

Announcements:

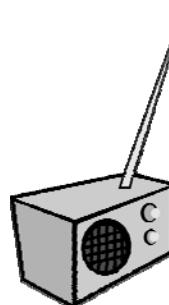
1. Announcement

EECS 70A: Network Analysis

Lecture 13

Wireless Communications

Broadcast Radio:



Telecom:



Internet:



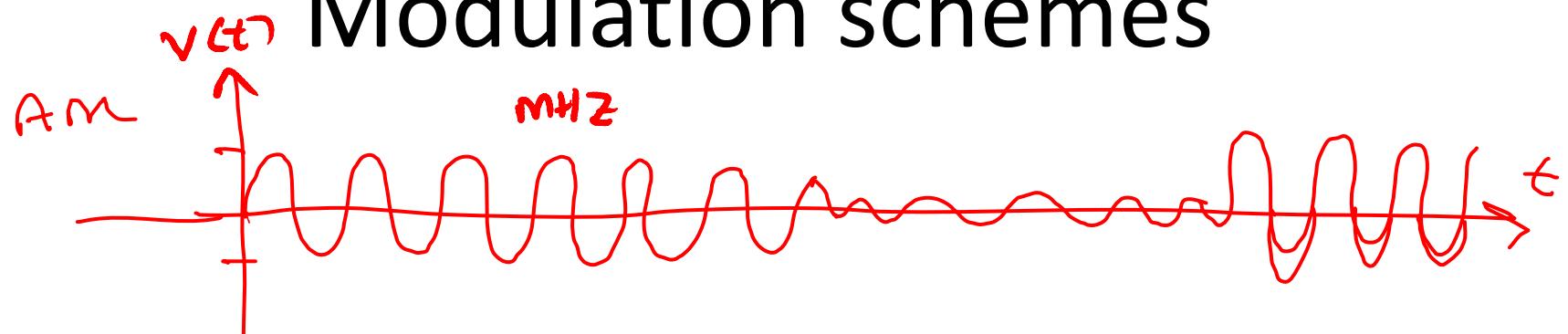
3G data:



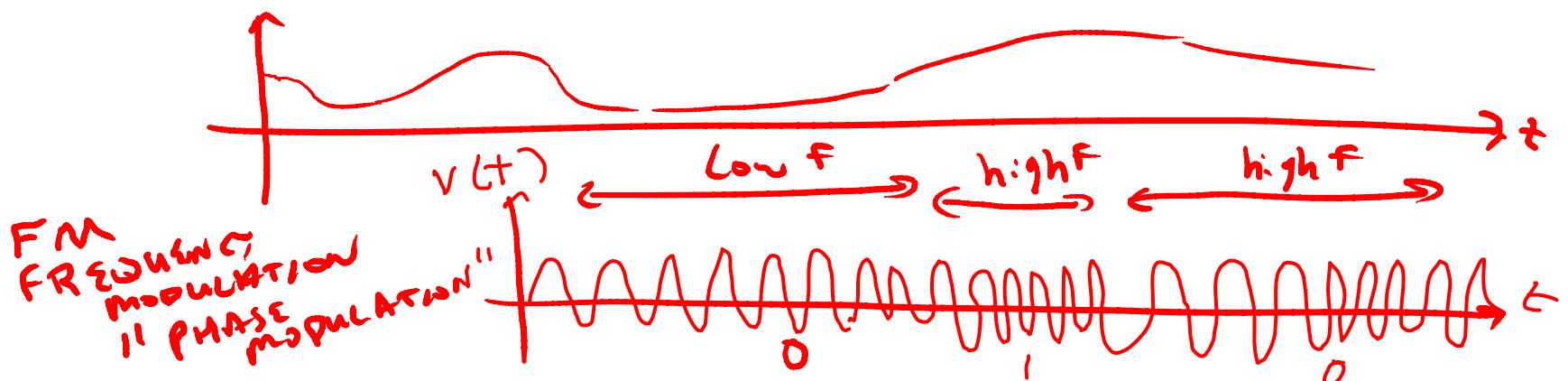
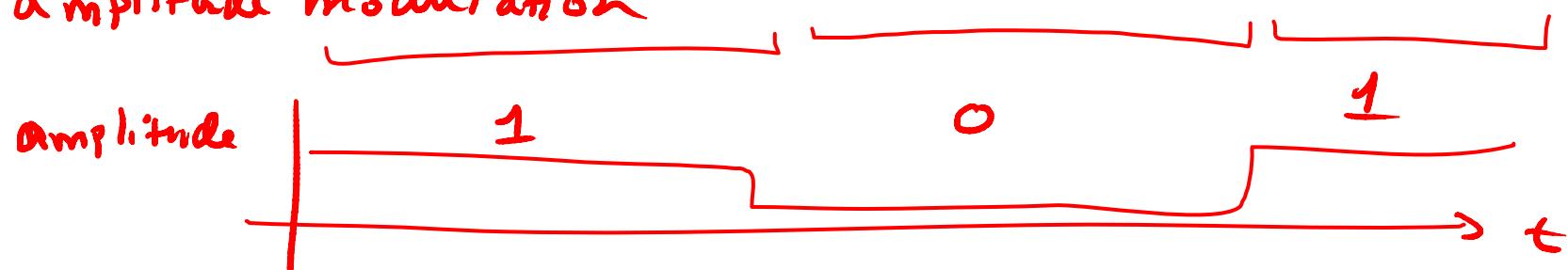
All use sine waves (phasors) as way to describe signals and circuits.

