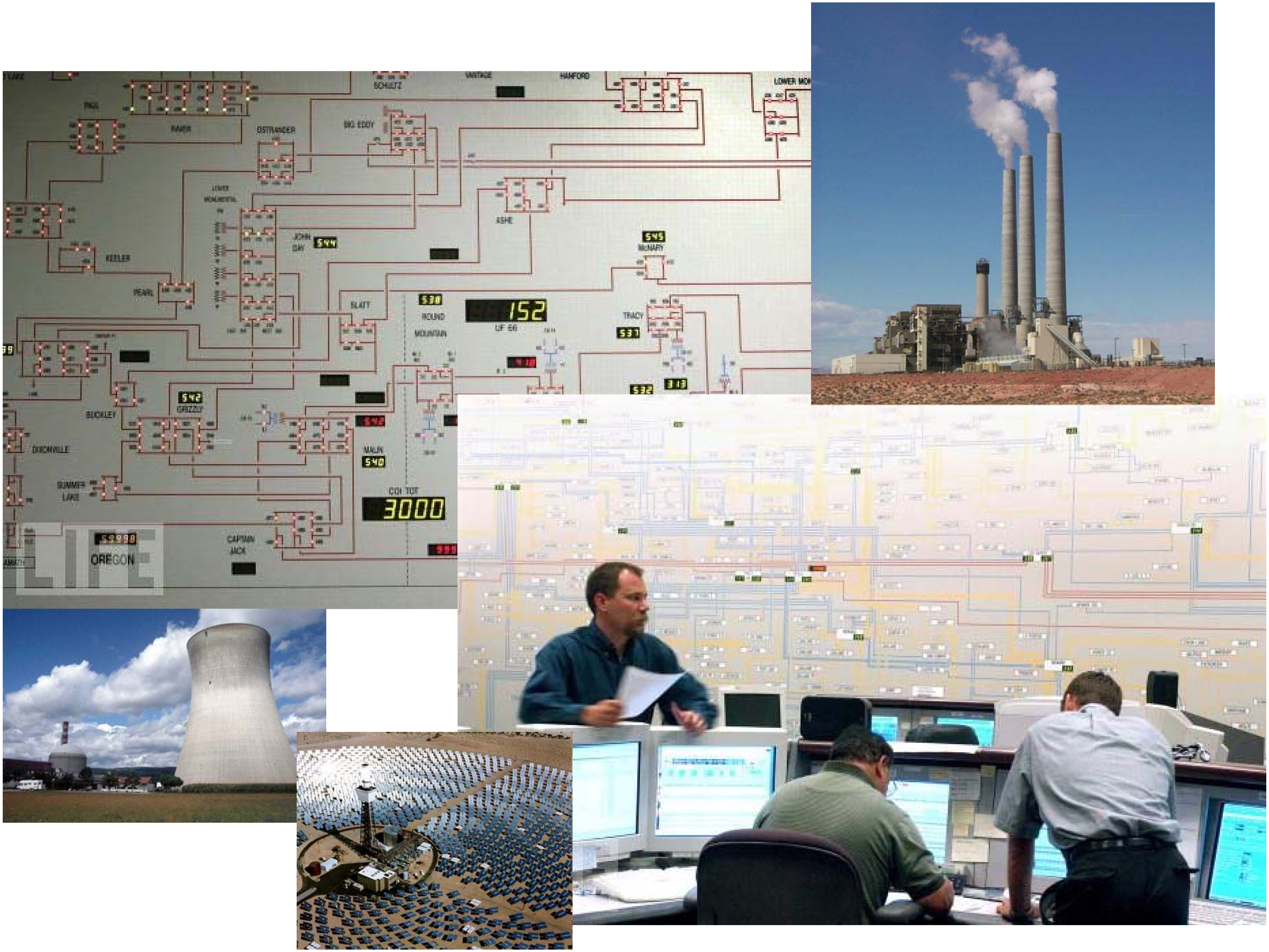
Announcements:

1. HW4 due Friday this week

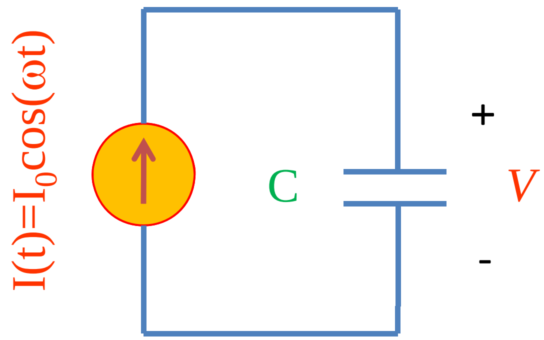
EECS 70A: Network Analysis

Lecture 10


SCHULTZ
VANTAGE
HANFORD
PAUL
RAVER
OSTRANDER
BIG EDDY
ASHE
KEELER
PEARL
JOHN DAY
McNARY
SLATT
ROUND MOUNTAIN
152
TRACY
GRIZZLY
BUCKLEY
DIXONVILLE
SUMMER LAKE
MALIN
COI TOT
3000
CAPTAIN JACK
OREGON
LIFE

