Announcements:
1. HW5 due Wednesday
2. Midterm #2 is Thursday, covers:
    1. Ch 1-6 (not 5)
    2. RC, LR circuits (small part of ch.7)
    3. Impedance calculation (like HW5) (small part of Ch 9)
    4. No diodes, transistors

EECS 70A: Network Analysis

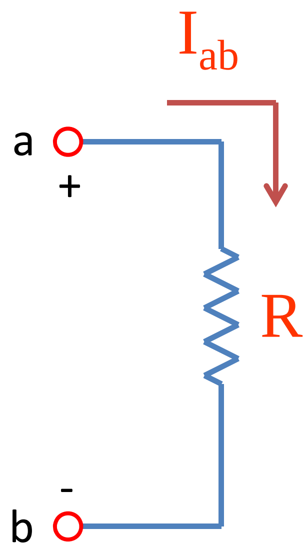
Lecture 12

Today's Agenda

$V = IR$ $\quad$ $Q = CV$, $i = C\frac{dv}{dt}$ $\quad$ $v = L\frac{di}{dt}$

- KCL, KVL ①
- Nodal analysis ③
- Mesh analysis ③
- Thevenin/Norton theorem ②
- R,L,C series, parallel ②
- Impedances ① sin cos

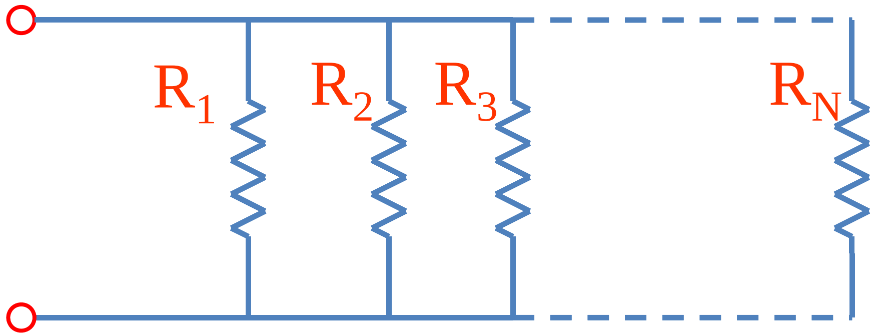
Resistors



$$V_{ab} = I_{ab} \times R$$

Resistance units: Ohms [Ω]

Generalize: N resistors in parallel



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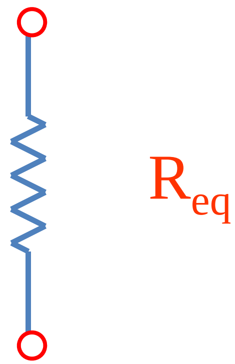
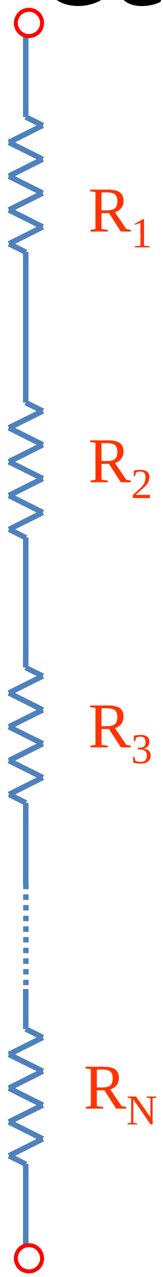


$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

$R_1 \parallel R_2$ is notation for "R_1 in parallel with R_2"

Generalize: N resistors in series



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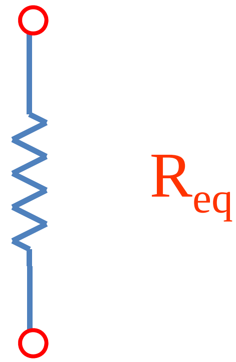
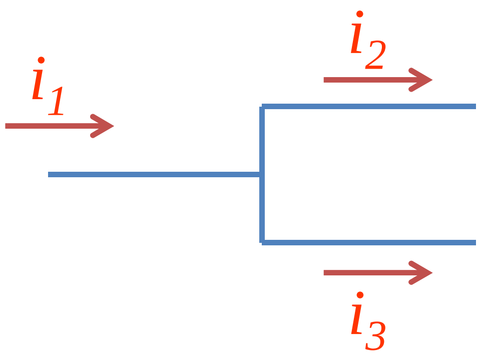


$$R_{eq} = \sum_{i=1}^{N} R_i$$

Kirchoff's current law

You have already seen:



$$i_1 = i_2 + i_3$$

Like water in a river…

More generally:

Sum of currents *entering* node = sum of currents *leaving* node.

Stated as Kirchoff's current law (KCL):

$$\sum_{n=1}^{N} i_n = 0$$

Current *entering* a node: i_n positive
Current *leaving* a node: i_n negative

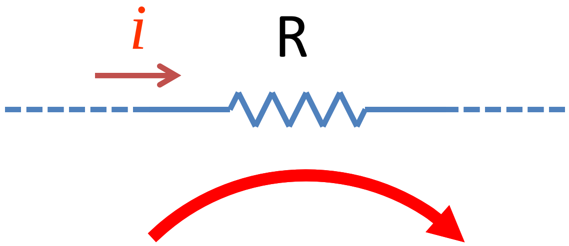
Kirchoff's voltage law

$$\sum_{n=1}^{N} v_n = 0$$

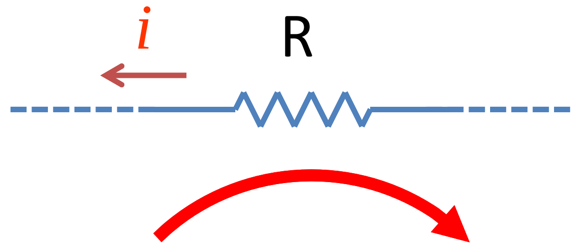
around *any* closed loop.