$$z = r e^{j\theta}$$

Modulation schemes



amplitude modulation



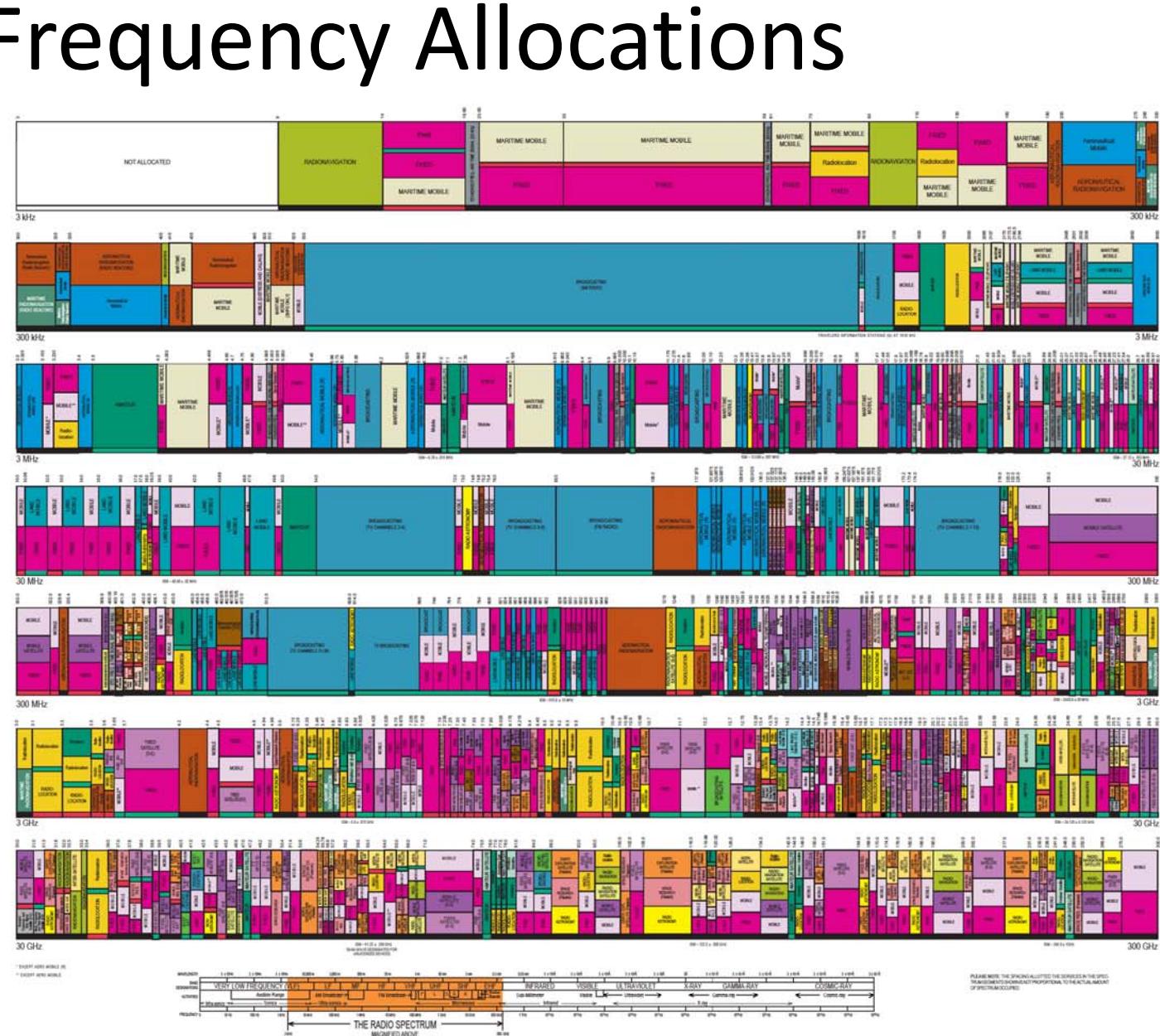
UNITED STATES FREQUENCY ALLOCATIONS

THE RADIO SPECTRUM



This chart is a graphic representation of the U.S. Frequency Allocations used by the FCC and NTIA. No such chart can completely reflect all aspects, i.e., technical and legal changes. For more detailed information, refer to the source documents cited at the bottom of this chart.

