Problem 1:
Time to frequency and back

A) Given $v(t) = 4\cos(\omega t + \pi/4)$ find the phasor **V** that represents $v(t)$. Express **V** as $x+jy$ and as $re^{j\phi}$.
B) Given $i(t) = 4\sin(\omega t + \pi/2)$ find the phasor **I** that represents $i(t)$. Express **I** as $x+jy$ and as $re^{j\phi}$.
C) Given $\mathbf{V} = 3 + j4$ find $v(t)$.
D) Given $\mathbf{I} = 1.5\, e^{j\pi/3}$ find $i(t)$.

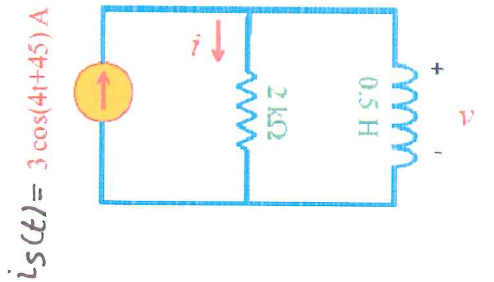
A) $v(t) = 4\cos(\omega t + \frac{\pi}{4})\,\mathrm{v} \Rightarrow V = 4e^{j\pi/4} = 4\cos\frac{\pi}{4} + j\cdot 4\sin\frac{\pi}{4}$
$= 2\sqrt{2} + j\cdot 2\sqrt{2}$

B) $i(t) = 4\sin(\omega t + \frac{\pi}{2})$
$= 4\cos\omega t \quad \Rightarrow V = 4e^{j0} = 4$
$V = 4(\cos 0 + j\sin 0) = 4 + j\cdot 0 = 4\,\mathrm{v}$

C) $V = 3 + j4$
$|V| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5$
$\varphi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53^\circ, \quad \varphi = 0.927\ \mathrm{rad}$
$v(t) = Re\left(Ve^{j\omega t}\right) = Re\left(5e^{j\cdot 0.927}\cdot e^{j\omega t}\right) = 5\cos(\omega t + 0.927\,(\mathrm{rad}))$
$= 5\cos(\omega t + 53^\circ)\,\mathrm{v}$

D) $I = 1.5\,e^{i\pi/3}$
$i(t) = Re\left(Ie^{j\omega t}\right) = Re\left(1.5e^{j\pi/3}\cdot e^{j\omega t}\right) = Re\left(1.5e^{j(\omega t + \pi/3)}\right)$
$= 1.5\cos(\omega t + \pi/3)\ A$

Problem 2

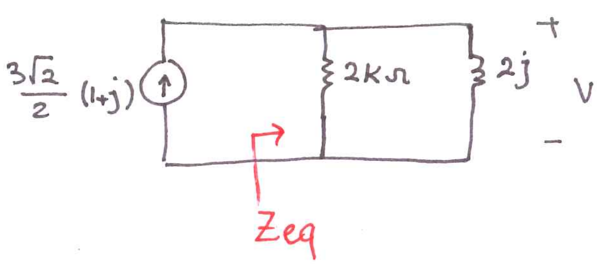


Find i(t) and v(t). Hint: convert the current source into a phasor, then find the current and voltage phasors for the whole circuit, then convert back to the time dependent i(t), v(t).

$$i_s(t) = 3\cos(4t + 45^\circ)\ A \qquad 45^\circ = \pi/4 \text{ rad}$$

$$I_s = 3e^{j\pi/4} = 3\cos\frac{\pi}{4} + 3j\sin\pi/4 = \frac{3\sqrt{2}}{2}(1+j) \quad A$$

$$Z_L = j\omega L = j\times 4\times\frac{1}{2} = 2j\ \Omega$$



$$v = I_s . Z_{eq}$$

$$Z_{eq} = \frac{2k\Omega . 2j\ \Omega}{2k\Omega + 2j\Omega} = \frac{2\times 10^3 \times 2j}{2\times 10^3 + 2j}\ \Omega$$

$$Z_{eq} = \frac{2j}{1+0.001j}\ \Omega$$

$$V = I_s . Z_{eq} = \frac{3\sqrt{2}}{2}(1+j)\times\frac{2j}{1+0.001j} = \frac{3\sqrt{2}(-1+j)}{1+0.001j}\times\frac{1-0.001j}{1-0.001j}$$

$$= 3\sqrt{2}\,\frac{(-1+j)(1-0.001j)}{(1+10^{-6})\approx 1} = 3\sqrt{2}\,\frac{-1+0.001j+j+0.001}{1}$$

$$= 3\sqrt{2}(-0.999+1.001j) = -4.238+4.246j \text{ (V)} \approx 6e^{j(2.35)} \text{ (V)}$$