voltage *drops*

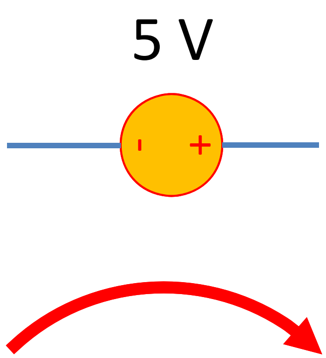
Sign of voltage drop



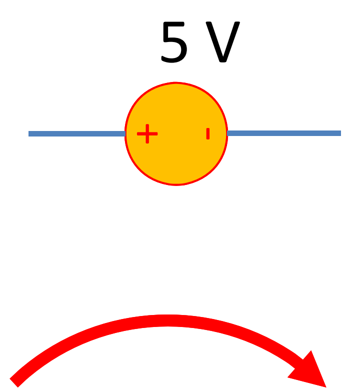
Voltage drop
= + i R



Voltage drop
= - i R



Voltage drop
= - 5 V

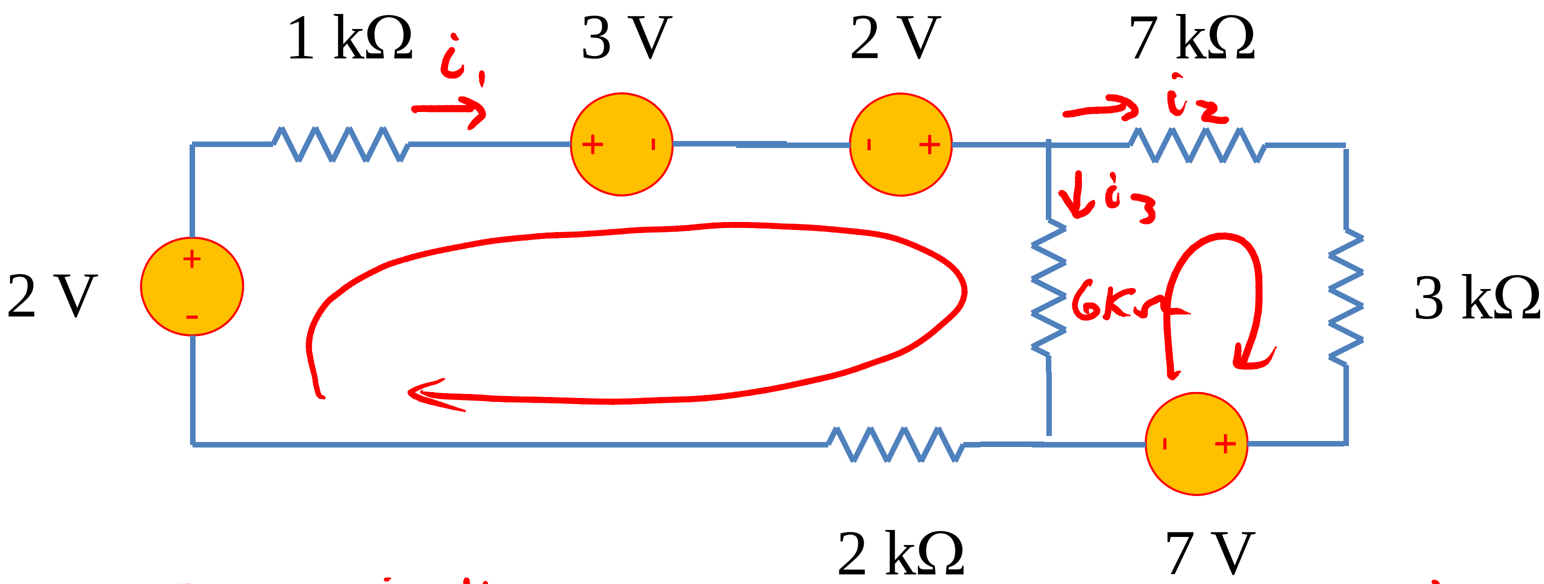


Voltage drop
= + 5 V

KVL example

If the voltage is *dropping* as you go around the loop, the voltage drop v_n is *positive*.

Apply KVL to the circuit below



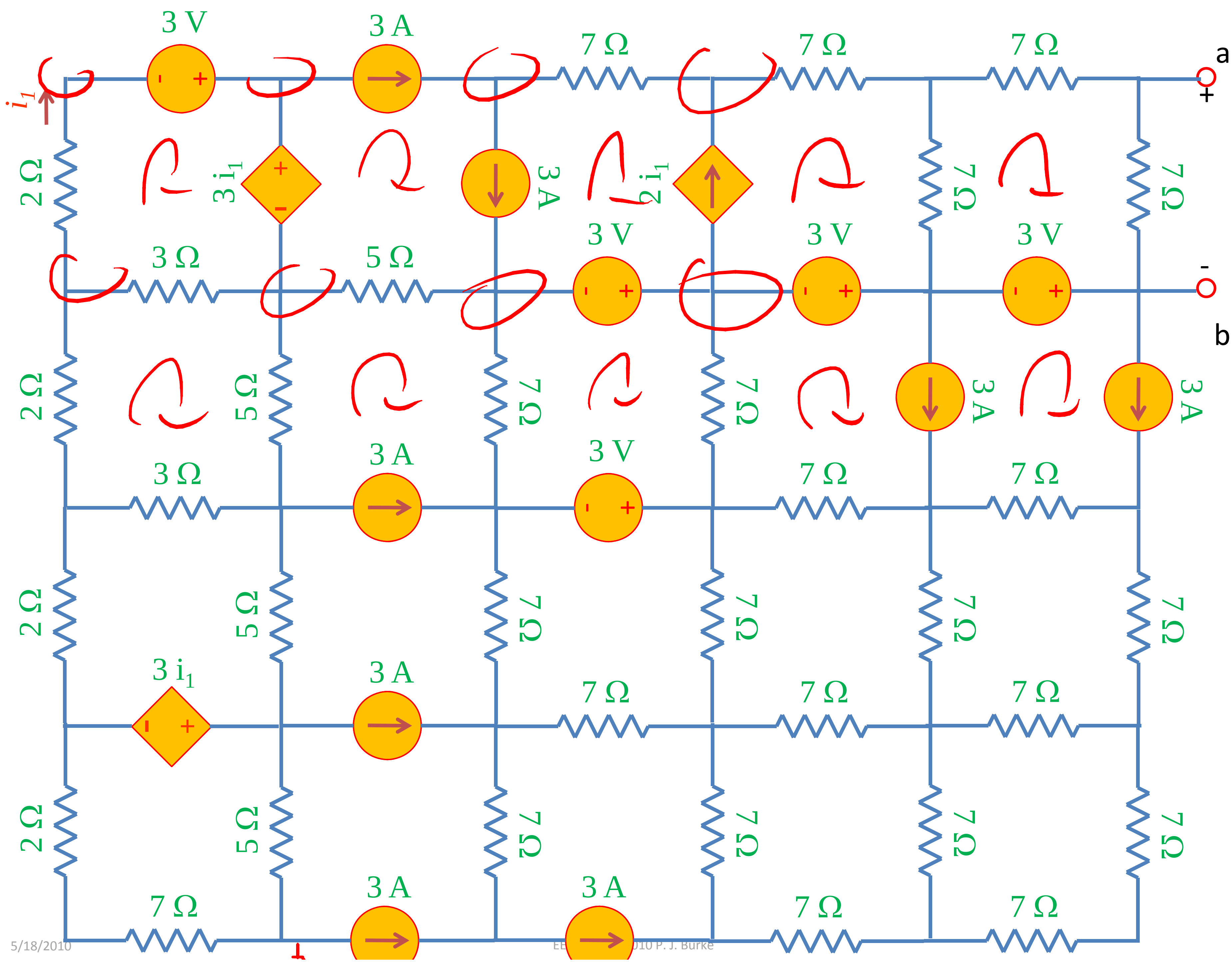
$$-2V + i_1\,1k\Omega + 3V - 2V + 6k\Omega\, i_3 + 2k\Omega\, i_1 = 0$$

$$-6k\Omega\, i_3 + 7k\Omega\, i_2 + 3k\Omega\, i_2 + 7V = 0$$

Ex #2

$$V = \underline{10A\ 1k\Omega} = 10000V \qquad V = A\Omega$$

$$= 10A\ 1000\Omega = 10,000V$$



3 V
3 A
7 Ω
7 Ω
7 Ω
a
+
i_1
2 Ω
$3 i_1$
3 A
$2 i_1$
7 Ω
7 Ω
3 Ω
5 Ω
3 V
3 V
3 V
-
b
2 Ω
5 Ω
7 Ω
7 Ω
3 A
3 A
3 Ω
3 A
3 V
7 Ω
7 Ω
2 Ω
5 Ω
7 Ω
7 Ω
7 Ω
7 Ω
$3 i_1$
3 A
7 Ω
7 Ω
7 Ω
2 Ω
5 Ω
7 Ω
7 Ω
7 Ω
7 Ω
7 Ω
3 A
3 A
7 Ω
7 Ω

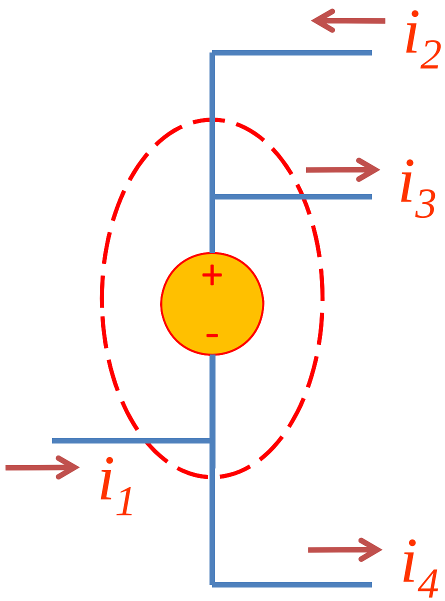
Nodal Analysis(Review)

~~Based on KCL~~, use node voltages as circuits variables.

1. Define a reference node.
2. Label remaining nodes (n-1 nodes) e.g. V_1, V_2, V_3, …
3. "Supernode":
   1. Case 1: Voltage source connected to reference node: solves one node.
   2. Case 2: Voltage source not connected to reference: Define supernode
4. Apply KCL all nodes (& supernodes)
   1. Express all i's in terms of v's using Ohm's law
5. Apply KVL to loops with voltage source
6. Solve the n-1 simultaneous equations, to find V's
   (e.g. using Kramer's rule)
7. Use Ohm's law to find the currents.

