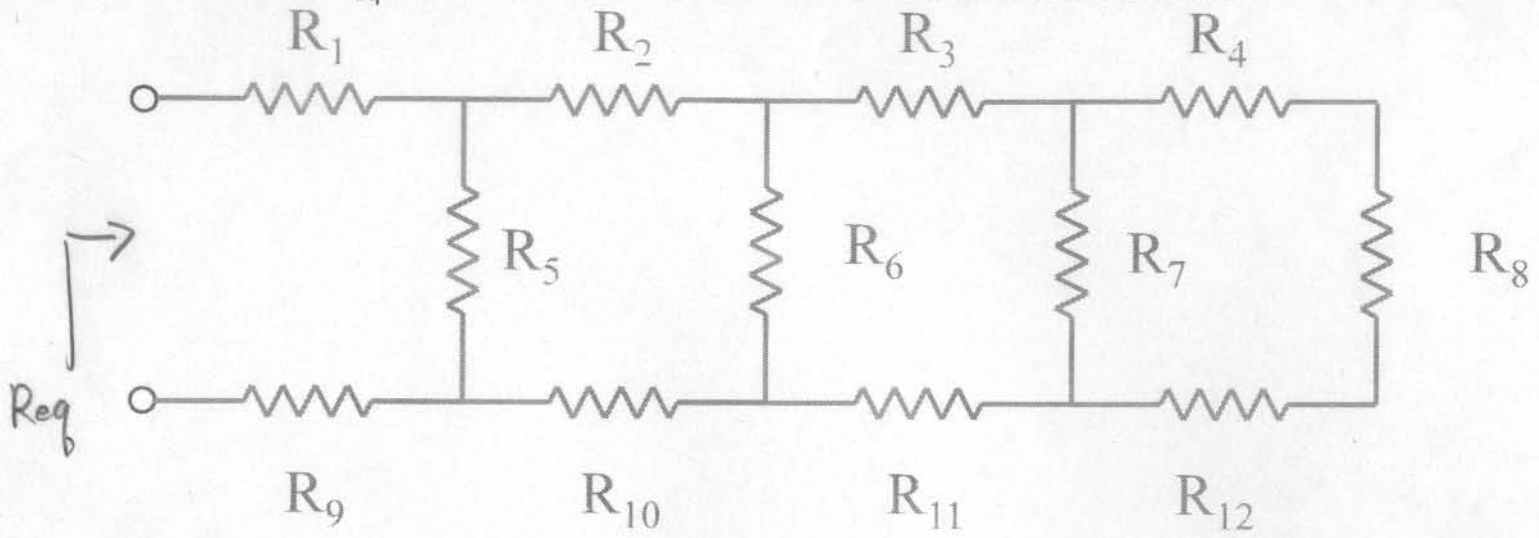
EECS 70A: Network Analysis

Homework #2

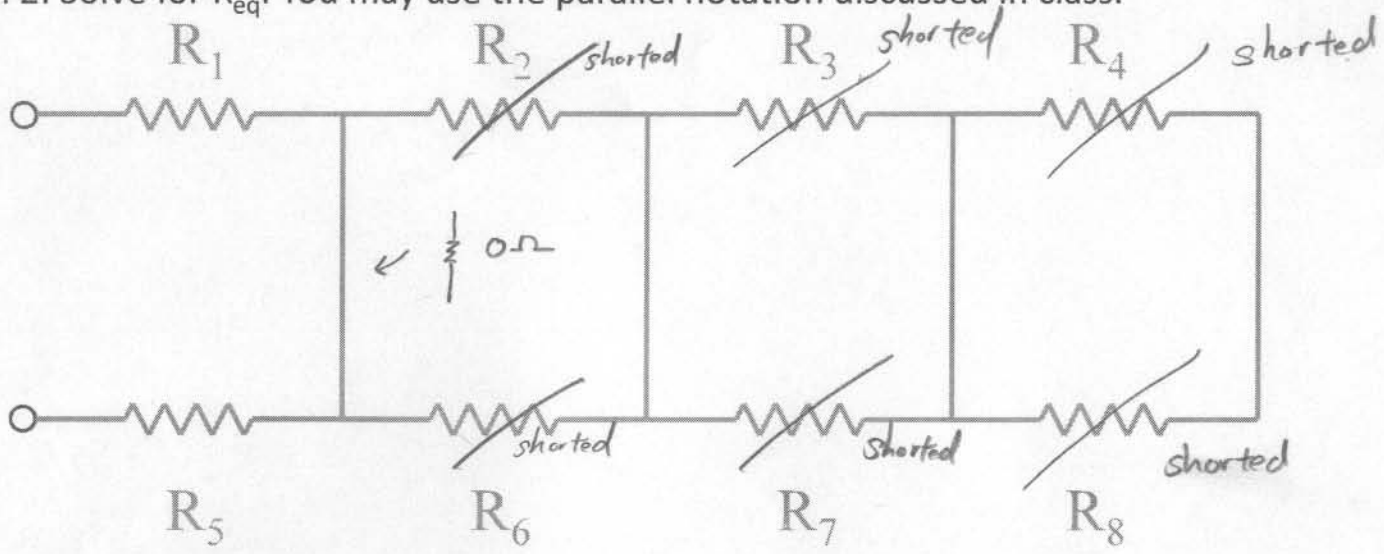
Due in discussion section,
Wednesday, April 14, 2010.