Example Capacitor Problem #2

Find V(t), q(t)



$j = \sqrt{-1}$ $\quad q = CV \quad i = C\frac{dV}{dt}$

$$V(t) = \frac{1}{C}\int i(t)\,dt \qquad V = \frac{1}{C}\int i(t)\,dt$$

$$= \frac{1}{C}\int I_0 \cos(\omega t)\,dt = \frac{I_0}{C}\sin(\omega t)$$

Euler

$$I(t) = \mathrm{Re}\left[I_0 e^{j\omega t}\right] \quad \text{why? b.c. (because)} \qquad e^{j\theta} = \cos\theta + j\sin\theta$$

$$z = x + jy$$

$$\mathrm{Re}[z] = x$$

$$= \mathrm{Re}\left[I_0(\cos(\omega t) + j\sin(\omega t))\right] = I_0\cos(\omega t)$$

$$V(t) = \frac{1}{C}\int \mathrm{Re}\left[I_0 e^{j\omega t}\right]dt = \frac{I_0}{C}\mathrm{Re}\left[\int e^{j\omega t}\,dt\right]$$

$$= \mathrm{Re}\left[\frac{I_0}{C}\frac{1}{j\omega}e^{j\omega t}\right] = \mathrm{Re}\left[\frac{1}{j\omega C} I_0\, e^{j\omega t}\right]$$

OVER →

Phasor

$$\int e^{j\omega t}\,dt = \qquad\qquad \int e^{u}\,du = e^{u}$$

$$u = j\omega t$$

$$du = j\omega\,dt \Rightarrow dt = \frac{1}{j\omega}\,du$$

$$= \int e^{u}\,\frac{1}{j\omega}\,du = \frac{1}{j\omega}\int e^{u}\,du = \frac{1}{j\omega}e^{u} = \frac{1}{j\omega}e^{j\omega t}$$

Phasors

$$I(t) = \mathrm{Re}\left[\underbrace{I_0}_{\text{CURRENT PHASOR}} e^{j\omega t}\right]$$

$$V(t) = \mathrm{Re}\left[\underbrace{\frac{1}{j\omega C} I_0}_{\text{VOLTAGE PHASOR}} e^{j\omega t}\right]$$

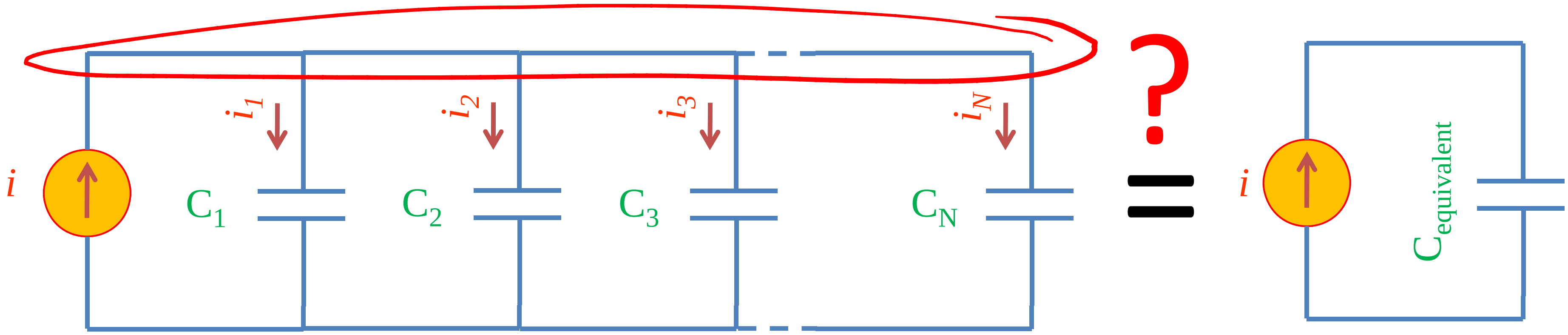
Voltage phasor $\mathbf{V}$

Current Phasor $\mathbf{I}$

For a capacitor we have $\mathbf{V} = \mathbf{I} \underbrace{\frac{1}{j\omega C}}$

Can apply KCL, KVL, nodal/mesh anal, "impedance" Norton, Thev. to cap. circuits if we use <u>phasors.</u>