"Supernode"

A node with a voltage source in it…



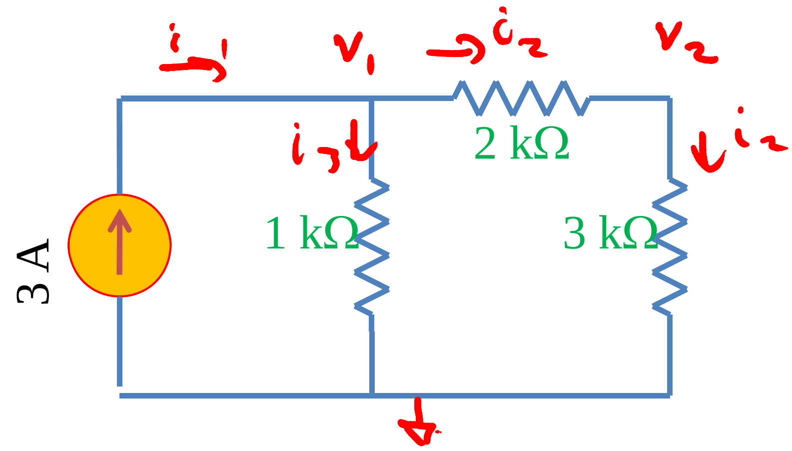
KCL:

IN = OUT

$i_1 + i_2 = i_3 + i_4$

Must define a supernode if a voltage source appears when doing nodal analysis…
(unless one end of voltage source connected to reference node)

Nodal analysis example 1



KCL @ N1

$i_1 = i_2 + i_3$

$3A = \frac{V_1 - V_2}{2k\Omega} + \frac{V_1}{1k\Omega}$

KCL @ N2

$i_2 = i_2$

$\frac{V_1 - V_2}{2k\Omega} = \frac{V_2}{3k\Omega}$

2 eq & 2 unknowns.

Solve for V_1, V_2.

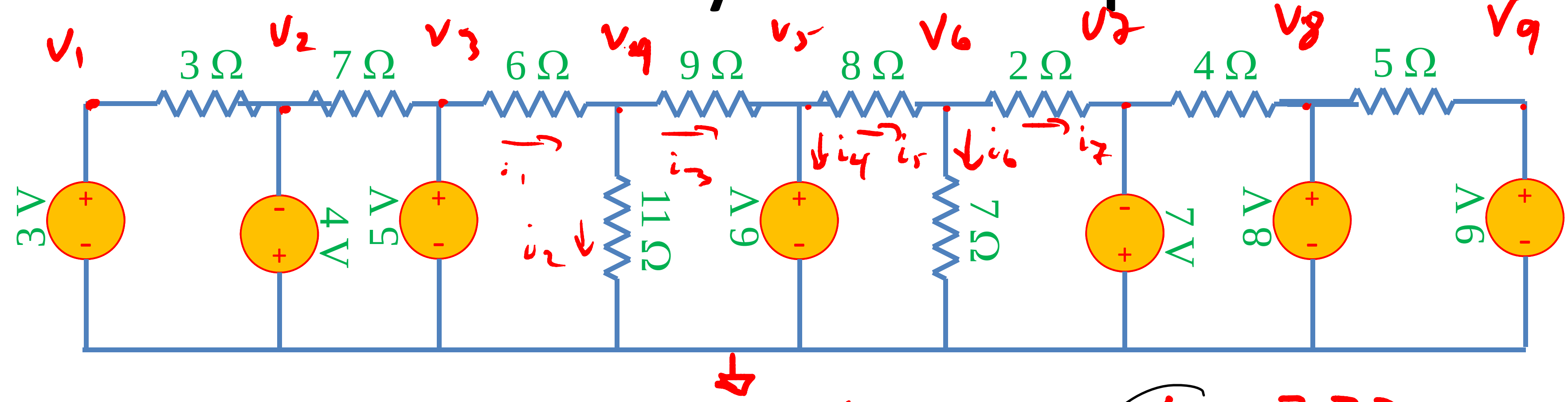
Back: Find

$i_1 = 3A$

$i_2 = \frac{V_1 - V_2}{2k\Omega}$

$i_3 = \frac{V_1}{1k\Omega}$

Nodal analysis example 2



$V_1 = 3V$ $V_2 = -4V$ $V_3 = 5V$ $V_4 = ???$

$V_5 = 6V$ $V_6 = ??$ $V_7 = -7V$

$V_8 = 8V$ $V_9 = 9V$

KCL @ NODE 4   IN = OUT

$i_1 = i_2 + i_3 \Rightarrow \frac{V_3 - V_4}{6} = \frac{V_4 - 0}{11} - \frac{(V_5 - V_4)}{9}$ FIND V_4

KCL @ N6

$i_5 = i_6 + i_7 \Rightarrow \frac{V_5 - V_6}{8} = \frac{V_6}{7} + \frac{V_6 - V_7}{2}$

FIND V_6

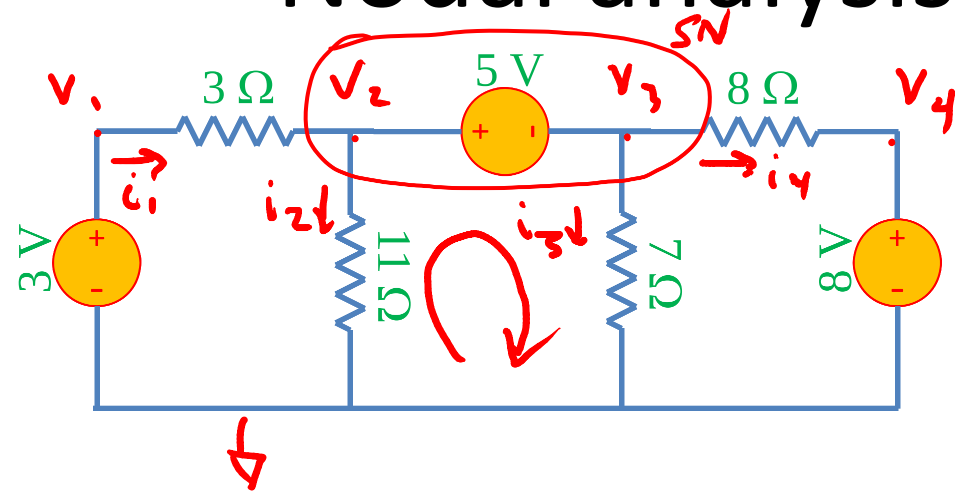
SIGN ? V_4 —9— V_5, $\rightarrow i_3$

$i_3 = \frac{V_4 - V_5}{9}$

GOT $V_1, V_2 \ldots V_9$

NEXT: FIND $i_1, i_2, \ldots i_7$

Nodal analysis example 3



$V_1 = 3V$

$V_2 =$

$V_3 =$

$V_4 = 8V$

KCL @ ~~NODE 1~~ SN

$i_1 = i_2 + i_3 + i_4$

$$\frac{V_1 - V_2}{3} = \frac{V_2}{11} + \frac{V_3}{7} + \frac{V_3 - V_4}{8}$$

5V 3Ω

5 3

NODE 1 NODE 4 IGNORE

KVL LOOP CONTAINING VOLTAGE

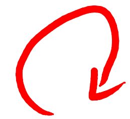
$-i_2 11 + 5V + i_3 7 = 0$

$$-\left(\frac{V_2}{11}\right)11 + 5V + \left(\frac{V_3}{7}\right)7 = 0$$

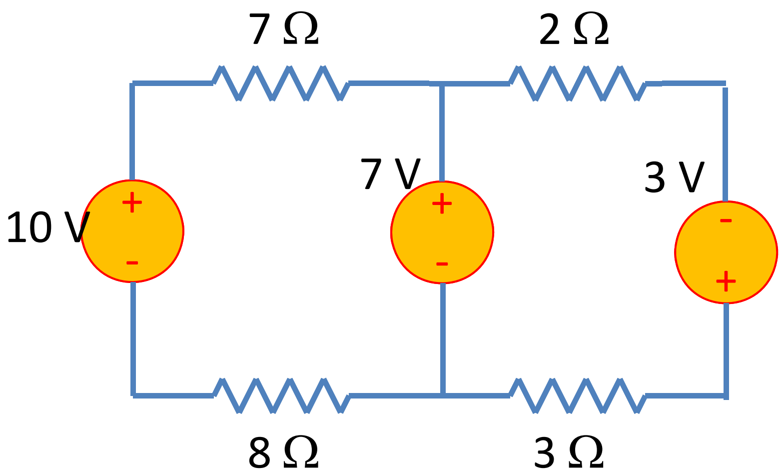
Solve V_2, V_3.