Problem 1: Solve for R_{eq}. You may use the parallel notation discussed in class.

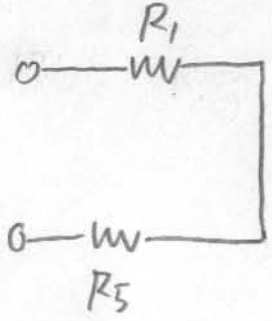


$$Req: \Big(\Big(\big((R_4 + R_8 + R_{12}) /\!/ R_7 \;+\; R_3 + R_{11}\big) /\!/ R_6\Big) + R_2 + R_2\Big) /\!/ R_5 \;+\; R_1 + R_9 .$$

Problem 2: Solve for R_{eq}. You may use the parallel notation discussed in class.

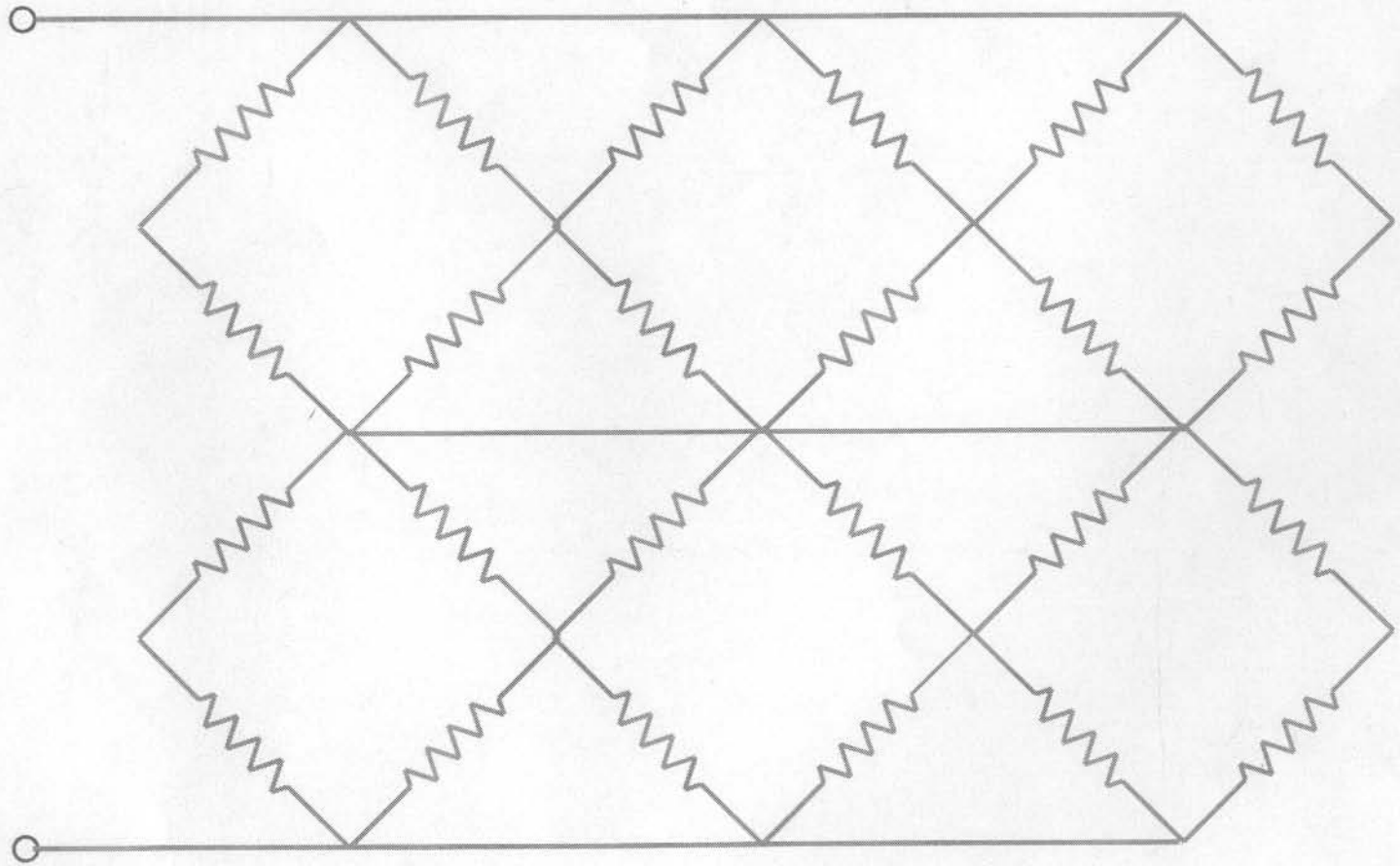