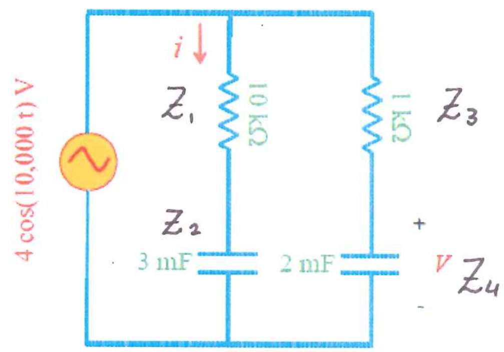
$$V(t) = 6\cos(4t+2.35) \text{ (V)}$$

problem 2.

$$I = \frac{V}{Z} = \frac{V}{R}$$

$$I = \frac{6e^{j(2.35)}\,V}{2\,K\Omega} = 3e^{j(2.35)}\,mA \Rightarrow i(t) = 3\cos(4t + 2.35\,(rad))\,mA$$

Problem 3



$\omega = 10^4$

$Z_1 = 10\ k\Omega$

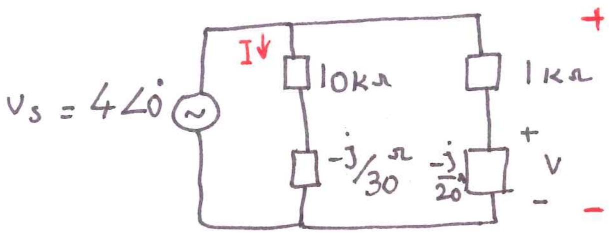
$Z_2 = \dfrac{1}{j\omega C} = \dfrac{1}{j\times 10^4 \times 3\times 10^{-3}} = \dfrac{1}{30j} = \dfrac{-j}{30}\ \Omega$

$Z_3 = 1\ k\Omega$

$Z_4 = \dfrac{1}{j\omega C} = \dfrac{1}{j\times 10^4 \times 2\times 10^{-3}} = \dfrac{-j}{20}\ \Omega$

Find i(t) and v(t). Hint: convert the voltage source into a phasor, then find the current and voltage phasors for the whole circuit, then convert back to the time dependent i(t), v(t).

$V_s = 4\angle 0^\circ\ V$



$V = \dfrac{V_s \times Z_4}{Z_4 + Z_3}$

$V = \dfrac{4V \times -j/20\,\Omega}{1000\,\Omega - \frac{j}{20}\,\Omega} \xrightarrow{\times -5} = \dfrac{(j)\times 4}{(-5000 + \frac{j}{4})\times 4} = \dfrac{4j}{-20000 + j}$

$V = \dfrac{4j}{-20000 + j} = \dfrac{4j(-20000 - j)}{(20000)^2 + 1^2} = \dfrac{-80000j + 4}{4000001 \approx 400000000} = \dfrac{-20000j + 1}{10^8}\ V$

-1.57 rad is almost equal to -90 degrees

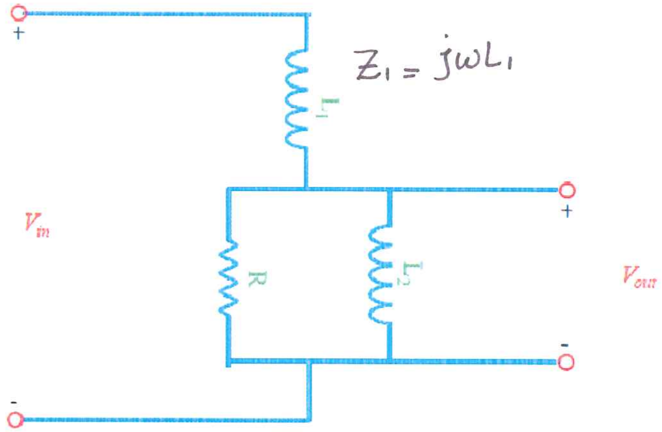
$V = \dfrac{2000\, e^{-1.57j}}{10^8}\ V \approx 0.2\, e^{-1.57j}\ mV \Rightarrow V(t) = 0.2\cos(10^4 t - 1.57)\ mV$