Then: get currents from voltages

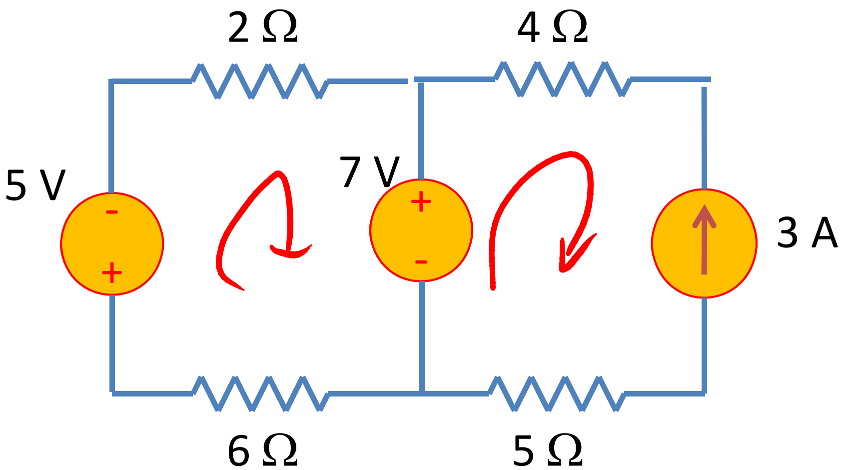
Mesh analysis summary

1. Assign mesh currents i_1, i_2, … i_n 
2. “Supermesh” (if current source present):
    1. Case 1: Source only on one side of mesh: Sets current
    2. Case 2: Create supermesh
3. Apply KVL to each mesh
4. Apply KCL to supermeshes
5. Solve for mesh currents (e.g. using Kramer’s rule)
6. Then solve for voltages

Mesh Analysis Example 1



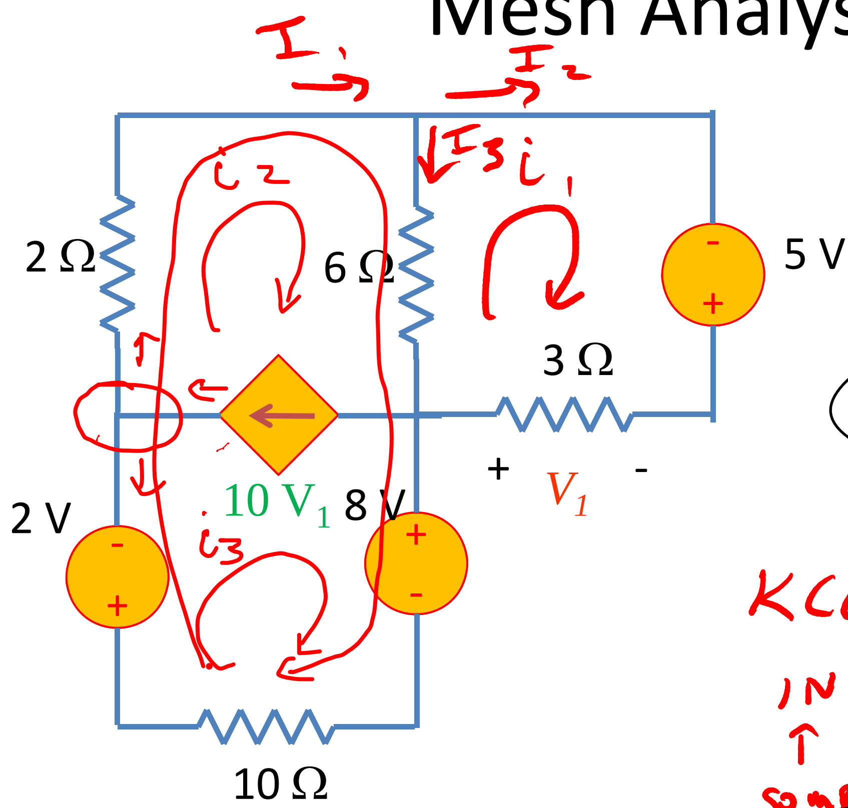
Mesh Analysis Example 2



no SM

Mesh Analysis Example 3

3 unknowns $i_1 \; i_2 \; i_3$



KVL MESH 1

$$(i_1 - i_2)6 - 5V + i_1 3 = 0$$

KVL SUPERMESH

$$2V + 2i_2 + 6(i_2 - i_1) + 8 + i_3 10 = 0$$

KCL to "+ junction"

IN = OUT

SOURCES MESH

$$10V_1 = i_2 - i_3$$

$V_1 = -3i_1$

$$I_3 = i_2 - i_1$$

$$I_1 = i_2$$

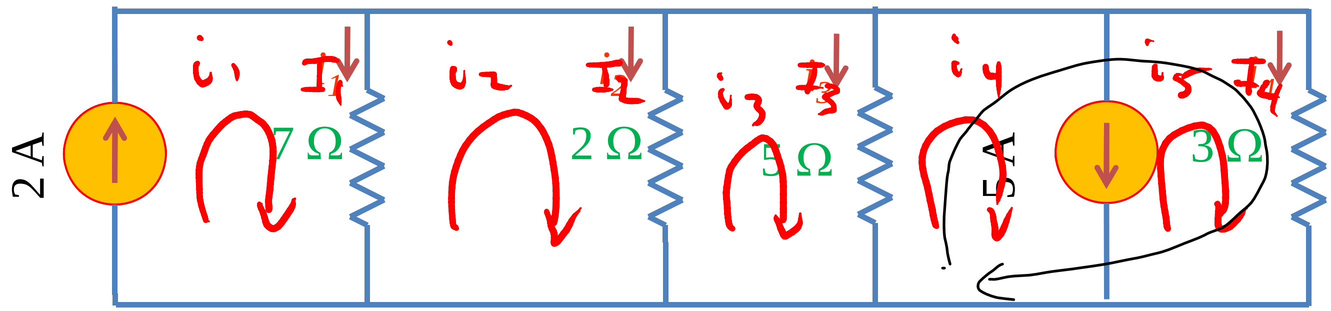
$$I_2 = i_1$$

Find $i_1 \; i_2 \; i_3$

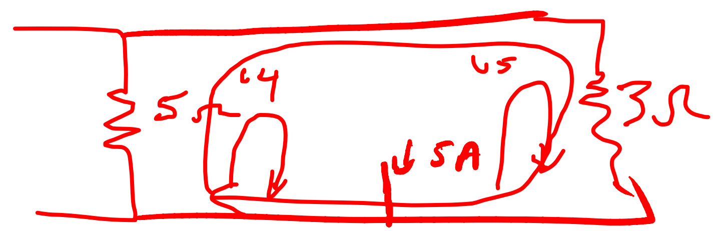
Find $I_1 \; I_2 \; I_3$

Ohm Find voltages

KCL application to supermesh

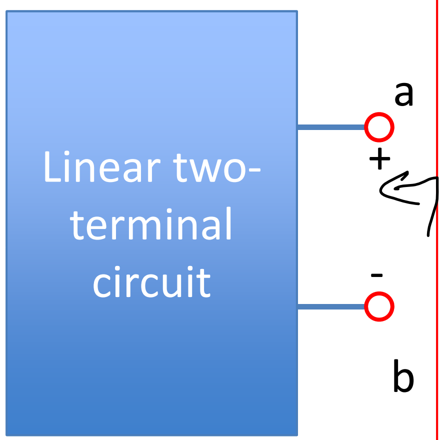


Supermesh

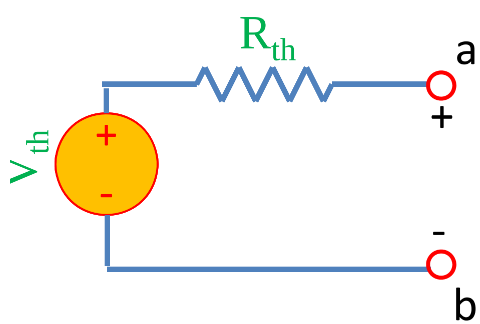


$$i_5 + 5A = i_4$$

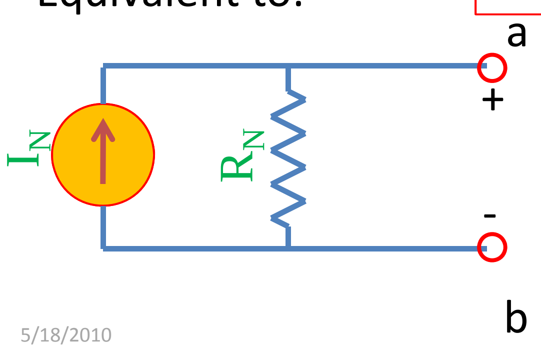
Thevenin, Norton Theorems:



Equivalent to:



Equivalent to:



Thevenin:

1. **Calculating V_{th}:**
   Connect nothing to a-b. Calculate voltage. This is V_{th}.
2. **Calculating R_{th}:**
   Method 1:
   Connect terminal a to b (short).
   Calculate the current from a to b. This is call $I_{short\ circuit}$.
   $R_{th}=V_{th}/I_{short\ circuit}$.
   Method 2:
   Find the input resistance looking into terminals a-b after all the independent sources have been turned off. (Voltage sources become shorts, current sources become opens.)
   Trick (if dependent sources present):
   Apply a 1 A current source to terminals a-b, find V_{ab}
   $R_{th} = V_{ab}/1A$.