Parallel Capacitors



KCL: $i = i_1 + i_2 + \ldots i_N$

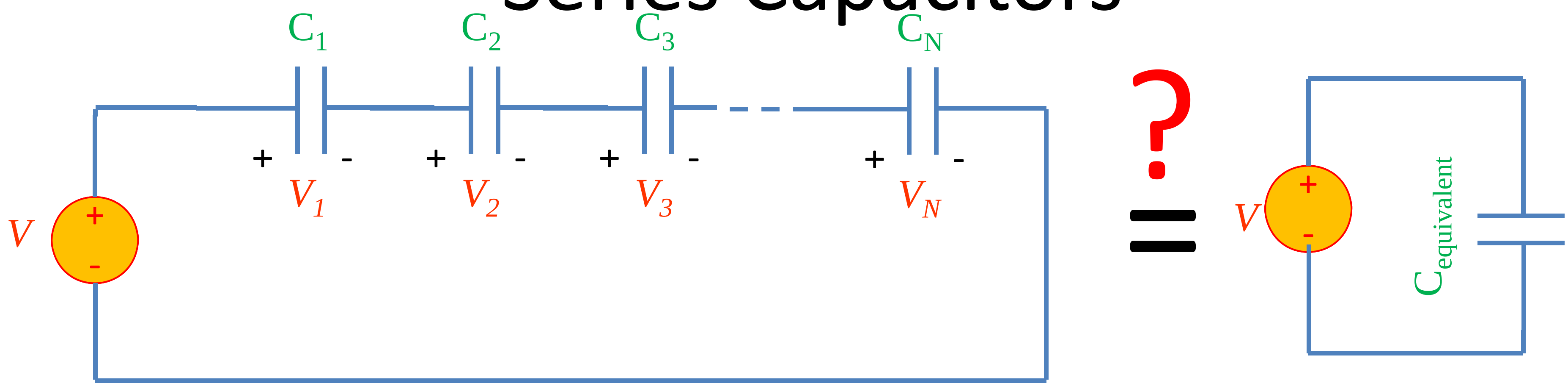
$i_1 = C_1 \frac{dV}{dt}$ $\quad i_2 = C_2 \frac{dV}{dt}$ $\quad \ldots$ <u>same v</u>

$i = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} + \ldots C_N \frac{dV}{dt}$

$$= \frac{dV}{dt} \underbrace{\sum_{k=1}^{N} C_k}_{C_{eq}}$$

$$C_{eq} = \sum_{k=1}^{N} C_k$$

Series Capacitors



KVL

$$V = V_1 + V_2 + \cdots V_N$$

$$V_1 = \frac{1}{C_1}\int i\,dt \qquad V_2 = \frac{1}{C_2}\int i\,dt \quad \cdots \quad \text{same } i$$

$$V = \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt + \cdots \frac{1}{C_N}\int i\,dt$$

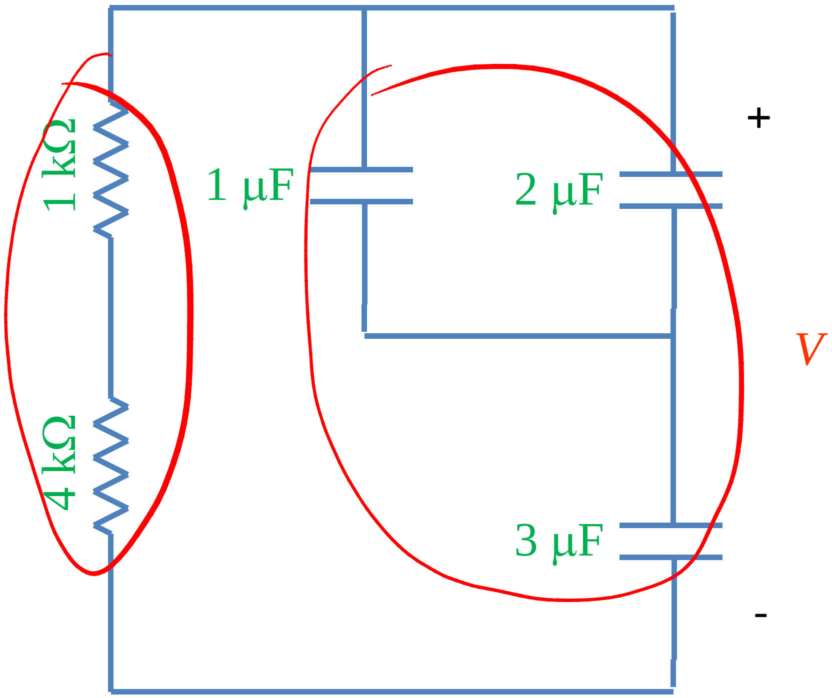
$$= \int i\,dt \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N}\right)}_{C_{eq}^{-1}}$$