U.S. DEPARTMENT OF COMMERCE
National Telecommunications and Information Administration
Office of Spectrum Management
October 2003



<http://www.ntia.doc.gov/osmhome/allochrt.PDF>

Phasors

Euler $\cos\theta + j \sin\theta = e^{j\theta}$

$$V(t) = \underline{V_m \cos(\omega t + \phi)} = \operatorname{Re}(V_m e^{j(\omega t + \phi)})$$

$$= \operatorname{Re}(\underline{V_m e^{j\phi}} e^{j\omega t})$$

“Voltage Phasor” $\textcircled{\textbf{V}}$

(Complex #)

$$i(t) = I_m \cos(\omega t + \phi) = \operatorname{Re}(I_m e^{j(\omega t + \phi)})$$

$$= \operatorname{Re}(\underline{I_m e^{j\phi}} e^{j\omega t})$$

“Current Phasor” $\textcircled{\textbf{I}}$

Circuits

$$V(t) = R i(t)$$

$$V = I R$$

$$V = \frac{q}{C} R$$

$$q(t) = C V(t)$$

$$i(t) = C \frac{dV(t)}{dt}$$

$$V = I/j\omega C$$

$$V = \frac{1}{j\omega C}$$

$$V = j\omega L I$$

“Impedance”

$$Z = R$$

$$Z = 1/j\omega C$$

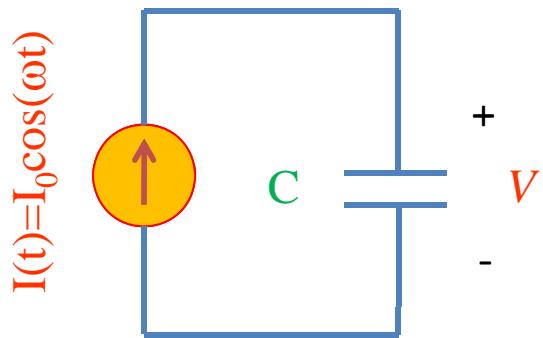
$$Z = j\omega L$$

Can think of this as a “generalized Ohm’s law for ac circuits”

KCL, KVL hold for relationship
between V , I .

Phasor to voltage conversion

Find $V(t)$, $q(t)$



Problem gives us:

$$I(t) = I_0 \cos(\omega t)$$

Compare to definition of current phasor:

$$i(t) = I_m \cos(\omega t + \phi) = \operatorname{Re}(I_m e^{j\phi} e^{j\omega t}) = \operatorname{Re}(I e^{j\omega t})$$
$$\Rightarrow \mathbf{I} = I_0$$

Find voltage phasor using generalized Ohm's law:

$$\mathbf{V} = \mathbf{I} / j\omega C$$

Find $v(t)$ from voltage phasor:

$$v(t) = \operatorname{Re}[\mathbf{V} e^{j\omega t}] = \operatorname{Re}\left[\left(\frac{\mathbf{I}}{j\omega C}\right) e^{j\omega t}\right] = \operatorname{Re}\left[\left(\frac{I_0}{j\omega C}\right) e^{j\omega t}\right]$$
$$= \frac{I_0}{\omega C} \operatorname{Re}\left[\left(\frac{1}{j}\right) e^{j\omega t}\right] = \frac{I_0}{\omega C} \operatorname{Re}[-j e^{j\omega t}] = \frac{I_0}{\omega C} \operatorname{Re}[-j(\cos(\omega t) + j \sin(\omega t))]$$
$$= \frac{I_0}{\omega C} \operatorname{Re}[-j \cos(\omega t) + (-j) j \sin(\omega t)] = \frac{I_0}{\omega C} \operatorname{Re}[\sin(\omega t) - j \cos(\omega t)] = \frac{I_0}{\omega C} \sin(\omega t)$$

Phase vs. impedance (Z)

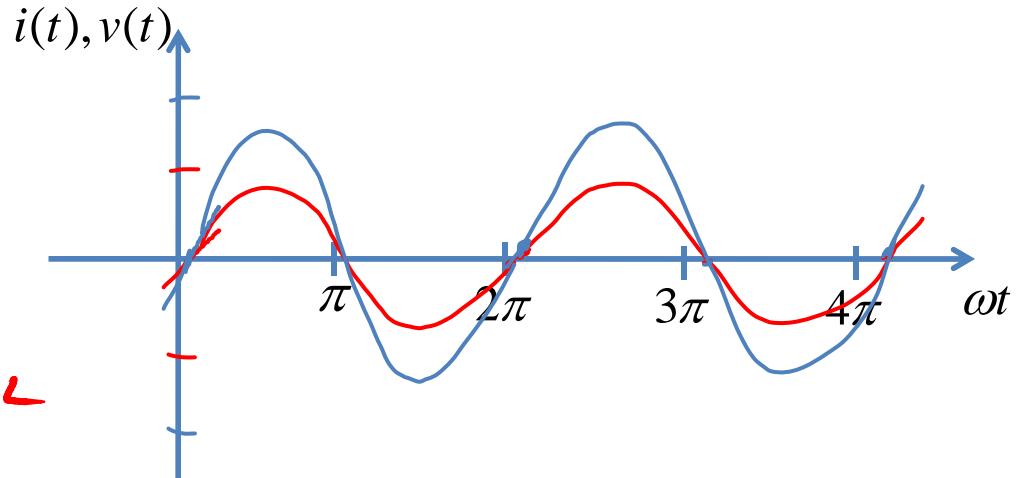
In general:

Z real i.e. $Z = x + jy$

$$\begin{array}{cc} \uparrow & \uparrow \\ \neq 0 & = 0 \end{array}$$

$\Rightarrow i(t), v(t)$ “in phase”

PROOF: TRIVIAL



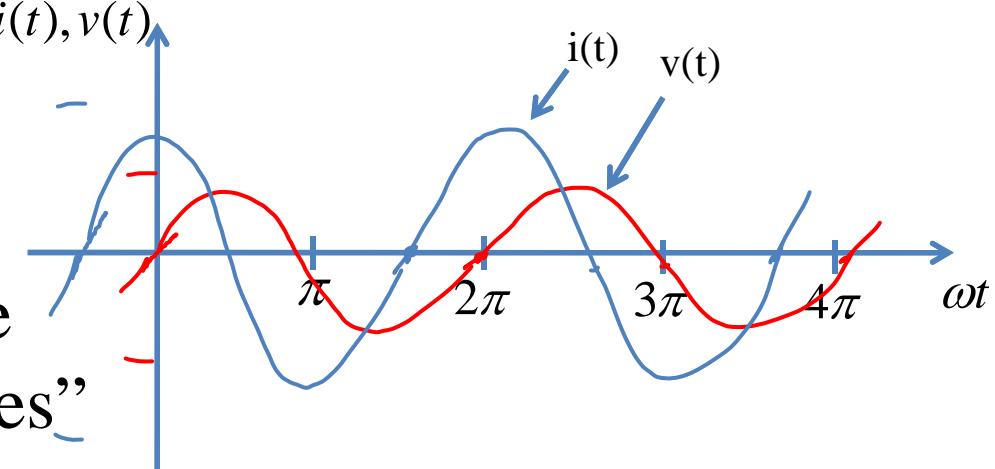
In general:

Z imag i.e. $Z = x + jy$

$$\begin{array}{cc} \uparrow & \uparrow \\ = 0 & \neq 0 \end{array}$$

$\Rightarrow i(t), v(t)$ “out of phase by 90 degrees”

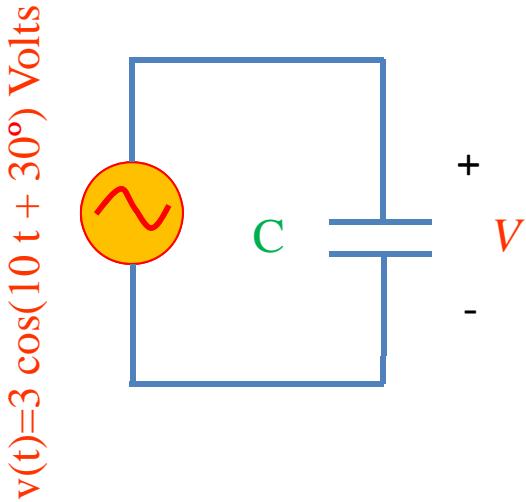
PROOF · EXERCISE



HW problem: Find relationship between phase shift and impedance (Z).

Example phasor problem

Find $i(t)$ (students)



Problem gives us:

$$v(t) = 3 \cos(10t + 30^\circ) \text{ Volts}$$

Compare to definition of voltage phasor:

$$v(t) = V_m \cos(\omega t + \phi) = \operatorname{Re}(V_m e^{j\phi} e^{j\omega t}) = \operatorname{Re}[V e^{j\omega t}]$$

$$\Rightarrow V = ? \quad (3) e^{j(30^\circ)} \quad \begin{matrix} \text{TPHASOR} \\ \text{RADIAN} \end{matrix} \quad \begin{matrix} \uparrow \\ \text{time dependence} \end{matrix}$$