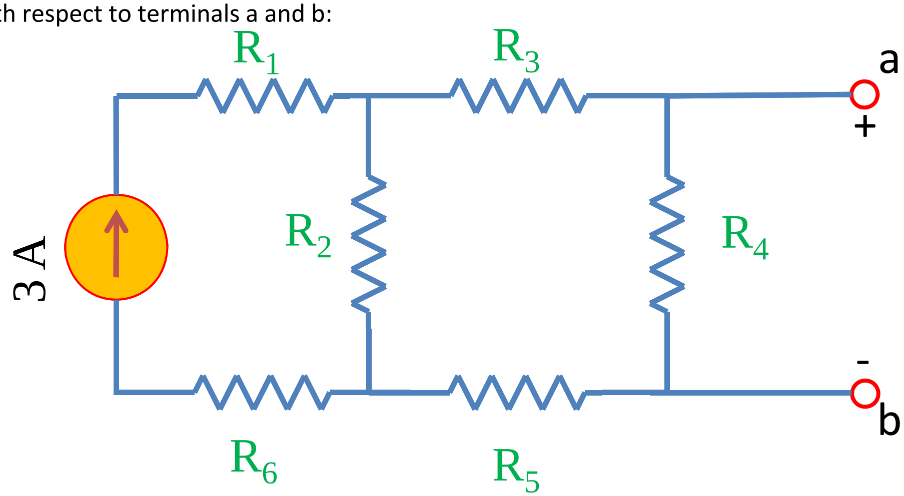
TRICK 2: APPLY 1V SOURCE TO ab
FIND I_{ab}
THEN $R_{ab} = \frac{1V}{I_{ab}} = R_{th}$

Norton:

1. **Calculating R_N:**
   $R_N=R_{th}$
2. **Calculating I_N:**
   $I_N=V_{th}/R_{th}$

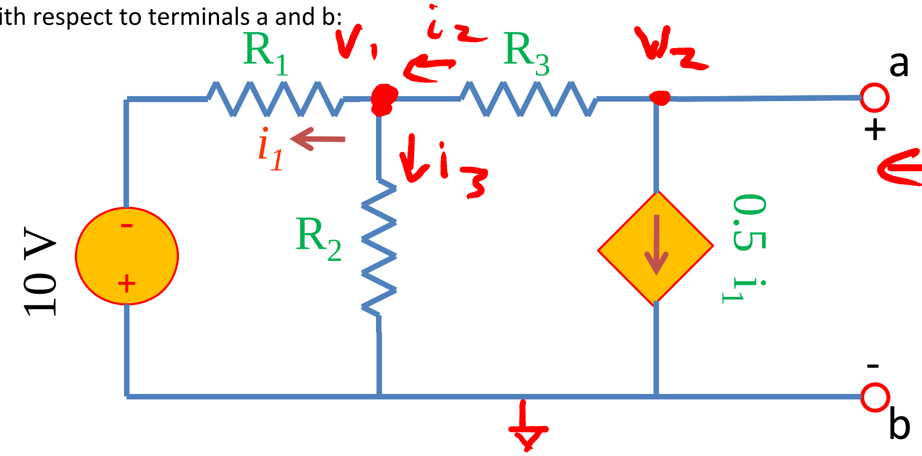
Thevenin/Norton example 1

Find the Thevenin & Norton equivalent circuit of the circuit below
with respect to terminals a and b:



Thevenin/Norton example 2

Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:



1) Vth: Find Vab

KCL @ N1:

$i_2 = i_1 + i_3$

$\frac{V_2 - V_1}{R_3} = \frac{V_1 + 10V}{R_1} + \frac{V_1}{R_2}$

KCL @ N2:

$i_2 = -0.5 i_1$

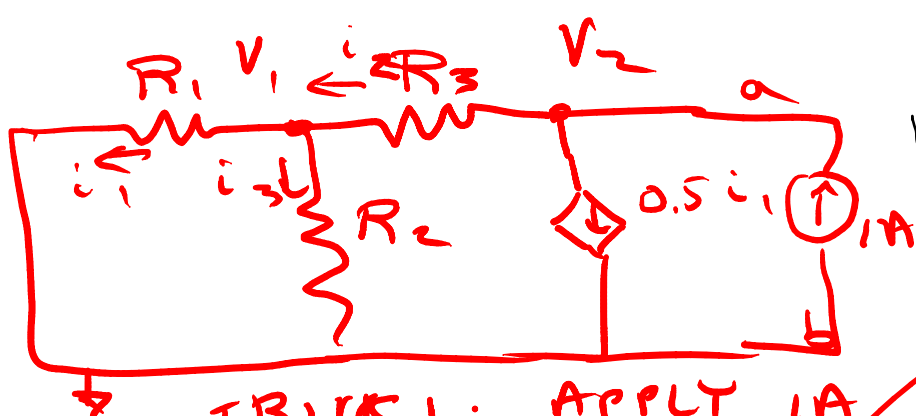
$\frac{V_2 - V_1}{R_3} = -0.5\left(\frac{V_1 + 10V}{R_1}\right)$

FIND V_1, V_2

$V_{ab} = V_2 = V_{th}$

STEP 2: FIND R_{th}

FIND R_{ab} WHEN ALL IND. SOURCES OFF



TRICK 1: APPLY 1A

2NODES KCL N1, N2

SOLVE V_1, V_2

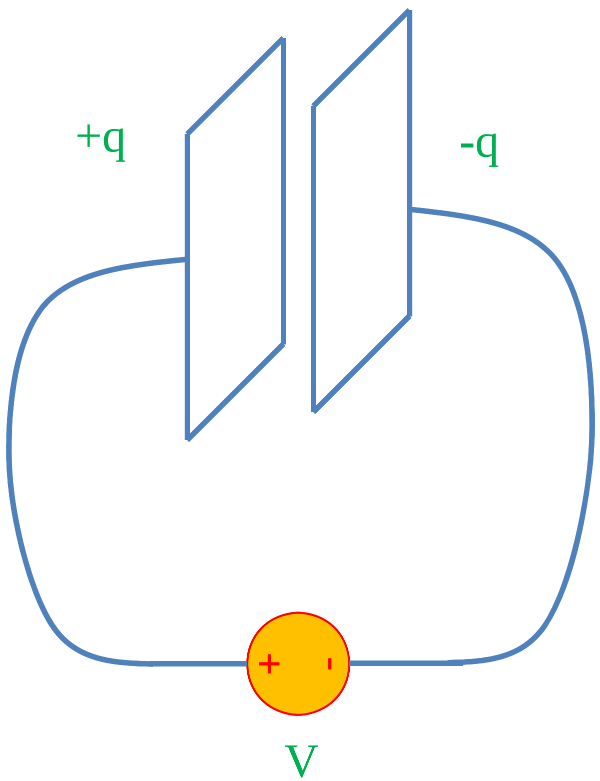
$V_2 = V_{ab}$

FIND V_{ab}, $R_{th} = \frac{V_{ab}}{1A}$

$i_1 + i_3 = i_2 \Rightarrow \frac{V_1}{R_1} + \frac{V_1}{R_2} = \frac{V_2 - V_1}{R_3}$

$1A = \frac{1}{2} i_1 + i_2 = \frac{V_2 - V_1}{R_3} + \frac{1}{2}\frac{V_1}{R_1}$

Capacitors



$$q = CV$$

$$C = \frac{\varepsilon A}{d}$$

A=area
d=plate separation

Farads[F] = Coulombs/Volt [C]/[V]