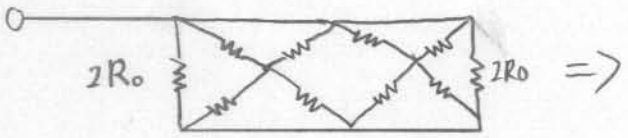
(1)



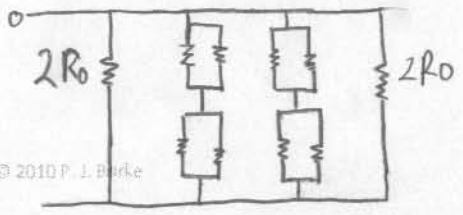
$R_{eq} = R_1 + R_5$

Problem 3: Solve for R_{eq}. All resistors have the same value R_0.

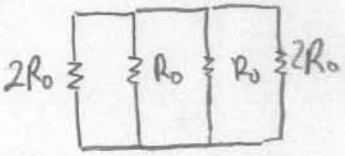




=>



=>



$\left(\frac{2}{3}R_0 // R_0\right) // \left(\frac{2}{3}R_0 // R_0\right)$

$= \frac{1}{3}R_0$ $\quad 2 \times \frac{1}{3}R_0 = \boxed{R_{eq} = \frac{2}{3}R_0}$

half Ckt:

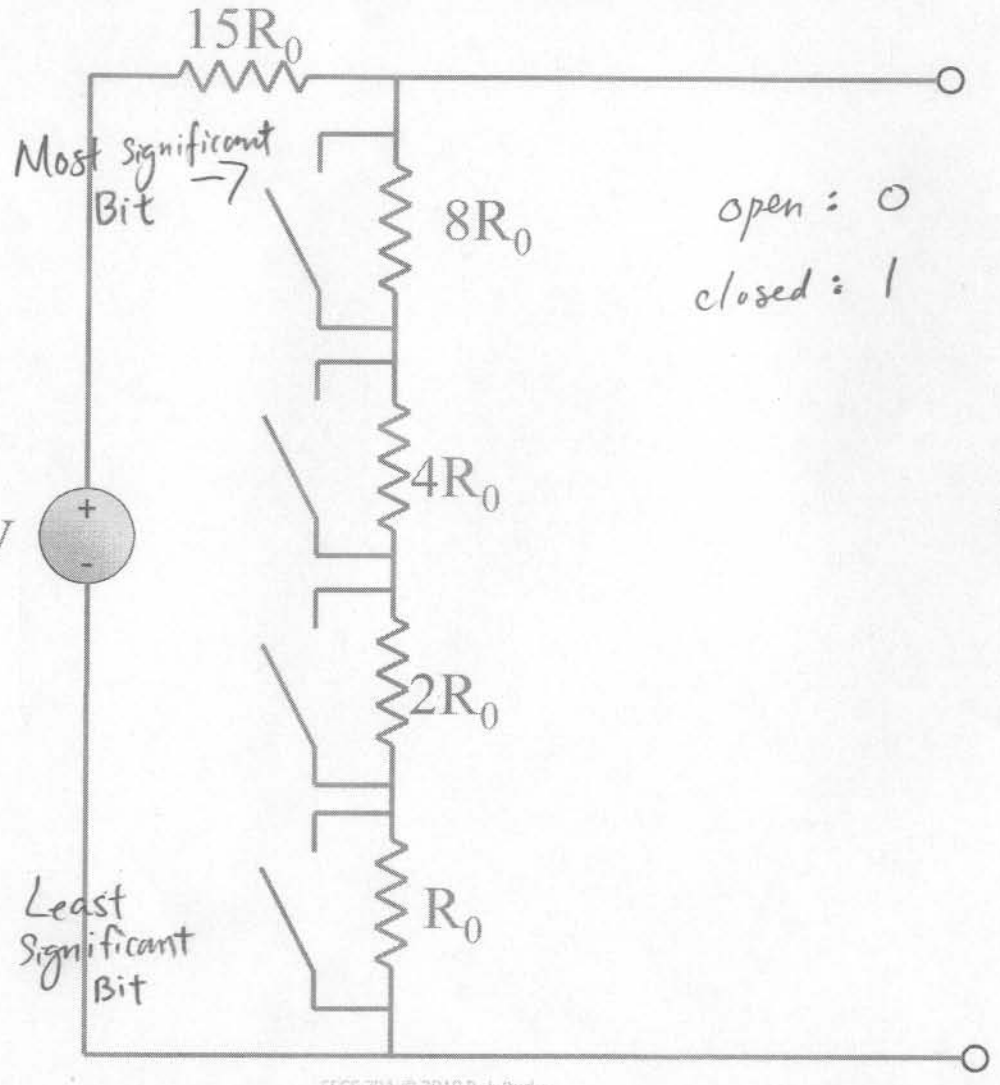
Problem 4: Digital to analog converter (D-A).

Background:

You may wonder how your iPod stores music as a string of bits and yet sends an analog voltage to the speakers in your earphones. At some point, there must be a circuit that converts the bits to a certain voltage to send to the earphones. As the computer changes the bits, the voltage changes, causing the membrane in the earphone to vibrate, causing the air pressure to change (sound waves) which your ear detects (music!). This circuit is called a *digital to analog converter*. Below is an example of one such circuit. The bits are represented by open (0) or closed (1) switches. (In reality, the switch could be implemented by a transistor.)

Problem:

Find a relationship between the number represented by the four bits (e.g. a 0001 is 1, 0010 is 2, 0011 is 3, etc.) and the actual voltage output by the circuit below.



Voltage Division

To Represent '0000'

$V_o = \frac{15R_o}{15R_o + 15R_o} \cdot 20 = 10\,V$

To Represent '0001'

$V_o = \frac{14R_o}{14R_o + 15R_o} \cdot 20 = 9.655\,V$

for: '0010':

$V_o = \frac{13R_o}{13R_o + 15R_o} \cdot 20 = 9.2\,V$

$\vdots$

$V_o = \frac{0}{0 + 15R_o} = 0\,V$ for

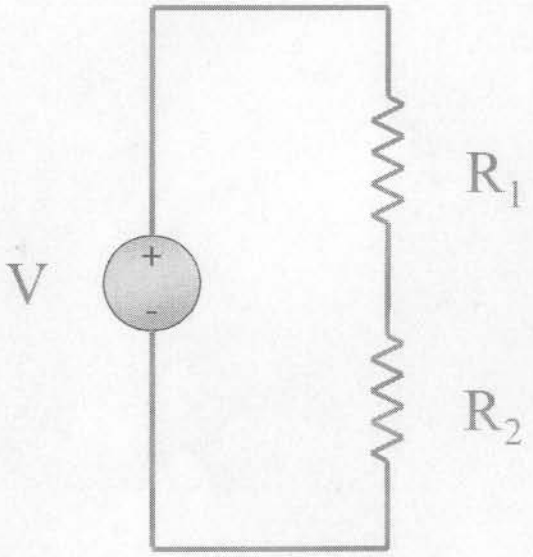
for: '1111'

open : 0

closed : 1

Problem 5: Voltage divider: Power analysis.

