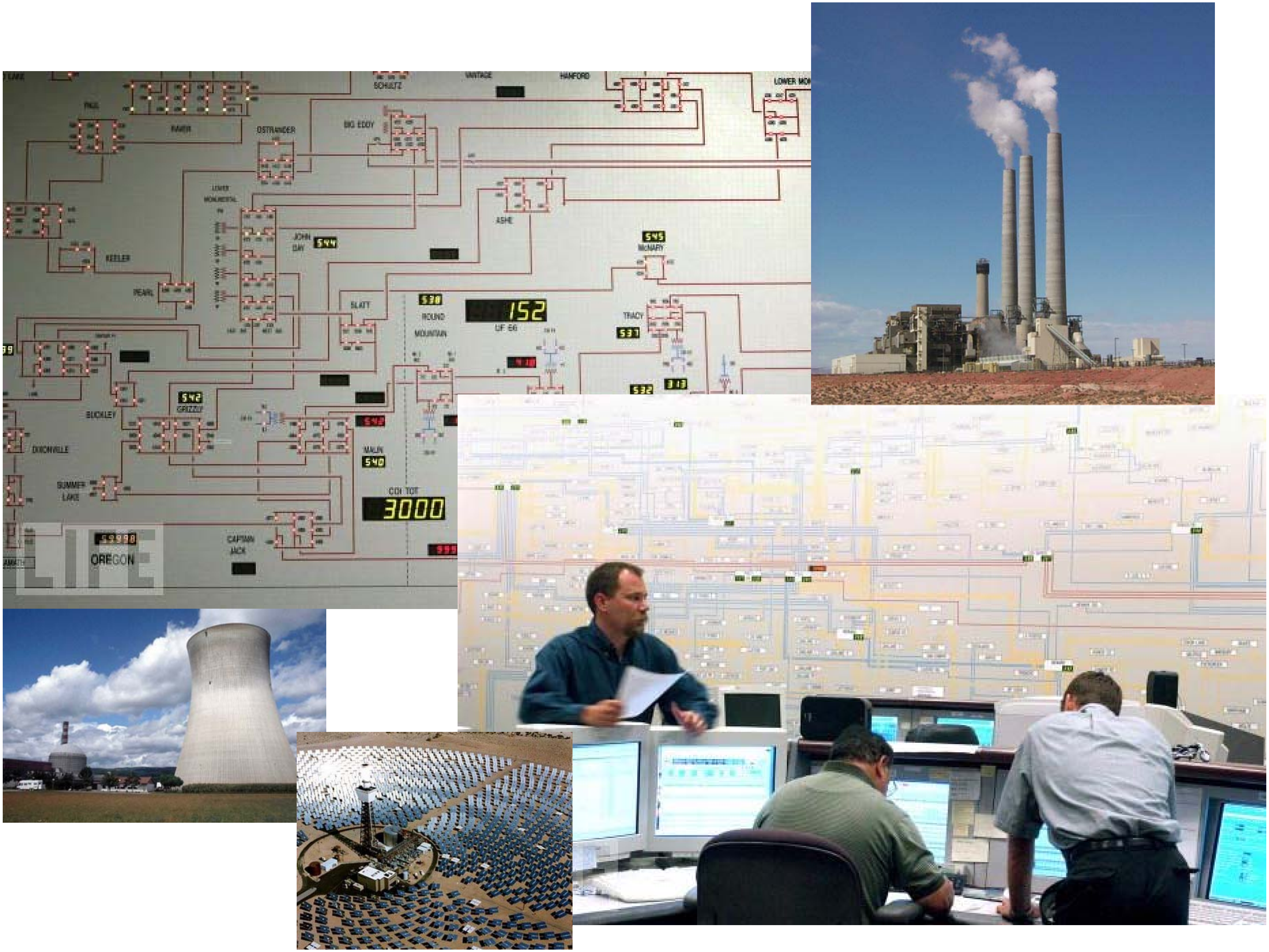
Announcements:

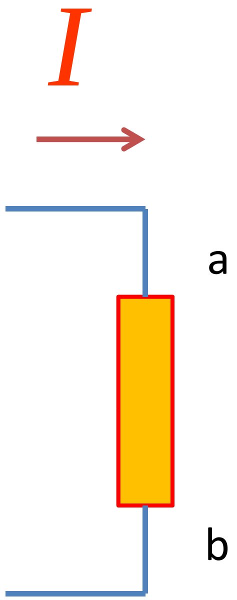
1. HW due tomorrow at 5.
   Hand in at MAE boxes (EG 2nd floor) OR
   To TA from 4-5 in discussion room ICF103

EECS 70A: Network Analysis

Lecture 11


SCHULTZ
VANTAGE
HANFORD
LOWER MON
PAUL
RAVER
OSTRANDER
BIG EDDY
LOWER MONUMENTAL
ASHE
JOHN DAY
544
545
McNARY
KEELER
PEARL
SLATT
538
ROUND MOUNTAIN
152
LIF 66
TRACY
533
542
BUCKLEY
DIXONVILLE
MALIN
540
SUMMER LAKE
COI TOT
3000
CAPTAIN JACK
OREGON
LIFE