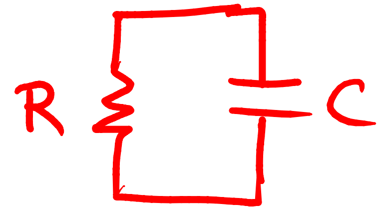
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N}$$

Example problem #4

$V_c = V(t=0)e^{-t/RC}$

(Students) Find V(t), given that V(t=0) = 5 Volts

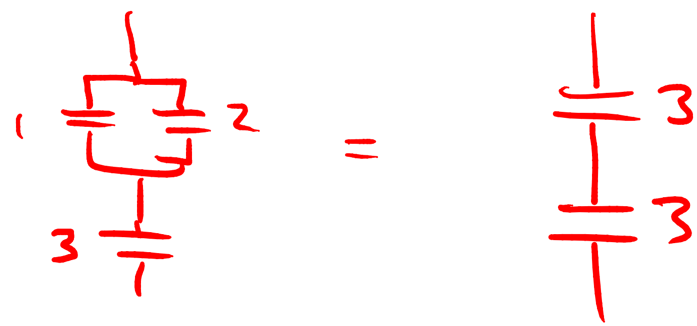




$V(t) = 5V e^{-t/5000\times\frac{3}{2}10^{-6}s}$

$R_{eq} = (4+1)k\Omega$

$C_{eq} =$

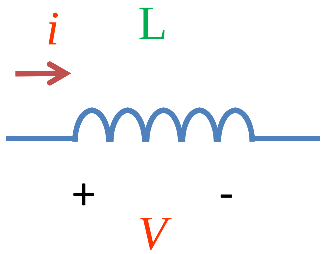


=  $\frac{1}{\frac{1}{3}+\frac{1}{3}} = \frac{3}{2}\mu F$

$\mu = 10^{-6}$

$K = 10^3$

Inductors



$$L = \frac{N^2 \mu A}{l}$$

A=area
l=wire length
N = # of turns
$\mu = 4\,\pi\, 10^{-6}$ H/m

$$V = L\frac{di}{dt}$$

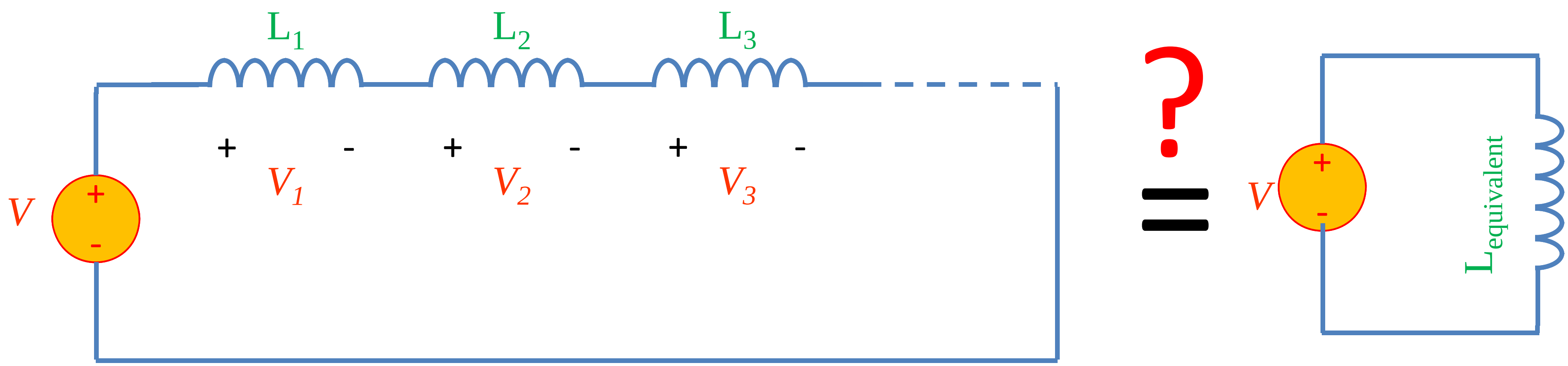
Henry[H]

