5\Omega} = \frac{(10.08 - 4.98)^V}{5\Omega} = 1.02 \text{ A}$$

$$i_5 = i_2 = 0.37 \text{ A}$$

$$i_6 = \frac{V_3 - V_4}{5\Omega} = \frac{(4.83 - 4.98)V}{5\Omega} = -0.03 \text{ A}$$

$$i_7 = \frac{V_4 - V_5}{3\Omega} = \frac{(4.98 - 5.32)V}{3\Omega} = -0.113 \text{ A}$$

$$i_8 = \frac{V_3}{3\Omega} = \frac{4.83\ v}{3\ \Omega} = 1.61 \text{ A}$$

$$i_9 = \frac{V_4 - V_6}{2\Omega} = \frac{(4.98 - 2.77)^V}{2\Omega} = 1.105 \text{ A}$$

$$i_{10} = \frac{V_5 - V_6}{3\Omega + 6\Omega} = \frac{(5.32 - 2.77)V}{9\Omega} = 0.28 \text{ A}$$

$$i_{11} = \frac{-V_6}{2\Omega} = \frac{-2.77\ v}{2\Omega} = -1.385 \text{ A}$$

$$i_{12} = -i_{10} = -0.28 \text{ A}$$

To solve the 6 simultaneous equations, you can use the elimination method or Cramer's rule to solve it by hand, or use a calculator or a software like MATLAB.

Here the equations have been solved by MATLAB. To do so, first enter the 6x6 matrix for voltage coefficients- Let's call it matrix G- using the following command:

G=[3 -2 -1 0 0 0; -20 37 0 -12 -5 0; -5 0 21 -6 0 0;0 -6 -6 37 -10 -15;0 -3 0 -12 19 -4; 0 0 0 -9 -2 20]

Also enter the 6x1 matrix on the right (I), If we call the 6x1 matrix for voltages V, Then the simultaneous equations in matrix form is: GxV=I

If both sides of the above equation are multiplied by the inverse of matrix G, i.e. G^{-1} we get G^{-1}xGxV=G^{-1}xI, or V=G^{-1}xI

after you have defined G and I, use the following command to find V:

V=(G^-1)xI