Power



$$I \text{ x } V_{ab} = power$$

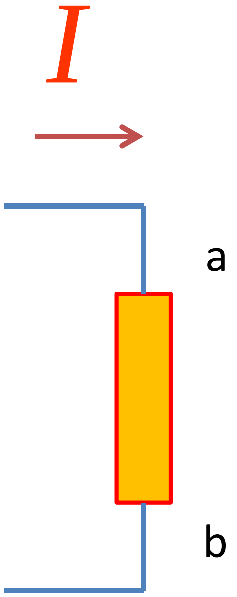
Watts [W] = Volt Amp [V-A]

Note: MKSA unit system:
Meters Kilogram Second Amp

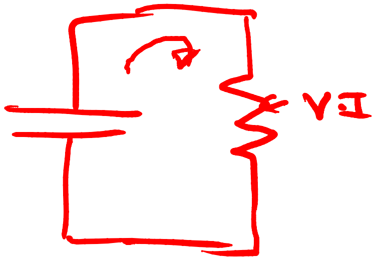
Resistor:
Energy lost to heat…

Inductor or capacitor:
Energy *STORED* and can be recovered…

Energy stored



$$I \text{ x } V_{ab} = power$$



Energy:

$$W = \int P dt = \int I \cdot V dt$$

$Q = CV$ CAP

$$I = C\frac{dV}{dt}$$

$$I \cdot V = C\frac{dV}{dt}V$$

Capacitor stored energy:

$$\int I \cdot V dt = \int C\frac{dV}{\cancel{dt}} \cdot V\cancel{dt} = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