Compare $q = CV$

$$i = C\frac{dV}{dt}$$

$$V = L\frac{di}{dt} \Rightarrow i(t) = \frac{1}{L}\int V(t)dt$$

Series Inductors



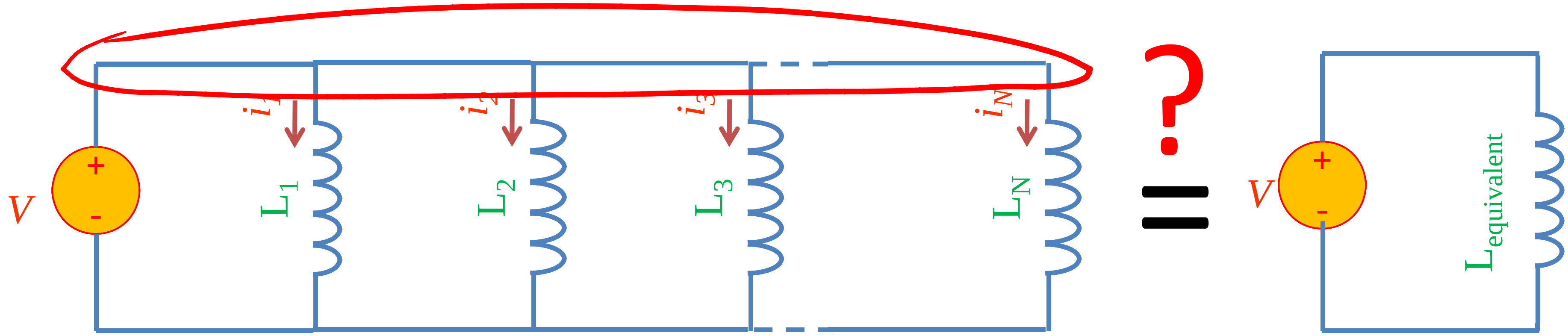
KVL $V = V_1 + V_2 + \cdots V_N$

$V_1 = L_1 \frac{di}{dt}$ $\quad V_2 = L_2 \frac{di}{dt}$ $\quad \cdots$ $\quad i$ same

$$V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots = \frac{di}{dt}\underbrace{\left(L_1 + L_2 + \cdots L_N\right)}_{L_{eq}}$$

$$L_{eq} = L_1 + L_2 + \cdots L_N$$

Parallel Inductors



KCL $\quad i = i_1 + i_2 + \cdots i_N$

$i_1 = \frac{1}{L_1}\int v\,dt \qquad i_2 = \frac{1}{L_2}\int v\,dt \quad \cdots$ same v

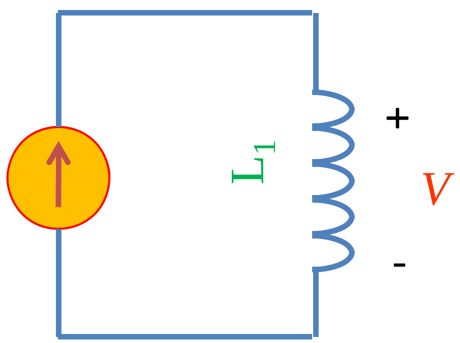
$$i = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt + \cdots$$

$$= \int v\,dt \underbrace{\left[\frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}\right]}_{\frac{1}{L_{eq}}}$$

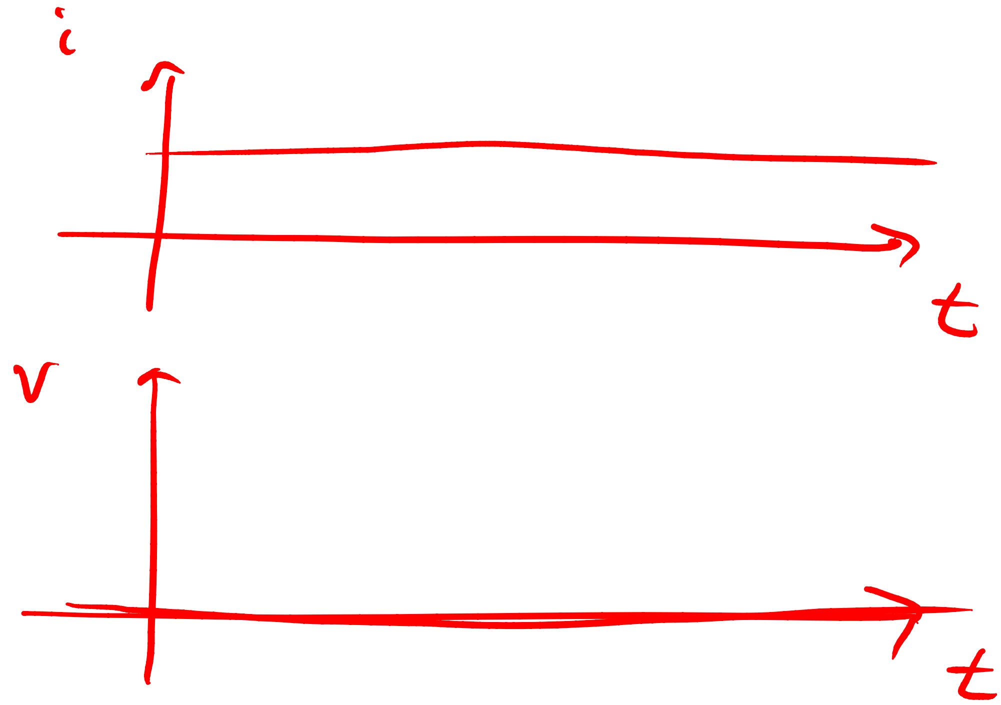
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}$$

Example Inductor Problem

(Students): Find V(t).