Find the power dissipated in R_1, in R_2, and the ratio of these two powers, in terms of V.



$$P = I \cdot V = I^2 \cdot R$$

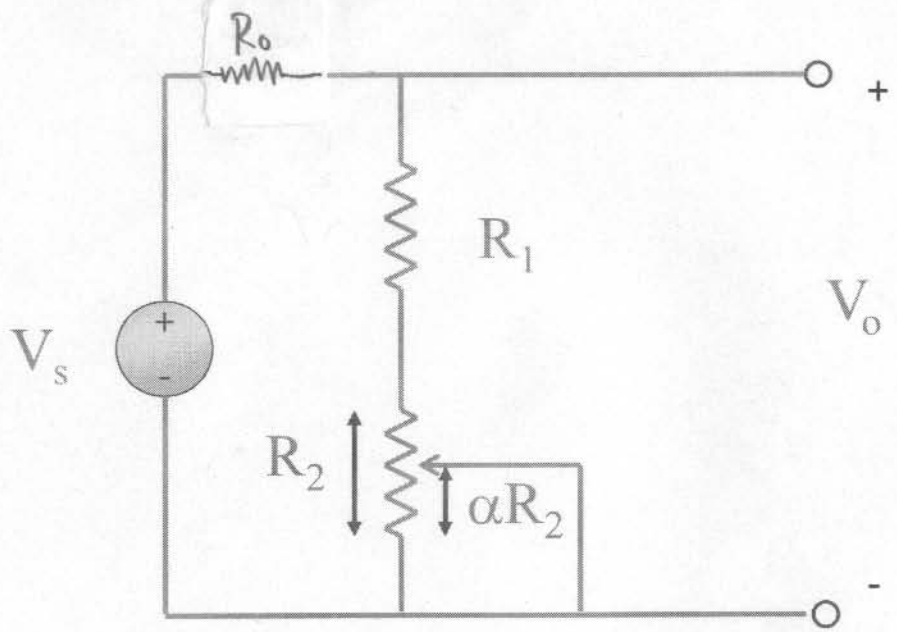
$$P_{R1} = \left(\frac{V}{R_1 + R_2}\right)^2 \cdot R_1$$

$$P_{R2} = \left(\frac{V}{R_1 + R_2}\right)^2 \cdot R_2$$

$$\frac{P_{R1}}{P_{R2}} = \frac{R_1}{R_2} \quad \text{or} \quad \frac{P_{R2}}{P_{R1}} = \frac{R_2}{R_1}$$

Problem 6: Potentiometer.

In the circuit below, the wiper divides the potentiometer resistance between αR_2 and $(1-\alpha)R_2$, where $0<\alpha<1$. Find V_o/V_s.



Voltage Division:

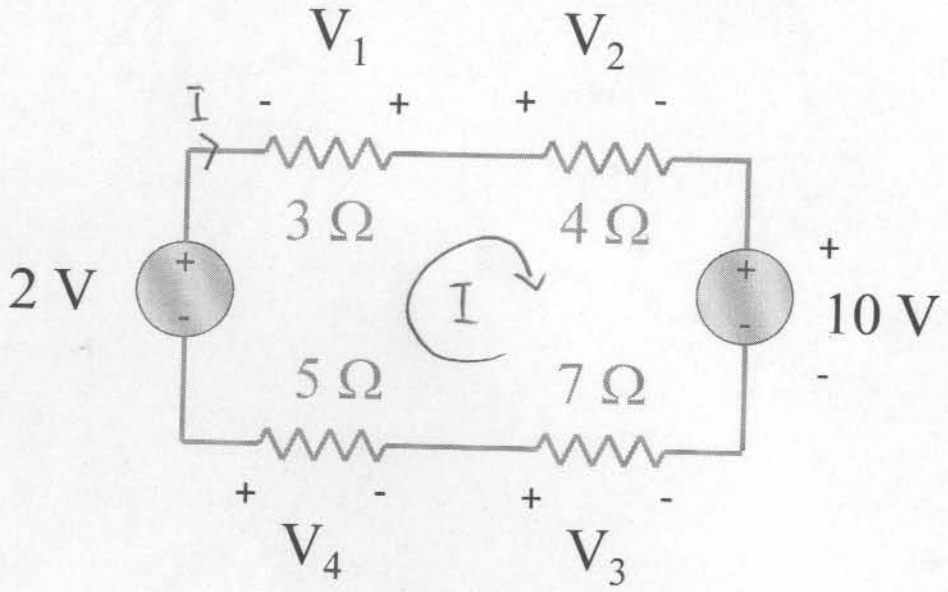
$$V_o = \frac{R_1 + (1-\alpha)R_2}{R_1 + (1-\alpha)R_2 + R_o} V_s$$