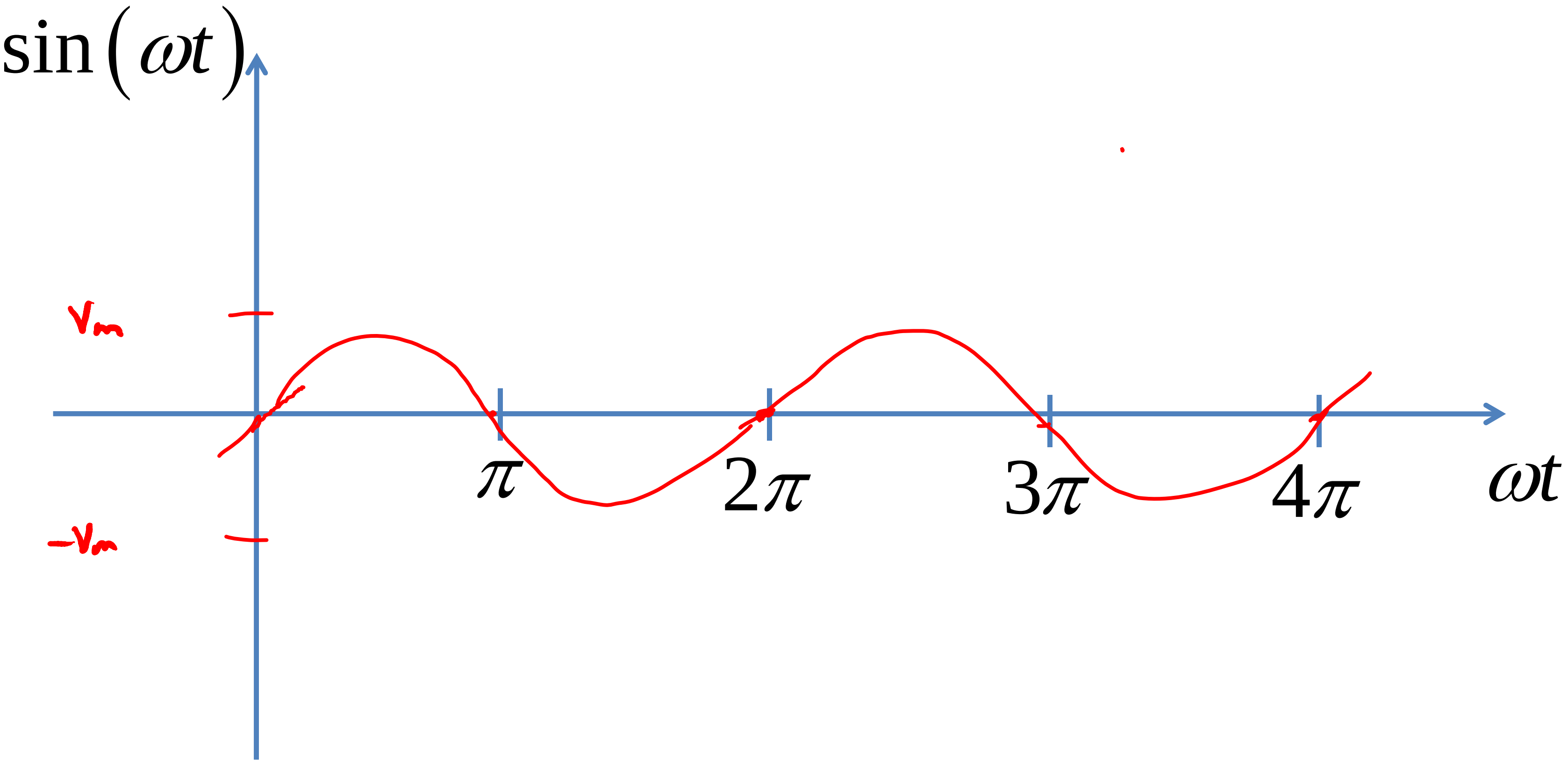
$$V = L\frac{dI}{dt}$$ INDUCTOR

Inductor stored energy:

$$\int I \cdot V dt = \int I \cdot L\frac{dI}{\cancel{dt}}\cancel{dt} = \frac{1}{2}LI^2$$

Sine waves

$$V(t) = \underline{V_m} \sin(\omega t)$$



Phase

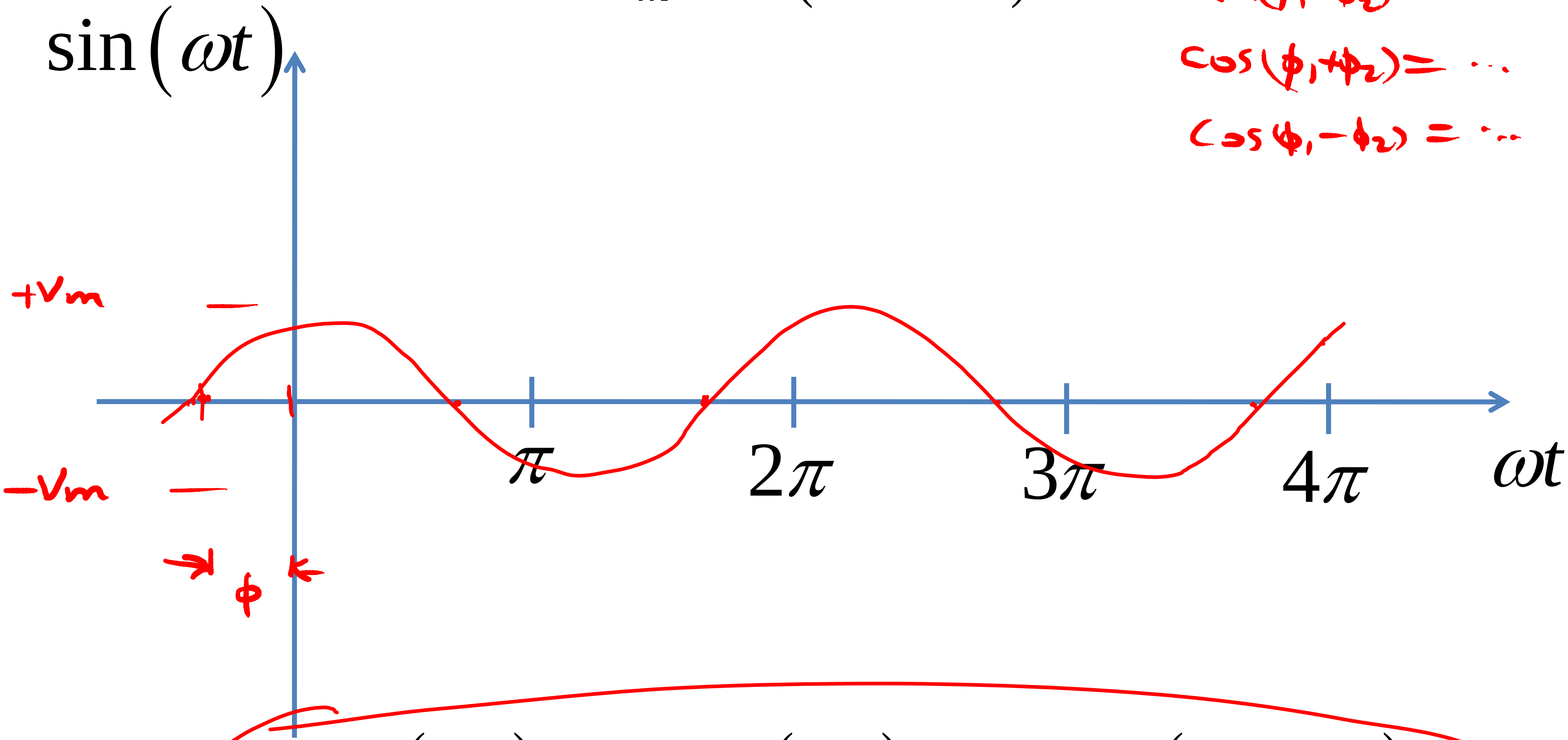
$$V(t) = V_m \sin\left(\omega t + \phi\right)$$