Find current phasor using generalized Ohm's law:

$$\underline{I} = j\omega C \underline{V} = \dots \quad (10jC) e^{j30^\circ}$$

Find $i(t)$ from current phasor:

$$i(t) = \operatorname{Re}(I e^{j\omega t}) = \dots$$

$$= \operatorname{Re}[10jC 3e^{j30^\circ} e^{j\omega t}] = \\ = 10 \times 3 \times C \operatorname{Re}[j e^{j30^\circ} e^{j\omega t}]$$

$$= 10 \times 3 \times C \operatorname{Re}[e^{j\frac{\pi}{6}} e^{j30^\circ} e^{j\omega t}]$$

$$= 10 \times 3 \times C \operatorname{Re}[e^{j\frac{\pi}{6}} e^{j\frac{\pi}{6}} e^{j\omega t}] = 30C \operatorname{Re}[e^{j(\frac{2\pi}{3} + \omega t)}] \\ = 30C \cos\left[\omega t + \frac{2\pi}{3}\right] \checkmark$$

TRICK:

$$j = e^{j\frac{\pi}{2}} = \frac{\cos \frac{\pi}{2}}{0} + j \frac{\sin \frac{\pi}{2}}{1} = j$$

TODAY !!!
PHASOR CONSTANT
IS DOES NOT
DEPEND ON
TIME

Trick #1

Euler

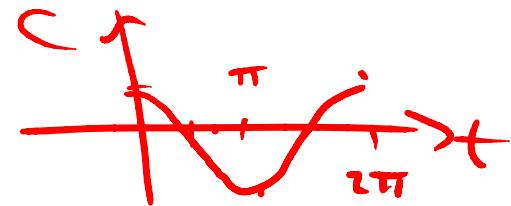
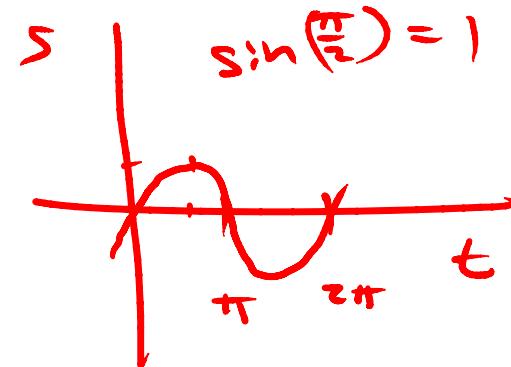
$$e^{j\theta} = \cos \theta + j \sin \theta$$

"Consider" $\theta = \frac{\pi}{2}$

$$e^{j\frac{\pi}{2}} = \cos \frac{\pi}{2} + j \sin \frac{\pi}{2}$$

$$= 0 + j(1) = j$$

$$\Rightarrow j = e^{j\frac{\pi}{2}}$$



$$\cos(\frac{\pi}{2}) = 0$$

Example problem #3

Find $i(t)$, $V_1(t)$, $V_2(t)$ for this circuit: (instructor)

$$V = V_0 \cos(\omega t)$$

$$V_1$$

$$V_2$$

$$Z_{eq} = (R + \frac{1}{j\omega C}) \frac{j\omega L}{j\omega L} = \frac{1 + j\omega RC}{j\omega C}$$

$$I = \frac{V}{Z_{eq}}$$

$$i(t) = \operatorname{Re}[I e^{j\omega t}] = \operatorname{Re}\left[\frac{V}{Z_{eq}} e^{j\omega t}\right]$$

$$= \operatorname{Re}\left[\frac{V_0}{R + j\omega C} e^{j\omega t}\right] = \operatorname{Re}\left[\frac{i V_0 j\omega RC}{R + j\omega RC} e^{j\omega t}\right]$$

$$= \frac{V_0}{R} \frac{\omega t}{\sqrt{1 + (\omega C)^2}} \cos(\omega t + \tan^{-1} \frac{1}{\omega C})$$

CASE 1: $\omega = 0$

$$I = \frac{V_0}{R} \cos \omega t$$

CASE 2: $\omega \rightarrow \infty$

$$I = \frac{V_0}{j\omega C} = \frac{V_0}{RC} \sin \omega t$$

$$i(t) = \text{Re} \left[\frac{1}{R} \frac{V_o j w L}{1 + j w R C} e^{j w t} \right] = \frac{V_o}{R} \text{Re} \left[\frac{j w L}{1 + j w L} e^{j w t} \right]$$

want

$$\frac{j w L}{1 + j w L} \text{ as } r e^{j \phi}$$

$$\frac{1 - j w L}{(1 + j w L)} \frac{1 - j w L}{(1 - j w L)} = \frac{1 - j w L}{1 + (w L)^2}$$

$$\begin{aligned} \frac{j w L}{1 + j w L} &= \frac{j w L (1 - j w L)}{1 + (w L)^2} = \frac{j w L - (w L)^2}{1 + (w L)^2} \\ &= w L \cdot \frac{w L + j}{1 + (w L)^2} \end{aligned}$$

$$i(t) = \frac{V_o}{R} \text{Re} \left[\frac{w L + j}{1 + (w L)^2} e^{j w t} \right]$$

$$\begin{aligned} &= \frac{V_o}{R} \frac{w L}{1 + (w L)^2} \text{Re} \left[(w L + j) e^{j w t} \right] \\ &= \frac{V_o}{R} \frac{w L}{1 + (w L)^2} \text{Re} \left[\sqrt{w L)^2 + 1} e^{j \phi} e^{j w t} \right] \end{aligned}$$

$$w\tau + j = |w\tau + j| e^{j\phi} \quad |w\tau + j| = \sqrt{(w\tau)^2 + 1} \quad e^{j\phi}$$

$$\phi \equiv \tan^{-1} \frac{\text{Im}(w\tau + j)}{\text{Re}(w\tau + j)} \equiv \tan^{-1}\left(\frac{1}{w\tau}\right)$$