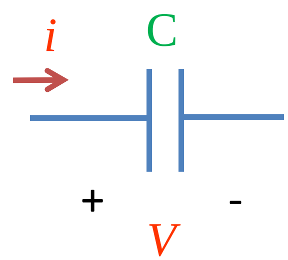
$$\varepsilon_0 = 8.85 \times 10^{-12} F/m$$

$$\varepsilon = \kappa \varepsilon_0$$

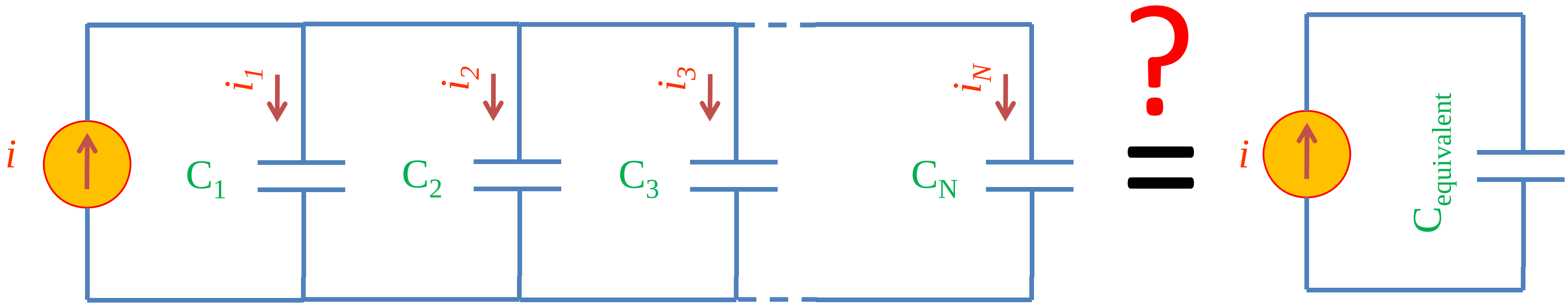
Dielectric constant:

$\kappa = 3.9$ SiO_2

$\kappa = 25$ HfO_2

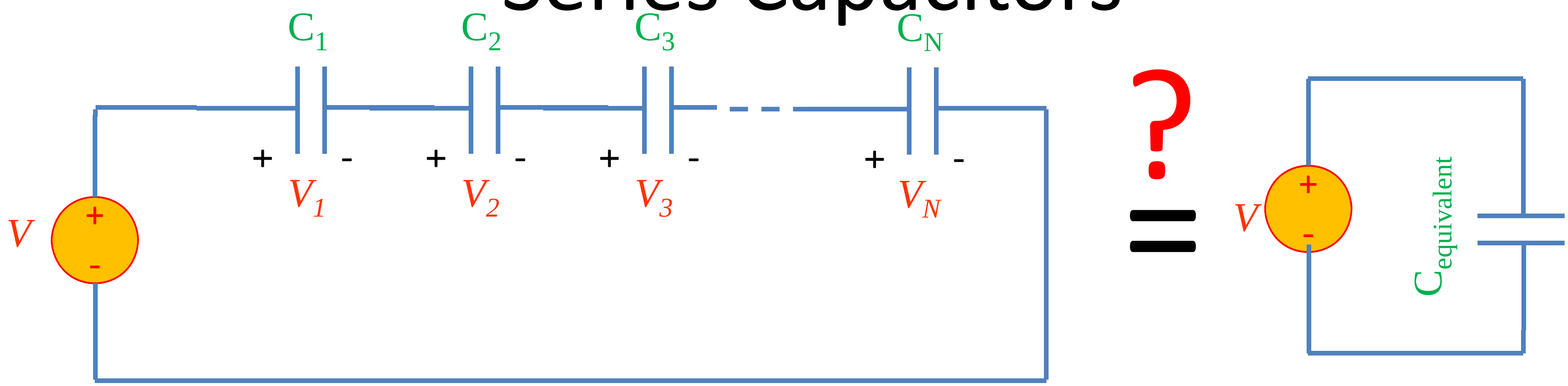


Parallel Capacitors



$$C_{eq} = \sum_{i=1}^{N} C_i$$

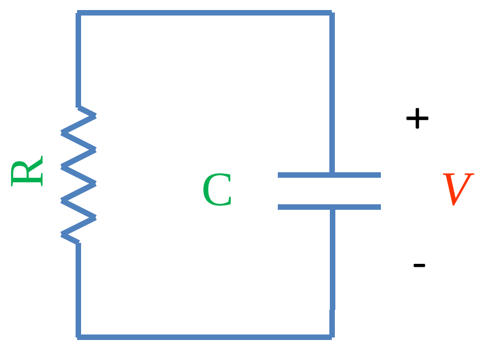
Series Capacitors



$$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$$

RC circuit

Find V(t), q(t), i(t)



RC circuit

Find V(t), q(t), i(t)

RESISTOR

$i_R R = V_R = V_c = (-i_c)R$

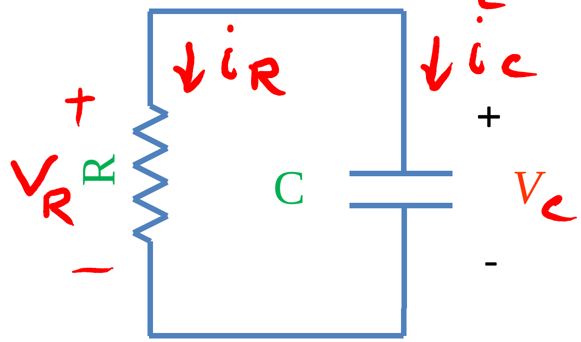
KVL $\Rightarrow V_R = V_c$

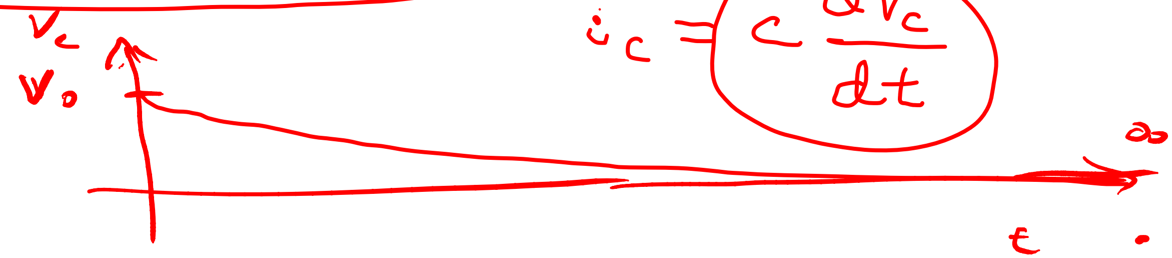
KCL $\Rightarrow i_R = -i_c$

CAPACITOR

$q = CV_c$

$i_c = C\frac{dV_c}{dt}$





$$\frac{V_c}{R} = -C\frac{dV_c}{dt}$$

$\tau \equiv RC$

$$\frac{dV(t)}{dt} = -\frac{1}{RC} V(t)$$

why?

Soln:

$$V(t) = V(t=0)\, e^{-t/RC}$$

Proof:

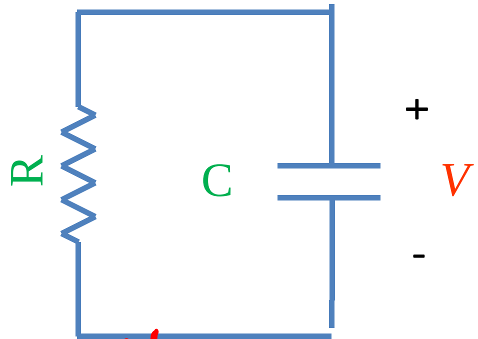
$$\frac{dV(t)}{dt} = \frac{d}{dt}\left[V(t=0)\, e^{-t/RC}\right]$$