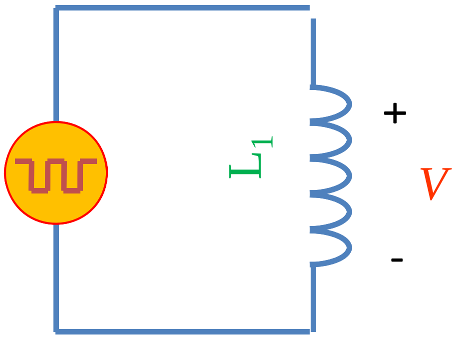


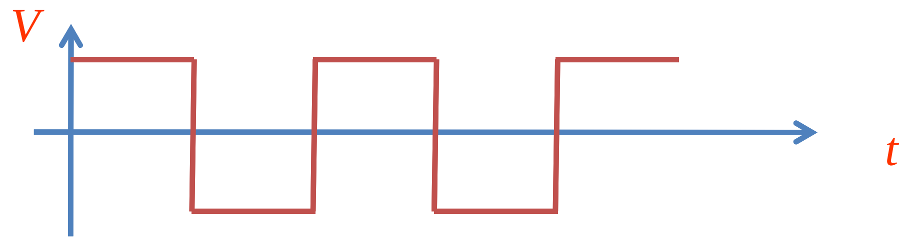
$$v = L\frac{di}{dt}$$

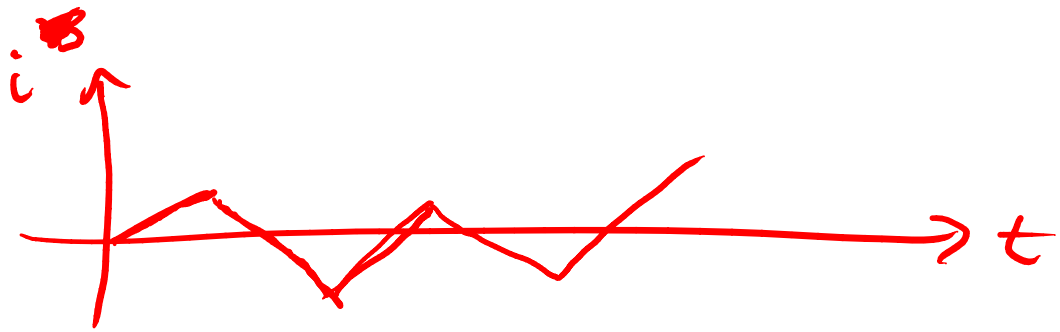


Example Inductor Problem #2

(Students): Find i(t)





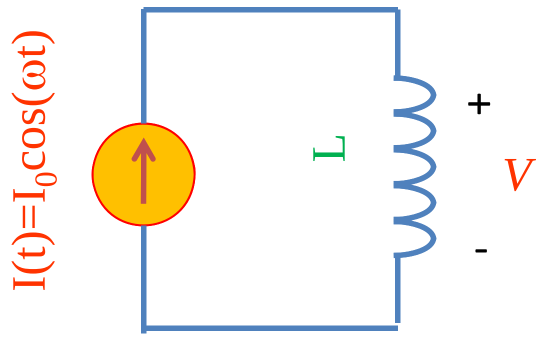


$$v = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v\,dt$$

Example Inductor Problem #3

Find V(t)



$$V = L\frac{di}{dt}$$

$$= L\frac{d}{dt}\left[I_0\cos(\omega t)\right]$$

$$= -I_0 L\omega \sin(\omega t)$$

$$I(t) = \mathrm{Re}\left[\underbrace{I_0}_{\text{current phasor } \mathbb{I}} e^{j\omega t}\right] = \mathrm{Re}\left[\mathbb{I}\, e^{j\omega t}\right]$$

$$V(t) = L\frac{d}{dt}\mathrm{Re}\left[I_0 e^{j\omega t}\right] = \mathrm{Re}\left[L I_0 \frac{d}{dt} e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[L I_0 j\omega e^{j\omega t}\right] = \mathrm{Re}\left[\underbrace{I_0\, j\omega L}_{\text{voltage phasor } \mathbb{V}}\, e^{j\omega t}\right]$$