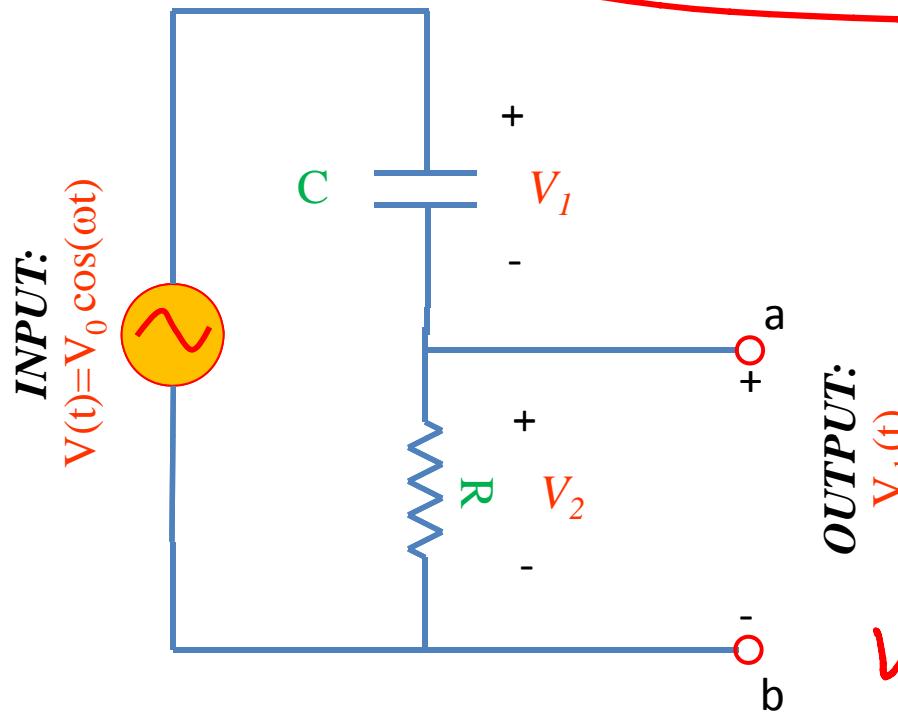
$$|w\tau + j| = \sqrt{(w\tau)^2 + 1}$$

$$i(t) = \frac{V_o}{R} \frac{w\tau}{\sqrt{1+(w\tau)^2}} \cancel{\text{Re}[e^{j\phi} e^{j\omega t}]} \quad \text{Re}[e^{j\phi} e^{j\omega t}]$$