$\sin(\phi_1 + \phi_2) = \cdots$

$\sin(\phi_1 - \phi_2) = \cdots$

$\cos(\phi_1 + \phi_2) = \cdots$

$\cos \phi_1 - \phi_2) = \cdots$



$$A\cos\left(\omega t\right) + B\sin\left(\omega t\right) = C\cos\left(\omega t - \theta\right)$$

$\underline{\theta = \tan^{-1}\left(\frac{B}{A}\right)}$ $\underline{C = \sqrt{A^2 + B^2}}$

* memorize

Complex numbers

Euler $\boxed{e^{j\phi} = \cos\phi + j\sin\phi}$

$$j \equiv \sqrt{-1} \qquad \frac{1}{j} = -j$$

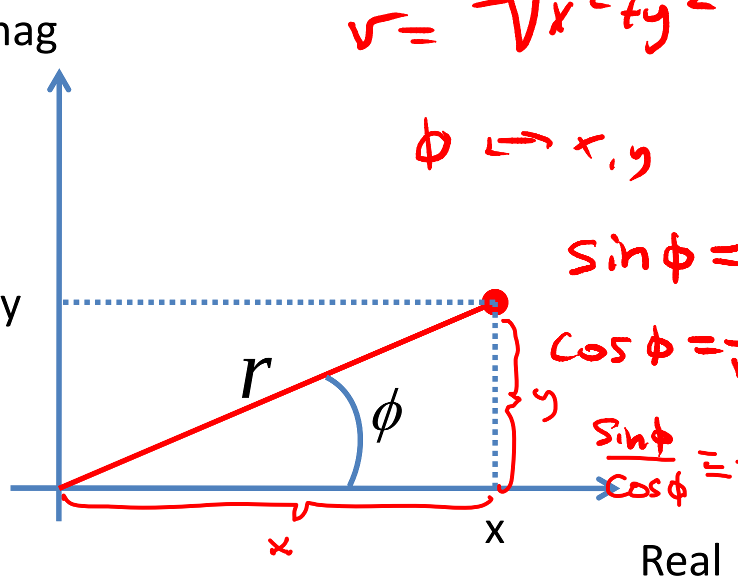
$$z = x + jy$$

$$z = re^{j\phi}$$

$$= r(\cos\phi + j\sin\phi)$$

$$= \underbrace{r\cos\phi}_{Re} + \underbrace{jr\sin\phi}_{Im}$$

$$z = r\angle\phi$$



$r = \sqrt{x^2+y^2}$

$\phi \leftrightarrow x, y$

$\sin\phi = \frac{y}{r}$

$\cos\phi = \frac{x}{r}$

$\frac{\sin\phi}{\cos\phi} = \tan\phi = \frac{y}{x}$

$Re(z) = Real(z) = x$

$Im(z) = Imag(z) = y$

$\phi = \tan^{-1}\left[\frac{y}{x}\right]$

$Re(z) = r\cos\phi \qquad Im(z) = r\sin\phi$

Complex algebra

$$\underline{z_1 = x_1 + jy_1 = \underline{r_1 e^{j\phi_1}}} \qquad z_2 = x_2 + jy_2 = r_2 e^{j\phi_2}$$

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication:

$$z_1 z_2 = \left(r_1 e^{j\phi_1}\right)\left(r_2 e^{j\phi_2}\right) = r_1 r_2 e^{j\phi_1} e^{j\phi_2} = r_1 r_2 e^{j(\phi_1+\phi_2)} = r_1 r_2 \angle(\phi_1 + \phi_2)$$

Division:

$$z_1 / z_2 = \frac{r_1 e^{j\phi_1}}{r_2 e^{j\phi_2}} = \frac{r_1}{r_2}\frac{e^{j\phi_1}}{e^{j\phi_2}} = \frac{r_1}{r_2} e^{j\phi_1} e^{-j\phi_2} = \frac{r_1}{r_2} e^{j\underline{(\phi_1-\phi_2)}}$$

Inversion:

$$1/z_1 = \frac{1}{r_1} e^{-j\phi_1} = \frac{1}{r_1 e^{j\phi_1}}$$

Square root:

$$\sqrt{z_1} = \left(r_1 e^{j\phi_1}\right)^{1/2} = r_1^{1/2}\left(e^{j\phi_1}\right)^{1/2} = r_1^{1/2} e^{j\phi_1/2}$$

Complex conjugate:

$$\boxed{Z^* = x - jy = re^{-j\phi}}$$

Euler relationship

$$\boxed{e^{j\phi} = \cos\phi + j\sin\phi}$$

$$\Rightarrow \cos\phi = \mathrm{Re}(e^{j\phi})$$

Phasors:

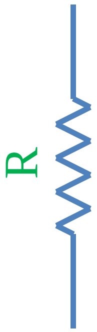
$$V(t) = V_m \cos\left(\omega t + \phi\right) = \mathrm{Re}\left(V_m e^{j(\omega t + \phi)}\right)$$

$$= \mathrm{Re}\left(\underbrace{V_m e^{j\phi}}_{\text{“Phasor”}} e^{j\omega t}\right)$$

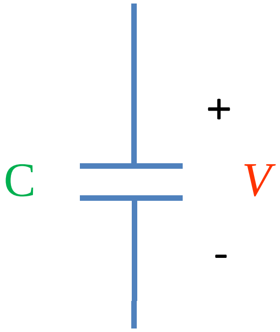
“Phasor” $\mathbf{V}$

(Complex #)

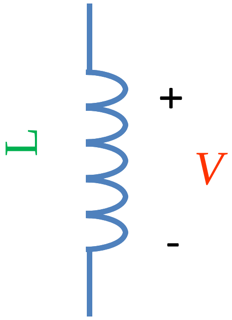
Circuits



$\mathbf{V} = \mathbf{I}\ R$



$\mathbf{V}=\mathbf{I}/j\omega C$



$\mathbf{V}=j\omega L\ \mathbf{I}$

“Impedance”

$Z = R$ $Z=1/j\omega C$ $Z = j\omega L$

KCL, KVL hold for relationship between **V**, **I**.

Series/Parallel Impedances

$$Z_{eq} = Z_1 + Z_2 + Z_3$$

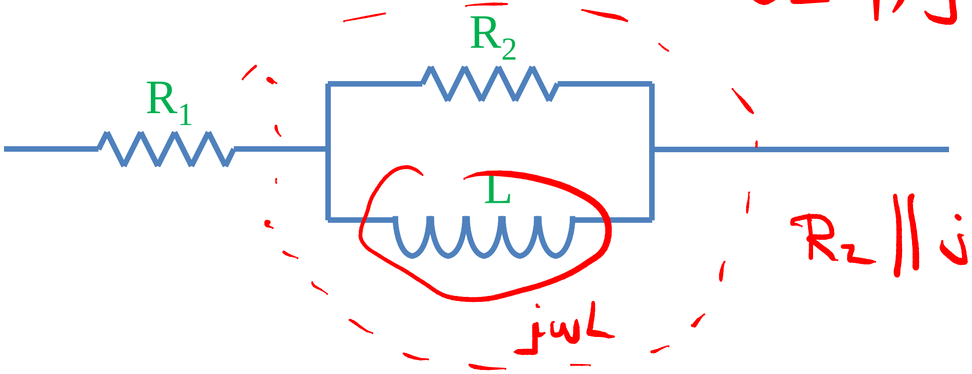
$$Z_{eq}^{-1} = Z_1^{-1} + Z_2^{-1} + Z_3^{-1}$$

Example problem #1

Find Z_{eq} for this circuit: (instructor)

$\left.\begin{matrix} Re(Z_{eq}) \\ Im(Z_{eq}) \end{matrix}\right]$ ex. @ home

$\tau \equiv \frac{L}{R_2}$



$R_2 \| j\omega L$

$$Z_{eq} = R_1 + (R_2 \| j\omega L) = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{j\omega L}}$$

$\cancel{R_2}\, j\omega L R_2$

$\cancel{R_2}\, j\omega L R_1$

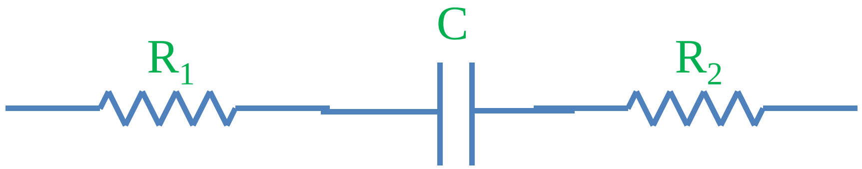
$$= R_1 + \frac{j\omega L}{j\omega \frac{L}{R_2} + 1} = R_1 + \frac{j\omega L}{1 + j\omega\tau}$$

$$= R_1 + \frac{R_2 j\omega L}{j\omega L + R_2}$$

$$\lim_{\omega \to \infty} Z_{eq}$$

Example problem #2

Find Z_{eq} for this circuit: (students)



$$\frac{1}{j} = -j$$

$$\frac{1}{j\omega C} = -j\frac{1}{\omega C}$$

$$Z_{eq} = R_1 + \frac{1}{j\omega C} + R_2 = \underbrace{(R_1+R_2)}_{Re} + j\underbrace{\left(-\frac{1}{\omega C}\right)}_{Im}$$

$$Re(Z_{eq}) = R_1+R_2$$

$$Im(Z_{eq}) = -\frac{1}{\omega C}$$

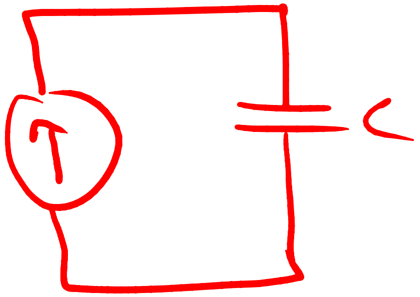
$$\lim_{\omega \to \infty} Z_{eq} = R_1+R_2$$