Inductor: $\mathbb{V} = \underbrace{j\omega L}_{\text{impedance}}\, \mathbb{I}$

“Impedance” = Z

R

$\frac{1}{j\omega C}$

$j\omega L$

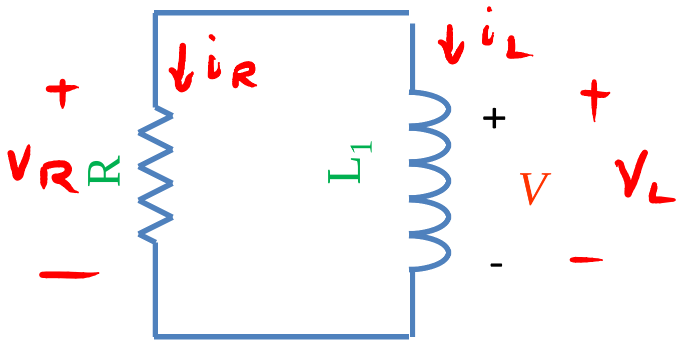
$$V = Z\,I$$

KVL, KCL, Node/Mesh, Thev, Norton

LR circuit

$V_R = i_R R$

Find V(t), i(t)



$V_L = L \frac{di_L}{dt}$

KVL $\Rightarrow$ $V_L = V_R$

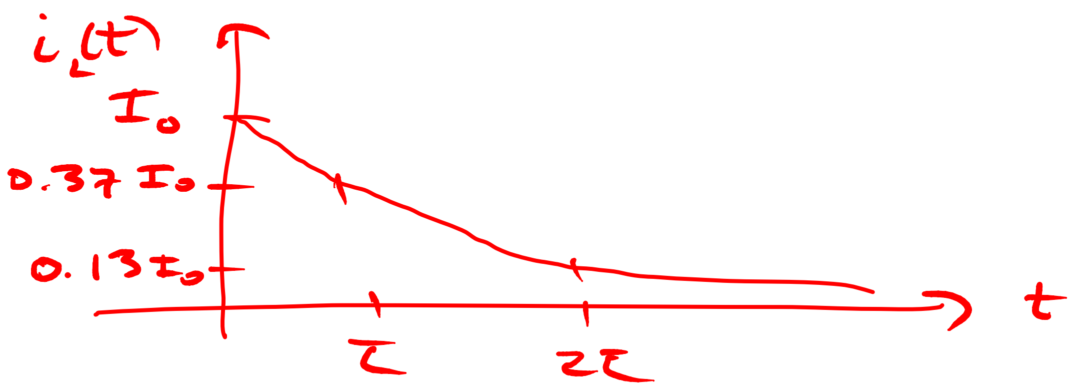
KCL $\Rightarrow$ $i_R = -i_L$

$$L \frac{di_L}{dt} = i_R R = \boxed{-i_L R = L \frac{di_L}{dt}}$$

$$\Rightarrow \quad \frac{di_L}{dt} = -\frac{1}{L/R} i_L$$

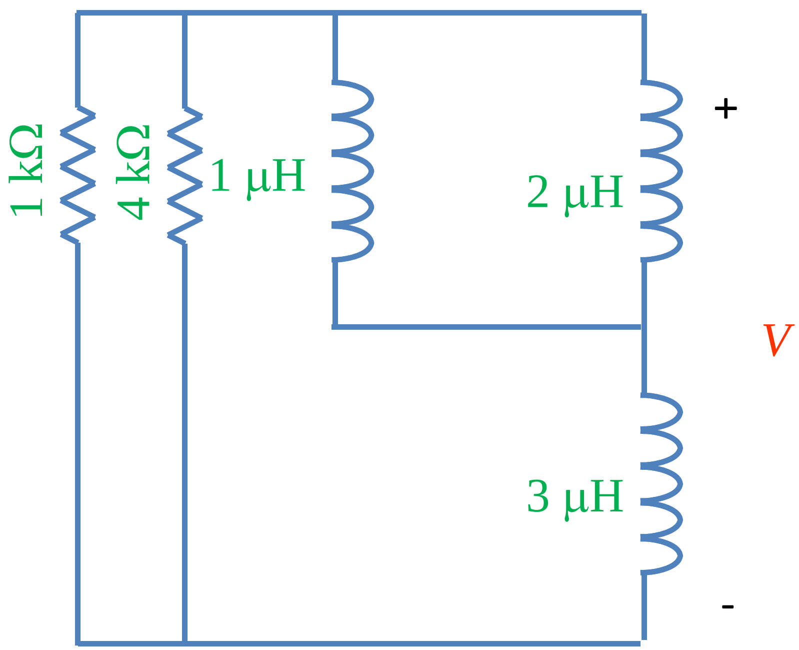
$\tau \equiv \frac{L}{R}$ time constant

$$i_L(t) = i_L(t=0)\, e^{-t/\tau}$$



Example LR problem

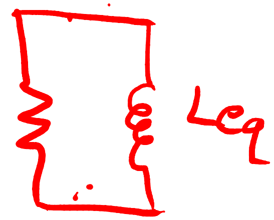
(Students) Find V(t), given that V(t=0) = 5 Volts