$$= V(t=0)\frac{d}{dt}\left[e^{-t/RC}\right] = -\frac{1}{RC} V(t=0) e^{-t/RC}$$

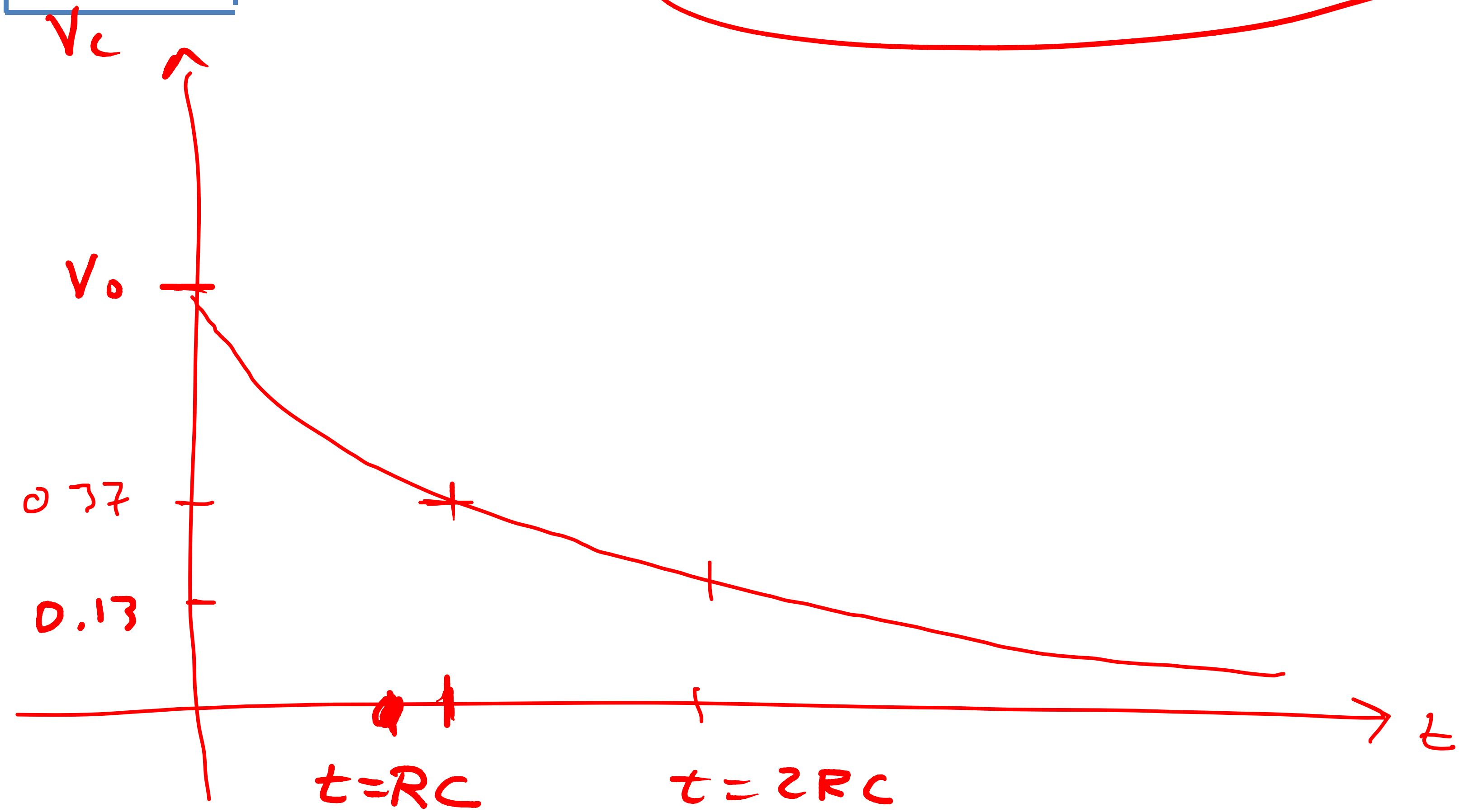
$$= -\frac{1}{RC} V(t)$$

RC circuit

Find V(t), q(t), i(t)

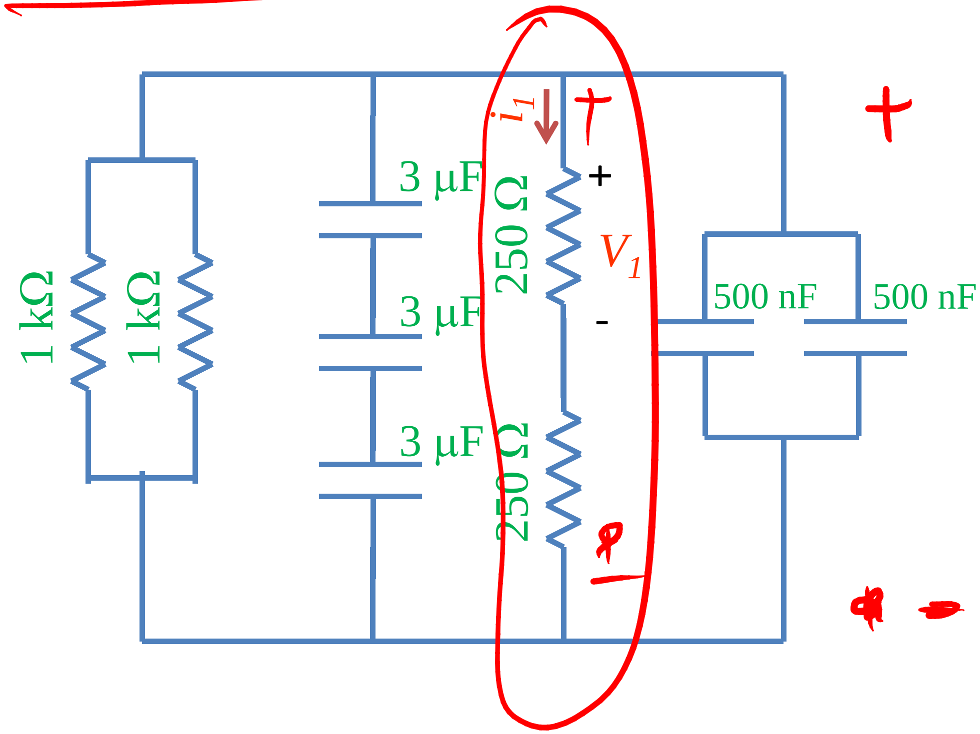


$$V(t) = V(t=0)\, e^{-t/RC}$$



Example RC problem 1

~~Find $i_1(t)$, given that $V_1(t=0) = 3$ Volts~~



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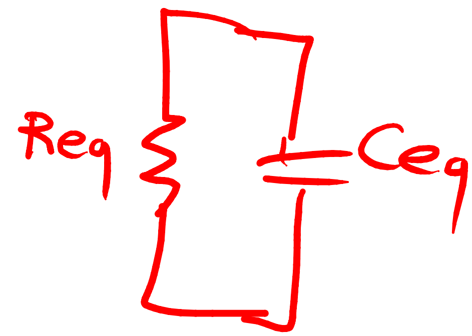
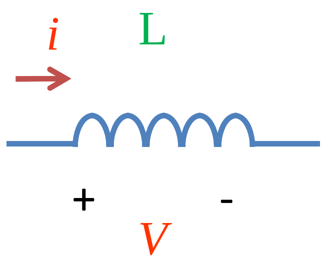


$$V(t) = V(t=0)\, e^{-t/R_{eq}C_{eq}}$$

Inductors



$$L = \frac{N^2 \mu A}{l}$$

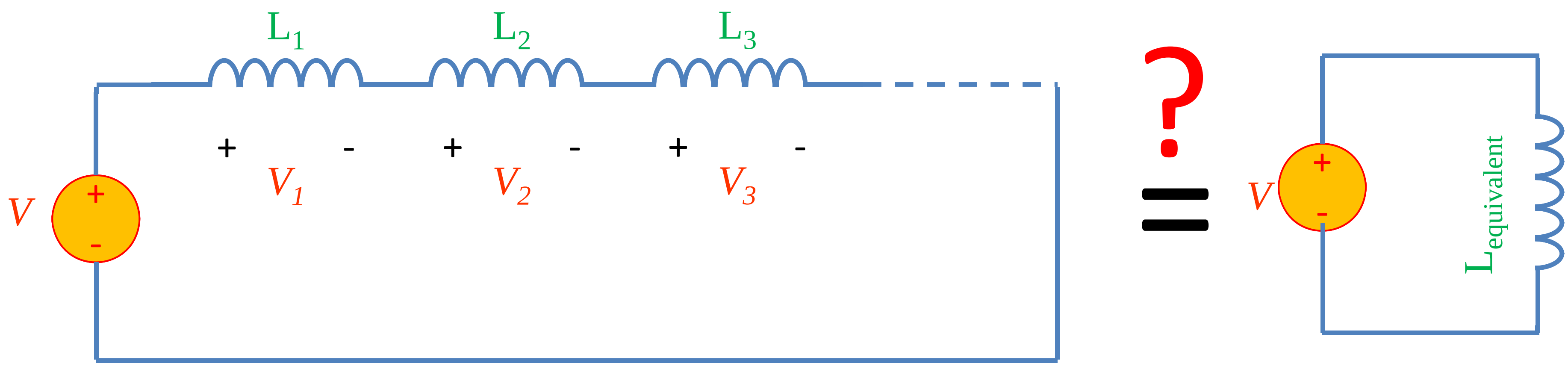
A=area
l=wire length
N = # of turns
$\mu = 4\,\pi\,10^{-6}$ H/m

$$V = L\frac{di}{dt}$$

Henry[H]