$Q = CV \qquad I = C\frac{dV}{dt} \qquad V = \frac{1}{C}\int I\,dt$

Phasor to voltage conversion

$j \times j = -1$

$I(t) = I_0 \cos(wt + 0)$



$$I = I_0 e^{j0} = I_0$$

$$Re[z] = x$$

$$V = \frac{1}{jwC} I$$

$$z = \boxed{x} + jy$$

$$\text{Find } V(t) = ? = Re\left[V e^{jwt}\right]$$

$$= Re\left[\frac{1}{jwC} I e^{jwt}\right] = Re\left[\frac{1}{jwC} I_0 e^{jwt}\right]$$

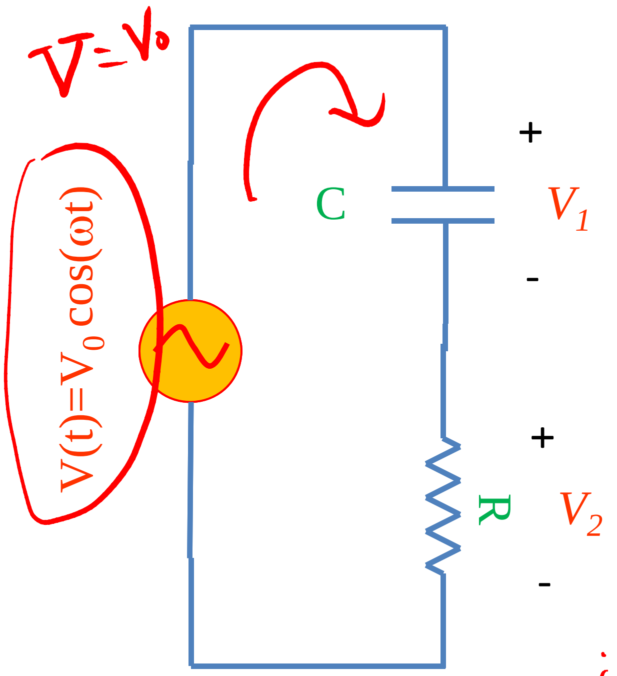
$$= Re\left[\left(-\frac{1}{wC}\right) j I_0 e^{jwt}\right] = -\frac{I_0}{wC} Re\left[j e^{jwt}\right]$$

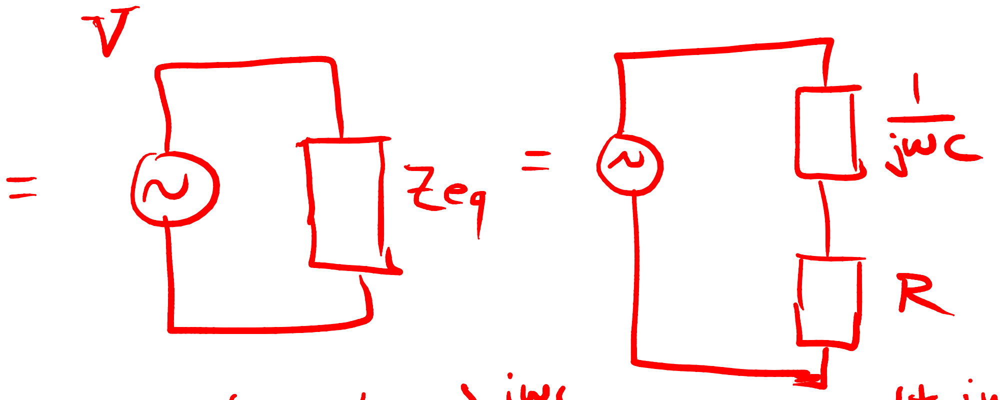
$$= -\frac{I_0}{wC} Re\left[j(\cos wt + j \sin wt)\right] = -\frac{I_0}{wC} \overset{Re}{\left[\underbrace{\boxed{-\sin(wt)}}_{x} + j\cos(wt)\right]}$$

$$= \frac{I_0}{wC} \sin(wt)$$

Example problem #3

Find i(t), $V_1(t)$, $V_2(t)$ for this circuit: (instructor)





$$Zeq = \left(R + \frac{1}{j\omega C}\right)\frac{j\omega C}{j\omega C} = \frac{1 + j\omega RC}{j\omega C}$$