$I = \dfrac{V_s}{Z_1 + Z_2} = \dfrac{4V}{10k\Omega + \frac{-j}{30}\Omega} = \dfrac{4}{10^4 - \frac{j}{30}}\ A = \dfrac{4(10^4 + j/30)}{\underbrace{(10^4)^2 + (\frac{1}{30})^2}_{\approx 10^8}} = \dfrac{4\times 10^4\, e^{j(3.33\times 10^{-6})}}{10^8}$

$I = 0.4\, e^{j(3.3\times 10^{-6})}\ mA$

$i(t) = 0.4\cos(10^4 t + 3.3\times 10^{-6})\ mA$

Problem 4: Transfer function

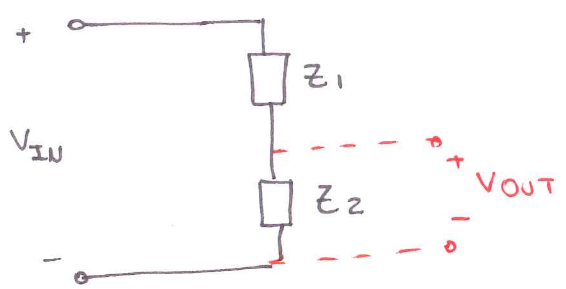


$Z_1 = j\omega L_1$

$Z_2 = R \parallel j\omega L_2$

$$Z_2 = \frac{R \cdot j\omega L_2}{R + j\omega L_2}$$

Calculate H(ω) for this circuit. Sketch the magnitude of H(ω) vs. ω.



Voltage divider :

$$V_{OUT} = \frac{V_{IN} \cdot Z_2}{Z_1 + Z_2}$$

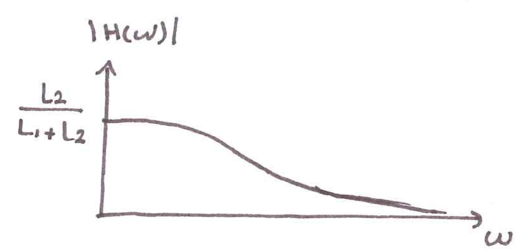
$$V_{OUT} = \frac{V_{IN} \cdot \dfrac{R \cdot j\omega L_2}{R + j\omega L_2}}{\dfrac{R \cdot j\omega L_2}{R + j\omega L_2} + j\omega L_1} \Rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{R \cdot j\omega L_2}{R \cdot j\omega L_2 + j\omega L_1 (R + j\omega L_2)}$$

$$H(\omega) = \frac{V_{OUT}}{V_{IN}} = \frac{j\omega R L_2}{j\omega R(L_2 + L_1) + j^2\omega^2 L_1 L_2} = \frac{R L_2}{R(L_1 + L_2) + j\omega L_1 L_2}$$

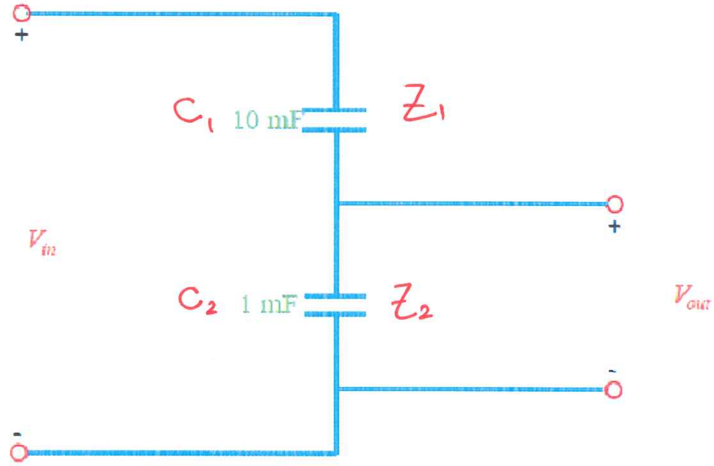
$$|H(\omega)| = \left| \frac{R L_2}{R(L_1 + L_2) + j\omega L_1 L_2} \right| = \frac{R L_2}{\sqrt{R^2 (L_1 + L_2)^2 + \omega^2 L_1^2 L_2^2}}$$

$$\lim_{\omega \to 0} |H(\omega)| = \frac{R L_2}{R(L_1 + L_2)} = \frac{L_2}{L_1 + L_2} \Rightarrow \text{Low pass filter}$$