$\tau = \frac{L_{eq}}{R_{eq}}$

$4k\Omega \,||\, 1k\Omega$

$4k\Omega || 1k\Omega = R_{eq}$

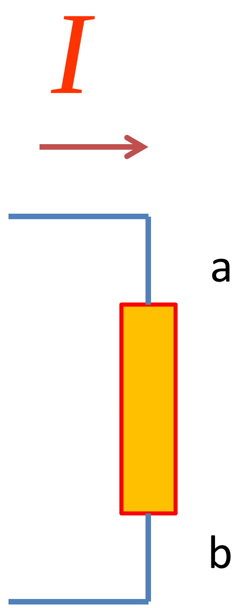


$(2\mu H || 1\mu H) + 3\mu H$

$i_L = i_L(t=0)\, e^{-t/\tau}$

$i_L(t=0) = \frac{5V}{R_{eq}}$

Power



$$I \times V_{ab} = power$$

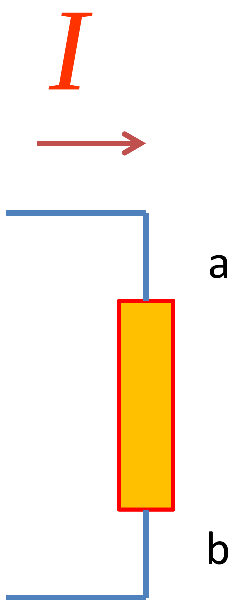
Watts [W] = Volt Amp [V-A]

Note: MKSA unit system:
Meters Kilogram Second Amp

Resistor:
Energy lost to heat…

Inductor or capacitor:
Energy *STORED* and can be recovered…

Energy stored



$$I \times V_{ab} = power$$

Energy:

$$W = \int P dt = \int I \cdot V dt$$

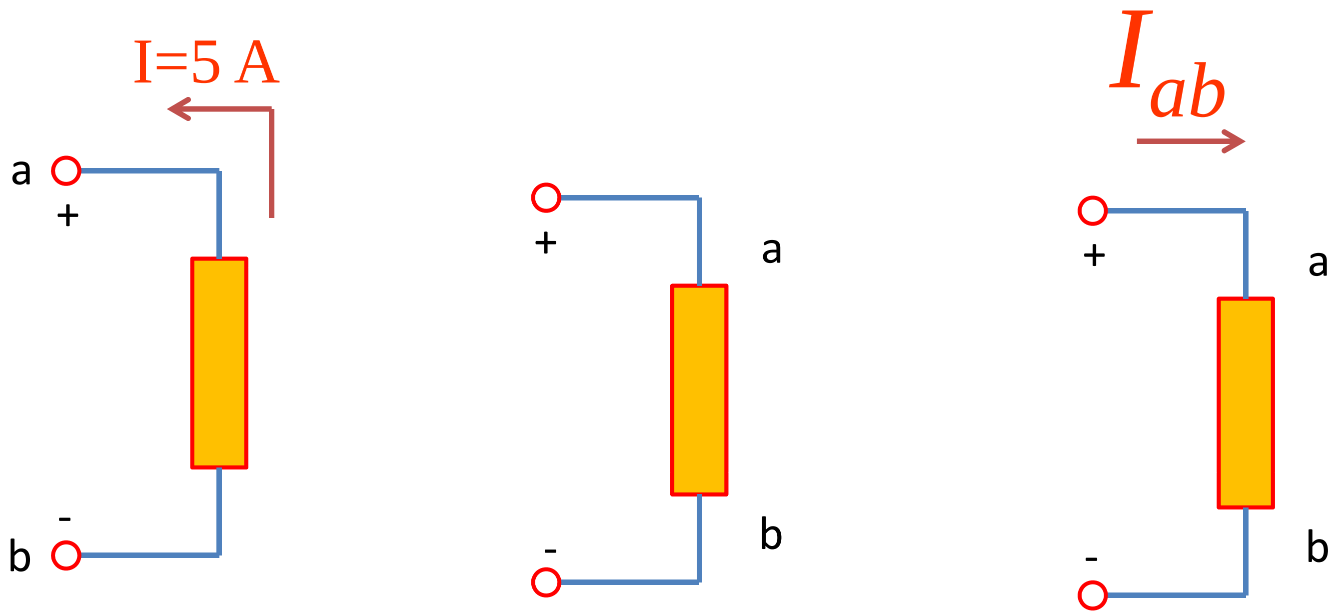
Capacitor stored energy:

$$\int I \cdot V dt = \int C \frac{dV}{dt} \cdot V dt = \frac{1}{2} C V^2 = \frac{Q^2}{2C}$$

Inductor stored energy:

$$\int I \cdot V dt = \int I \cdot L \frac{dI}{dt} dt = \frac{1}{2} L I^2$$

Symbol library



Symbol library