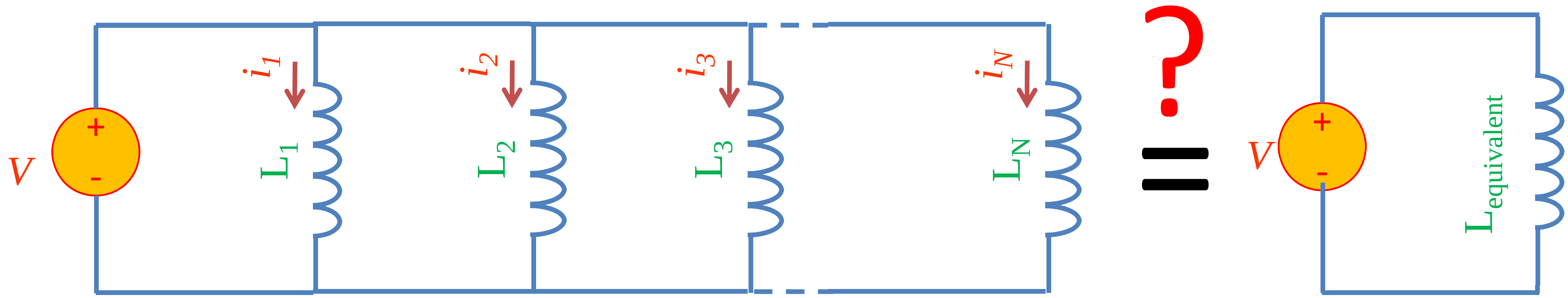
$$V = L\frac{di}{dt} \Rightarrow i(t) = \frac{1}{L}\int V(t)dt$$

Series Inductors



$$L_{eq} = \sum_{i=1}^{N} L_i$$

Parallel Inductors

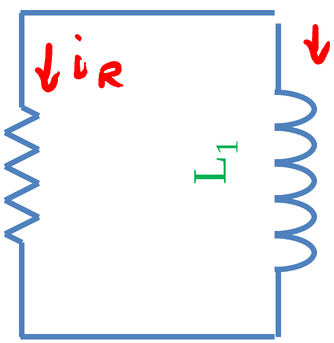


$$\frac{1}{L_{eq}} = \sum_{i=1}^{N} \frac{1}{L_i}$$

LR circuit

$V_R = i_R R$

Find V(t), i(t)



$V_L = L \frac{di_L}{dt}$

KVL $\Rightarrow V_L = V_R$

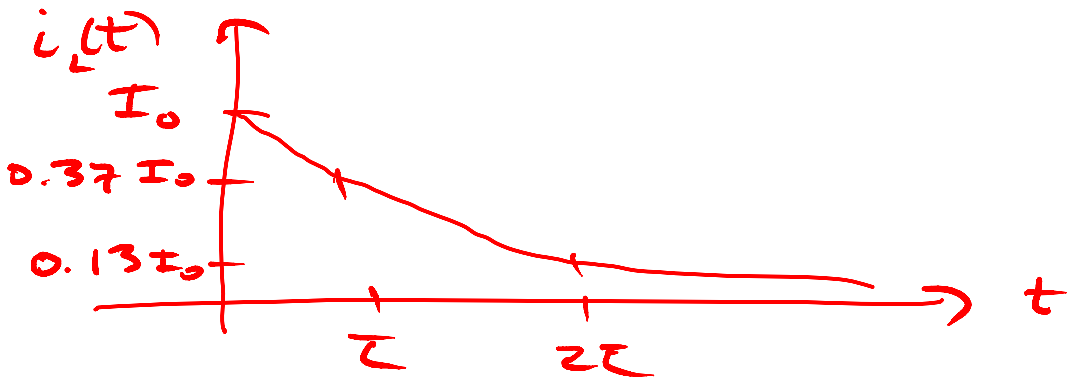
KCL $\Rightarrow i_R = -i_L$

$$L \frac{di_L}{dt} = i_R R = \boxed{-i_L R = L \frac{di_L}{dt}}$$

$$\Rightarrow \frac{di_L}{dt} = -\frac{1}{L/R} i_L$$

$\tau \equiv \frac{L}{R}$ time constant

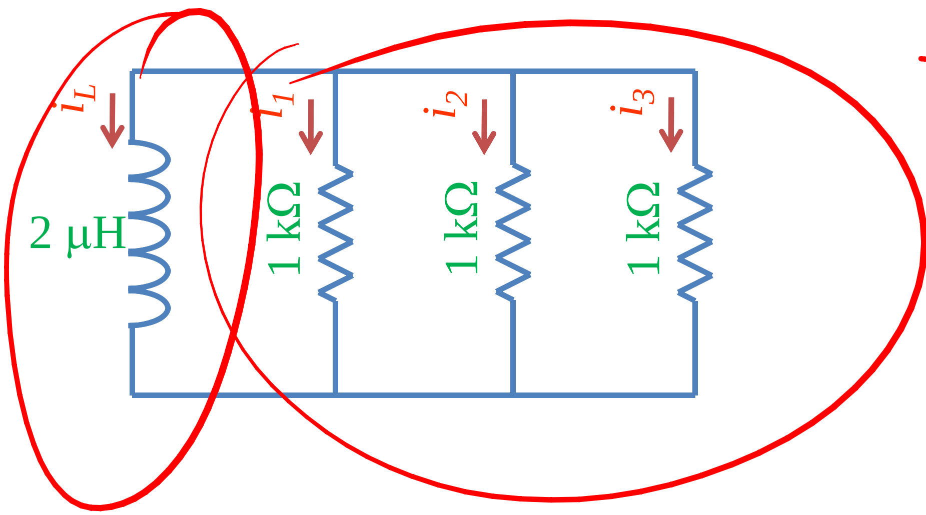
$$i_L(t) = i_L(t=0)\, e^{-t/\tau}$$



Example LR problem #1

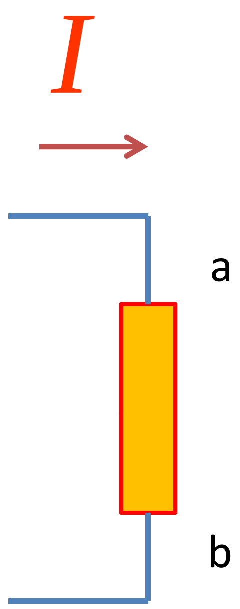
(Students) Find $i_1(t)$ given $i_L(t=0) = 5$ A.

Hint: $i_1+i_2+i_3= -i_L$. How are i_1, i_2, i_3 related?



$-i_L = i_2 + i_1 + i_3$

Power



$$I \times V_{ab} = power$$

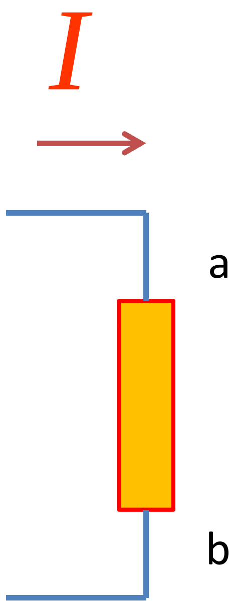
Watts [W] = Volt Amp [V-A]

Note: MKSA unit system:
Meters Kilogram Second Amp

Resistor:
Energy lost to heat…

Inductor or capacitor:
Energy *STORED* and can be recovered…

Energy stored



$$I \times V_{ab} = power$$

Energy:

$$W = \int P dt = \int I \cdot V dt$$

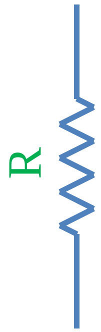
Capacitor stored energy:

$$\int I \cdot V dt = \int C \frac{dV}{dt} \cdot V dt = \frac{1}{2} C V^2 = \frac{Q^2}{2C}$$

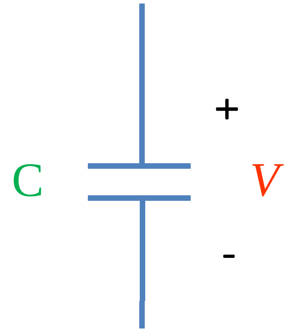
Inductor stored energy:

$$\int I \cdot V dt = \int I \cdot L \frac{dI}{dt} dt = \frac{1}{2} L I^2$$

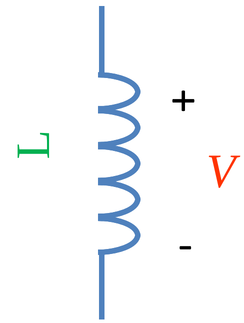
Circuits



$\mathbf{V} = \mathbf{I}\, R$



$\mathbf{V} = \mathbf{I}/j\omega C$



$\mathbf{V} = j\omega L\, \mathbf{I}$

"Impedance"

$Z = R$ $\qquad Z = 1/j\omega C$ $\qquad Z = j\omega L$

KCL, KVL hold for relationship between **V**, **I**.

Series/Parallel Impedances

$$Z_{eq} = Z_1 + Z_2 + Z_3$$

$$Z_{eq}^{-1} = Z_1^{-1} + Z_2^{-1} + Z_3^{-1}$$

Example Impedance Problem

Find Z_{eq} for this circuit: (students)