$$= \frac{V_o}{R} \frac{w\tau}{\sqrt{1+(w\tau)^2}} \cos(\omega t + \phi)$$

$$\phi \equiv \tan^{-1}\left(\frac{1}{w\tau}\right)$$

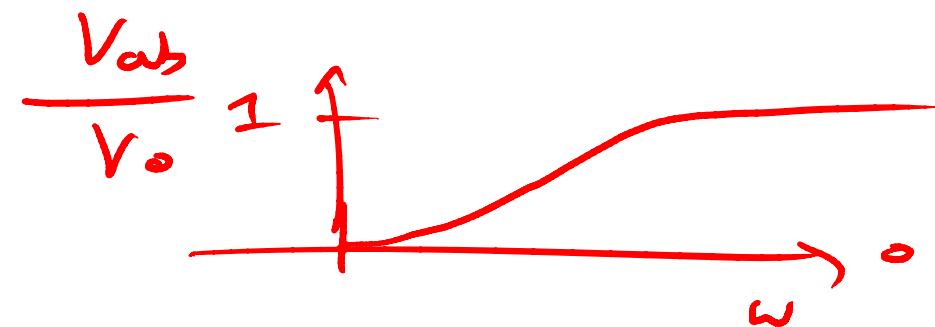
High pass filter



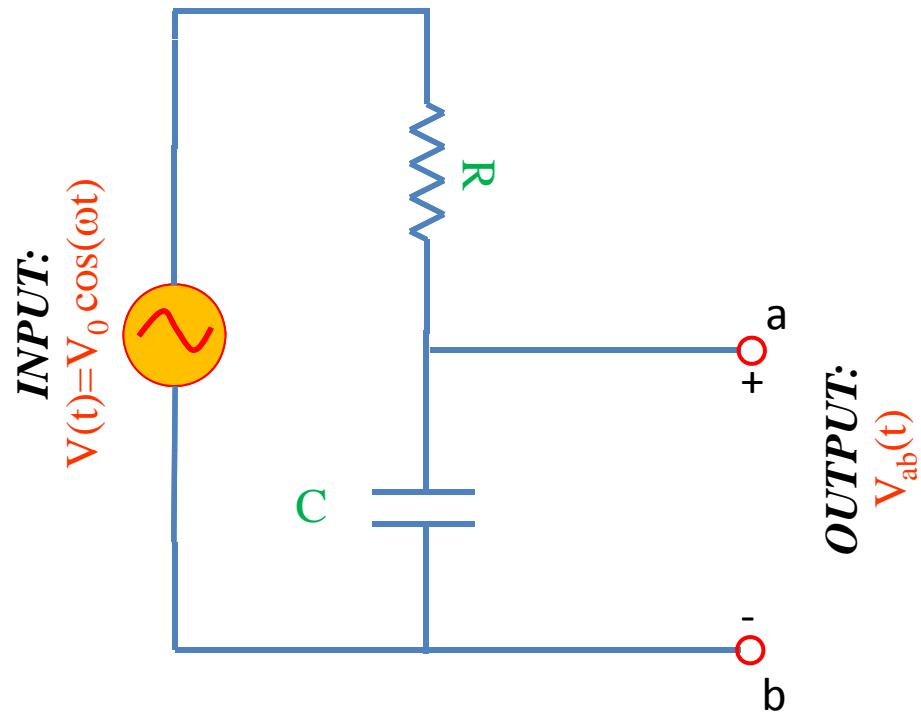
$$V_{ab}(t) = R i(t)$$

$$= R \frac{V_o}{R} \frac{\omega \tau}{\sqrt{1 + (\omega \tau)^2}} \cos(\omega t + \phi)$$

$$\frac{V_{ab}}{V_o} = \frac{\omega \tau}{\sqrt{1 + (\omega \tau)^2}} \cos(\omega t + \phi)$$



Low pass filter

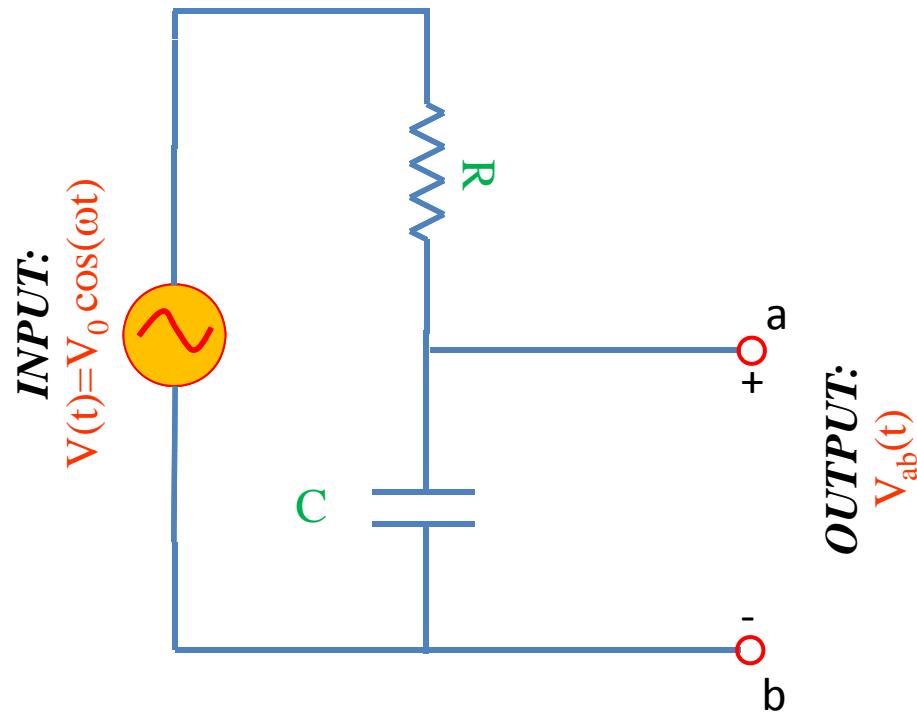


“Transfer function”