$$\lim_{\omega \to \infty} |H(\omega)| = 0$$



Problem 5: Transfer function

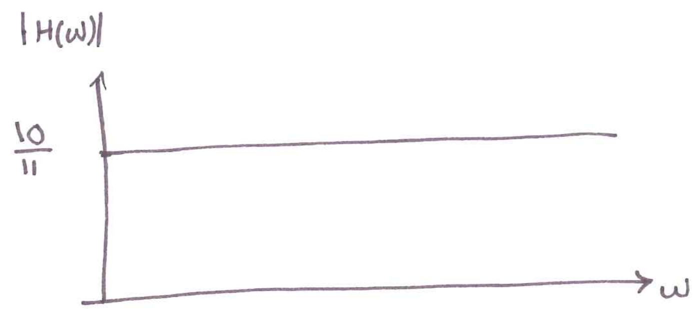


Calculate H(ω) for this circuit. Sketch the magnitude of H(ω) vs. ω.

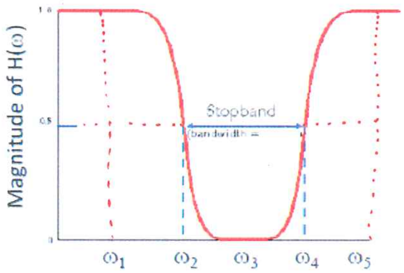
$$V_{OUT} = \frac{V_{IN} \times Z_2}{Z_1 + Z_2}$$

$$H(\omega) = \frac{V_{OUT}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2} = \frac{1/j\omega C_2}{1/j\omega C_1 + \frac{1}{j\omega C_2}} = \frac{1/C_2}{1/C_1 + 1/C_2} = \frac{1/1\times 10^{-3}}{\frac{1}{10\times 10^{-3}} + 1/10^{-3}} = \frac{10}{1+10}$$

$$H(\omega) = \frac{10}{11} \Rightarrow |H(\omega)| = 10/11$$



Problem 6: Band stop filter



A) Given $v(t) = 4\cos(\omega_1 t + \pi/4)$ find $v_{out}(t)$.
B) Given $v(t) = 3\cos(\omega_2 t + \pi/2)$ find $v_{out}(t)$.
C) Given $v(t) = 2\cos(\omega_3 t + \pi/3)$ find $v_{out}(t)$.
D) Given $v(t) = 2\cos(\omega_4 t + \pi/2)$ find $v_{out}(t)$.
E) Given $v(t) = 2\cos(\omega_5 t + \pi/4)$ find $v_{out}(t)$.

Note: You cannot determine the phase of $v_{out}(t)$ so leave that as unknown in your answer.

A) $v(t) = 4\cos(\omega_1 t + \pi/4) \Rightarrow V = 4e^{j\pi/4}$

$|H(\omega_1)| = 1 \Rightarrow \left|\frac{V_{out}}{V_{in}}\right|_{\omega=\omega_1} = 1 \Rightarrow |V_{out}| = |V_{IN}| = 4$

$V_{out}(t) = 4\cos(\omega_1 t + \varphi_1)$ (φ is unknown)

B) $v(t) = 3\cos(\omega_2 t + \frac{\pi}{2}) \Rightarrow V = 3e^{j\pi/2}$

$|V_{out}|_{\omega=\omega_2} = |H(\omega_2)||V_{in}| = 0.5 \times 3 = 1.5 \Rightarrow V_{out}(t) = 1.5\cos(\omega_2 t + \varphi_2)$

C) $v(t) = 2\cos(\omega_3 t + \pi/3) \Rightarrow V = 2e^{j\pi/3}$

$|H(\omega_3)| = 0 \Rightarrow \frac{|V_{out}|}{|V_{in}|} = 0 \Rightarrow |V_{out}| = 0 \Rightarrow V_{out}(t) = 0$

D) $v(t) = 2\cos(\omega_4 t + \pi/2) \Rightarrow V = 2e^{j\pi/2}$

$|H(\omega_4)| = 0.5 \Rightarrow |V_{out}| = |H(\omega_4)| \cdot |V_{in}| = 0.5 \times 2 = 1$

$V_{out}(t) = \cos(\omega_4 t + \varphi_3)$

E) $v(t) = 2\cos(\omega_5 t + \pi/4) \Rightarrow V = 2e^{j\pi/4}$, $|H(\omega_5)| = 1 \Rightarrow |V_{out}| = 2$

$V_{out}(t) = 2\cos(\omega_5 t + \varphi_4)$