$$\frac{V_o}{V_s} = \frac{R_1 + (1-\alpha)R_2}{R_1 + (1-\alpha)R_2 + R_o}$$

where $0<\alpha<1$

Problem 7: KVL & Ohm

Find V_1 through V_4 and the current flowing in the circuit below.



KVL:

$$-2V + 3I + 4I + 10V + 7I + 5I = 0$$

$$8V = -19I$$

$$\boxed{I = -\tfrac{1}{2}A}$$

$V_1 = (3\Omega)(\tfrac{1}{2}A) = 1.5V$

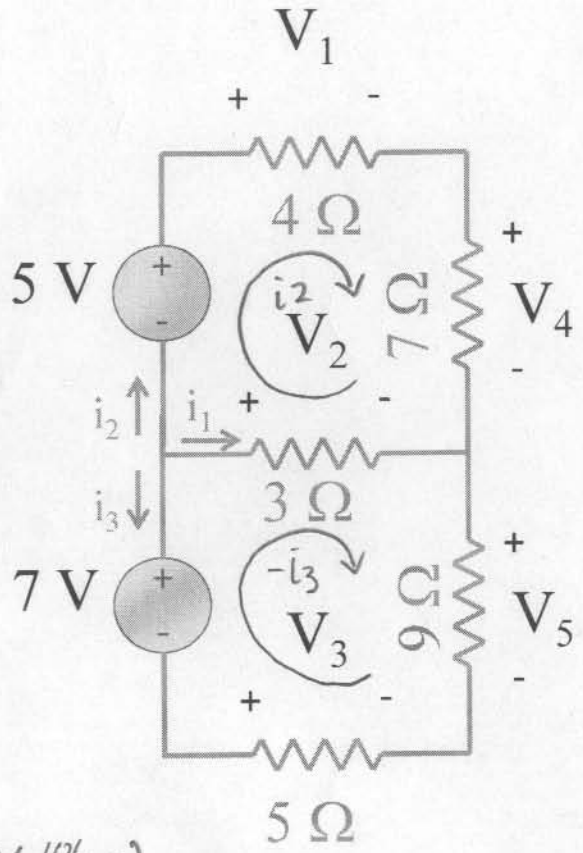
$V_2 = (4\Omega)(-\tfrac{1}{2}A) = -2V$

$V_3 = (7\Omega)(\tfrac{1}{2}A) = 3.5V$

$V_4 = (5\Omega)(\tfrac{1}{2}A) = 2.5V$

Problem 8: KVL, KCL & Ohm

Find V_1 through V_4 and i_1 through i_3 in the circuit below.



$V_1 = (4)\left(\frac{106}{229}A\right) = 1.85V$

$V_2 = (3)\left(-\frac{7}{229}A\right) = -0.01V$

$V_3 = (5)\left(-\frac{113}{229}\right) = -2.47V$

$V_4 = (7)\left(\frac{106}{229}\right) = 3.24V$

KVL @ Loop i_2:

$$-5 + 4I_2 + 7I_2 + 3(I_2 - (-I_3)) = 0$$

$$-5 + 11I_2 + 3I_2 + 3I_3 = 0$$

$$14I_2 + 3I_3 = 5$$

KVL @ Loop $-i_3$:

$$-7 + 3(-i_3 - i_2) + -i_3 \cdot 9 - i_3 \cdot 5 = 0$$

$$-7 - 17i_3 - 3i_2 = 0$$

$$17i_3 + 3i_2 = -7$$

2 equations, 2 unknowns

solving: $17i_3 + 3i_2 = -7$

$14I_2 + 3I_3 = 5$,

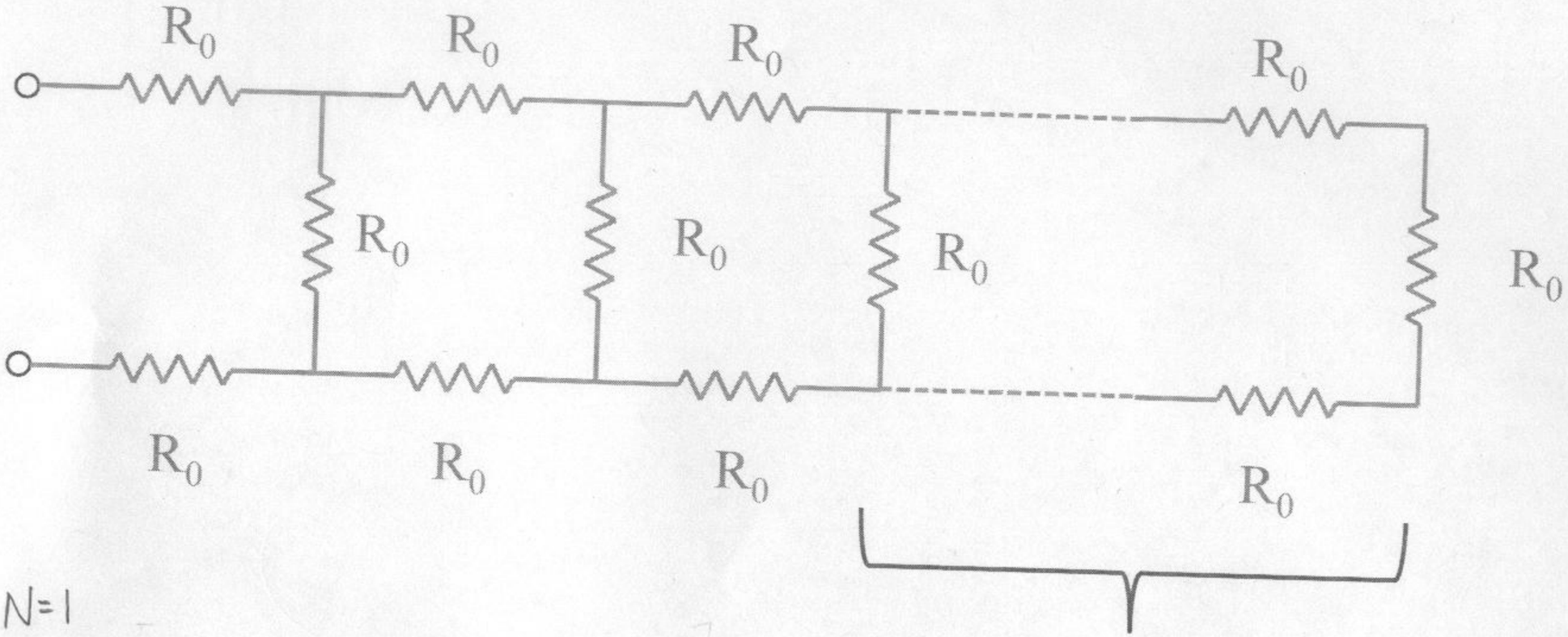
$\boxed{i_2 = \frac{106}{229}A}$, $\boxed{i_3 = -\frac{113}{229}A}$

$i_1 = -(i_2 + i_3)$

$= -\left(\frac{106}{229} - \frac{113}{229}\right)$

$= \boxed{-\frac{7}{229}A}$

Extra credit: Solve for R_{eq}.



Pattern repeats a total of N times.

N=1

$2R_0 + R_0$

N=2

$2R_0 + R_0 // 3R_0$

⋮

N = N times:

$R_{eq} = 2R_0 + [R_0 // R_{eqn-1}]$