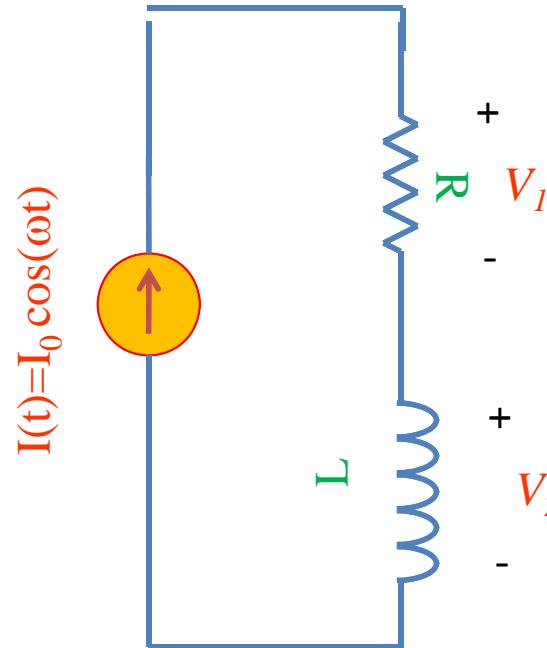
demo

RC transfer function

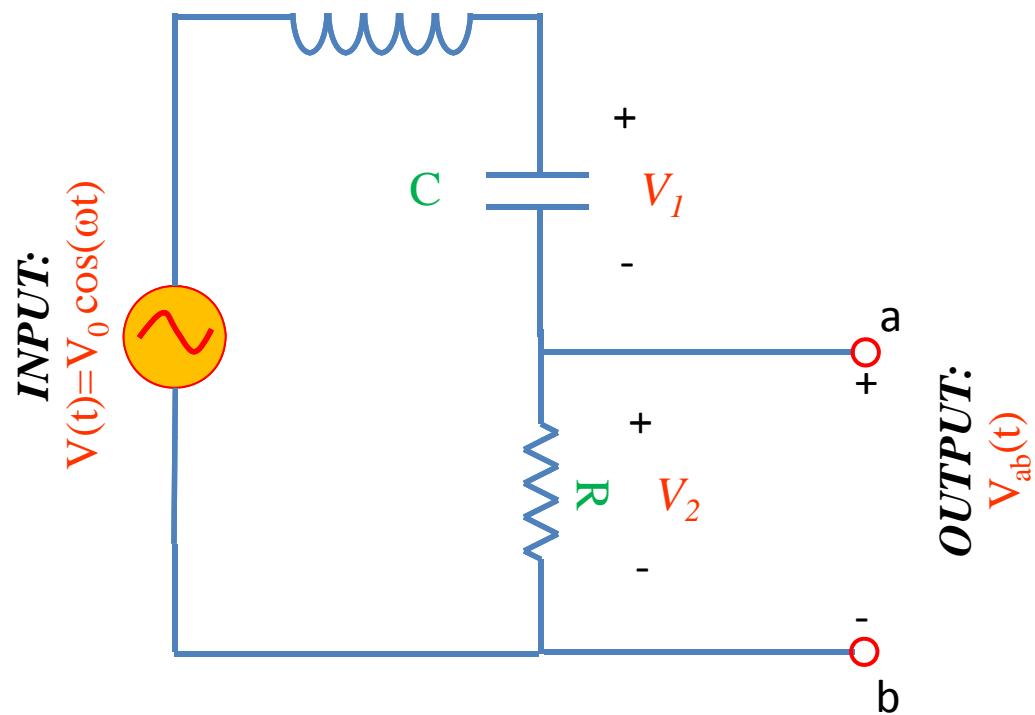


Example problem #4

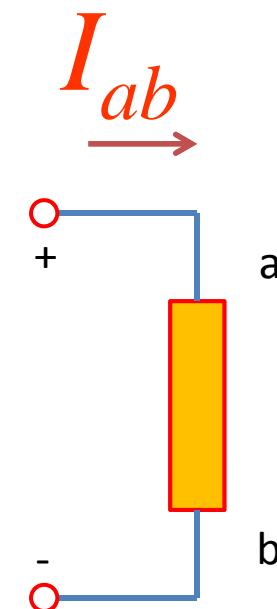
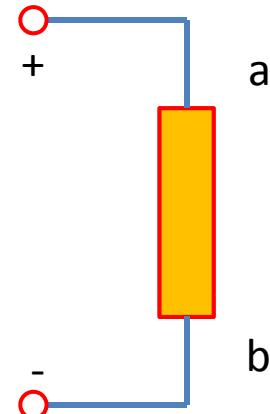
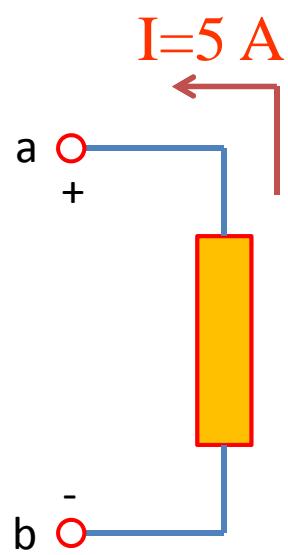
Find $i(t)$, $V_1(t)$, $V_2(t)$ for this circuit: (students)



Band pass filter (RLC)



Symbol library



Symbol library