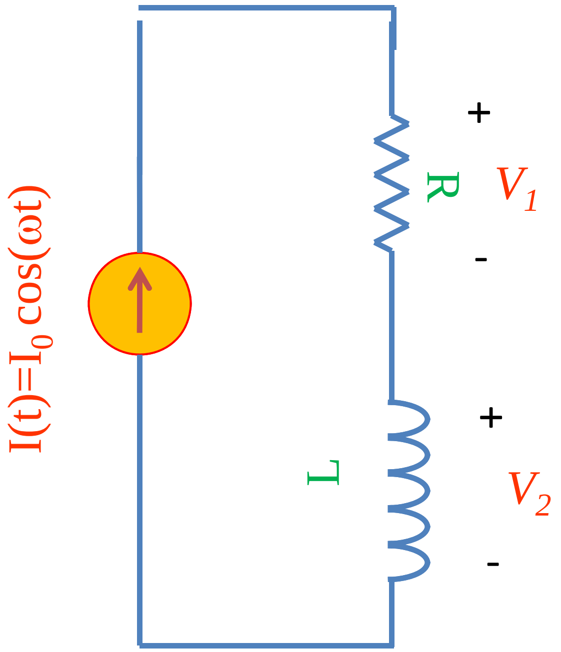
$$V = Zeq\, I \Rightarrow I = \frac{V}{Zeq}$$

$$i(t) = Re\left[I e^{j\omega t}\right] = Re\left[\frac{V}{Zeq} e^{j\omega t}\right]$$

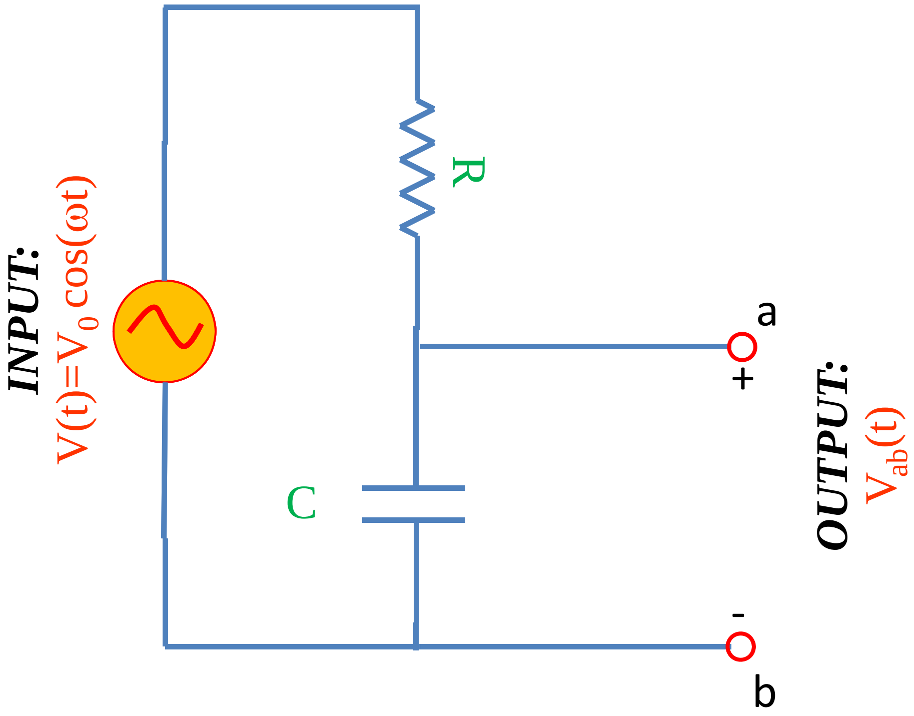
$$= Re\left[\underbrace{\frac{V_0}{R + (j\omega C)^{-1}} e^{j\omega t}}\right] = Re\left[\underbrace{\frac{V_0\, j\omega RC}{1 + j\omega RC} \underbrace{e^{j\omega t}}_{\cos\omega t + j\sin\omega t}}\right]$$

Example problem #4

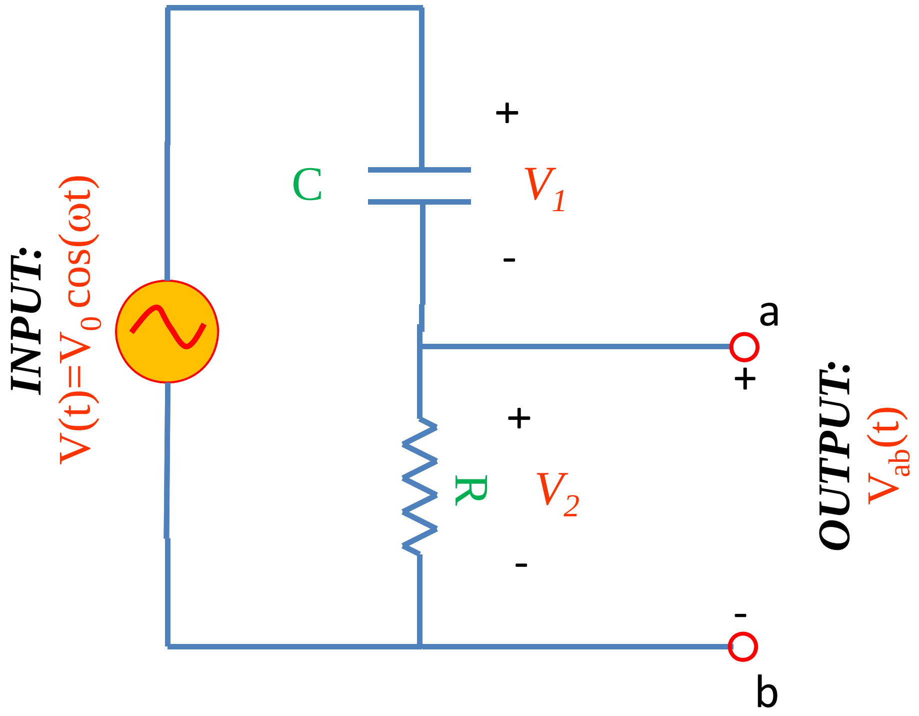
Find i(t), V_1(t), V_2(t) for this circuit: (students)



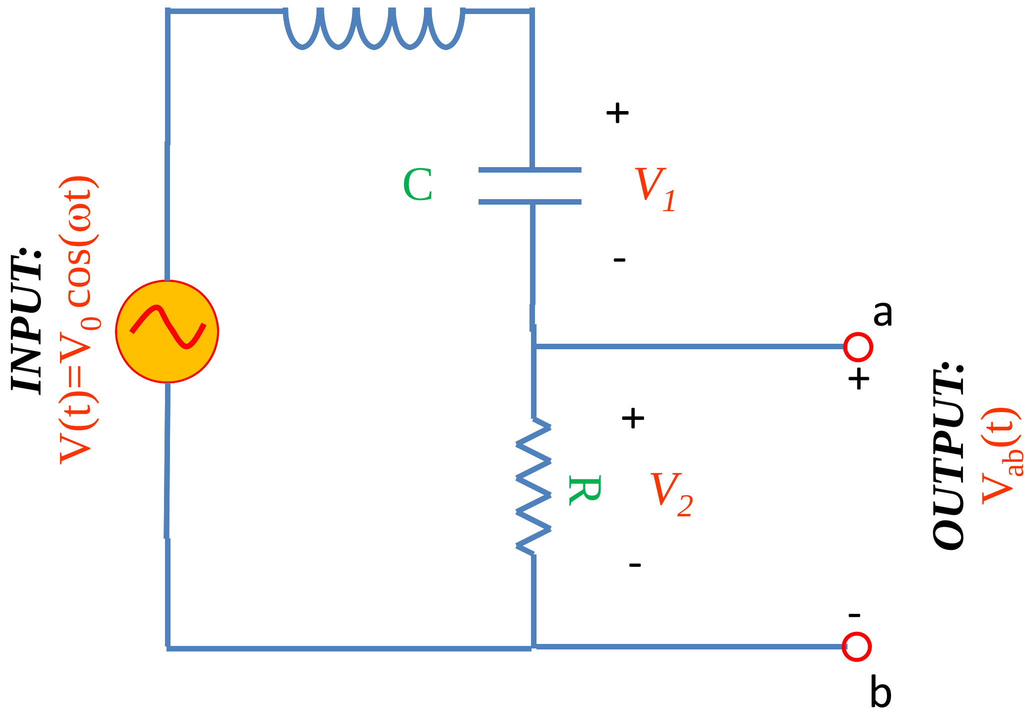
Low pass filter



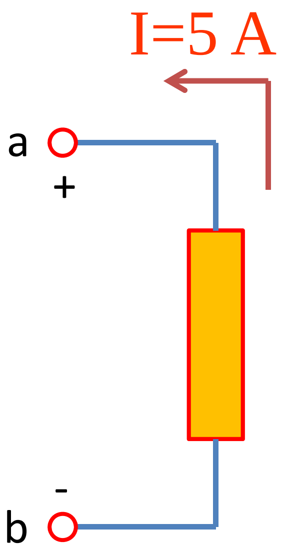
High pass filter

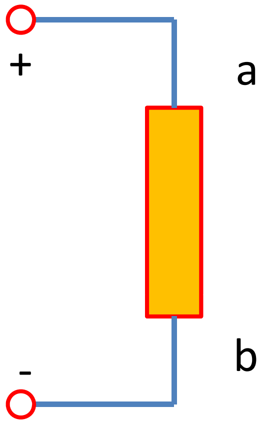


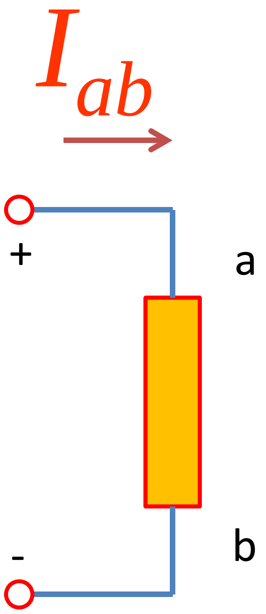
Band pass filter (RLC)



Symbol library







Symbol library