Announcements:

1. Announcement

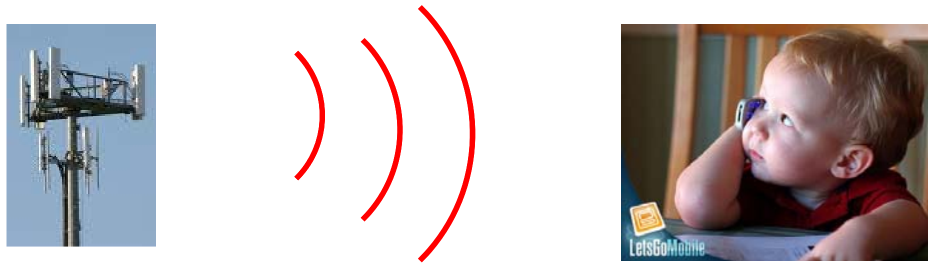
EECS 70A: Network Analysis

Lecture 13

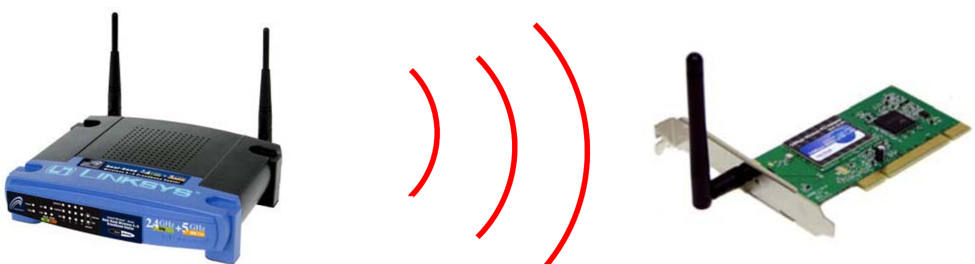
Wireless Communications

Broadcast Radio:

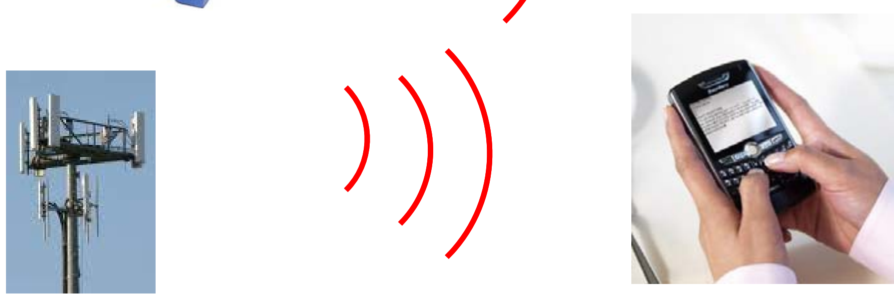
Telecom:



Internet:



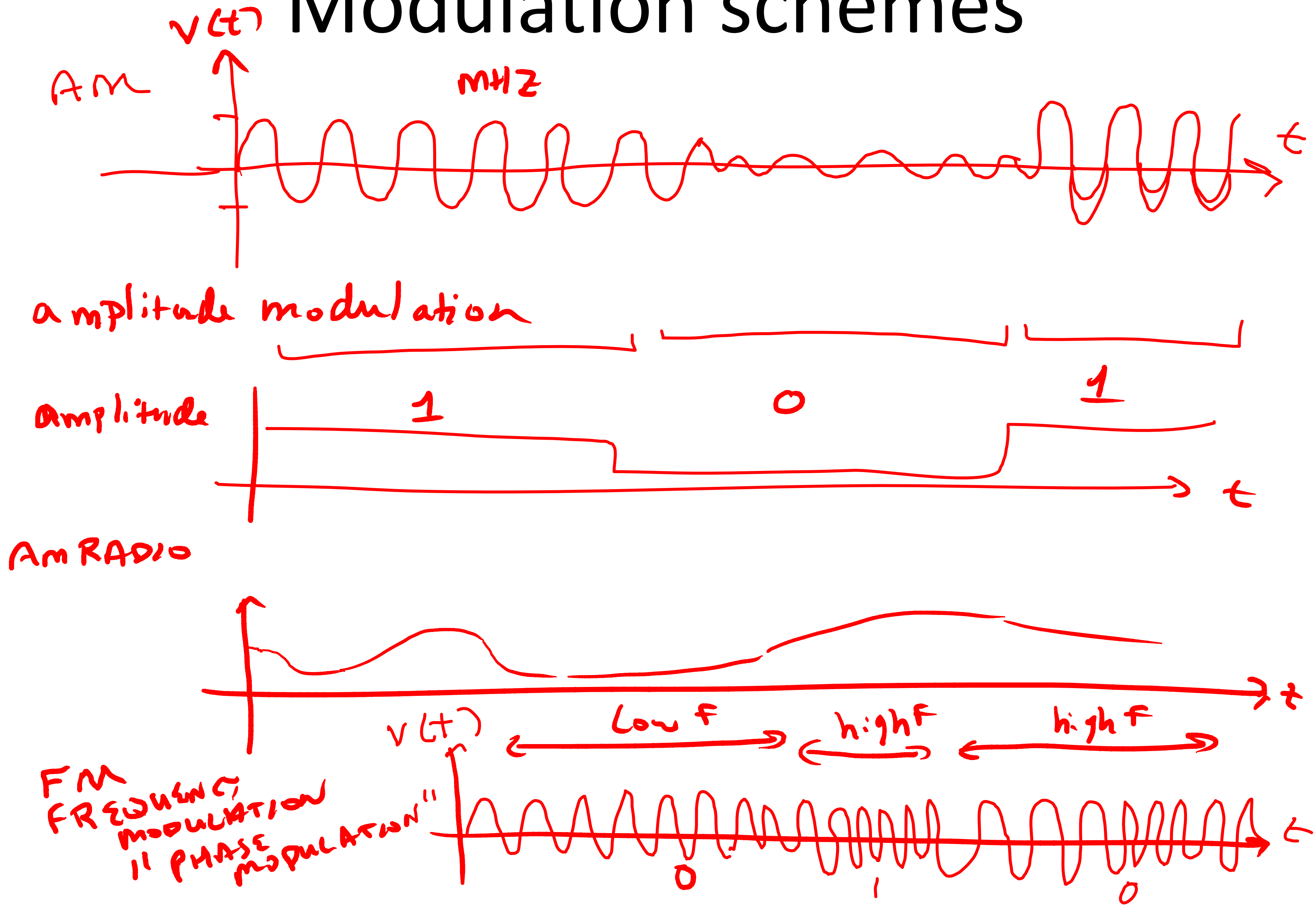
3G data:



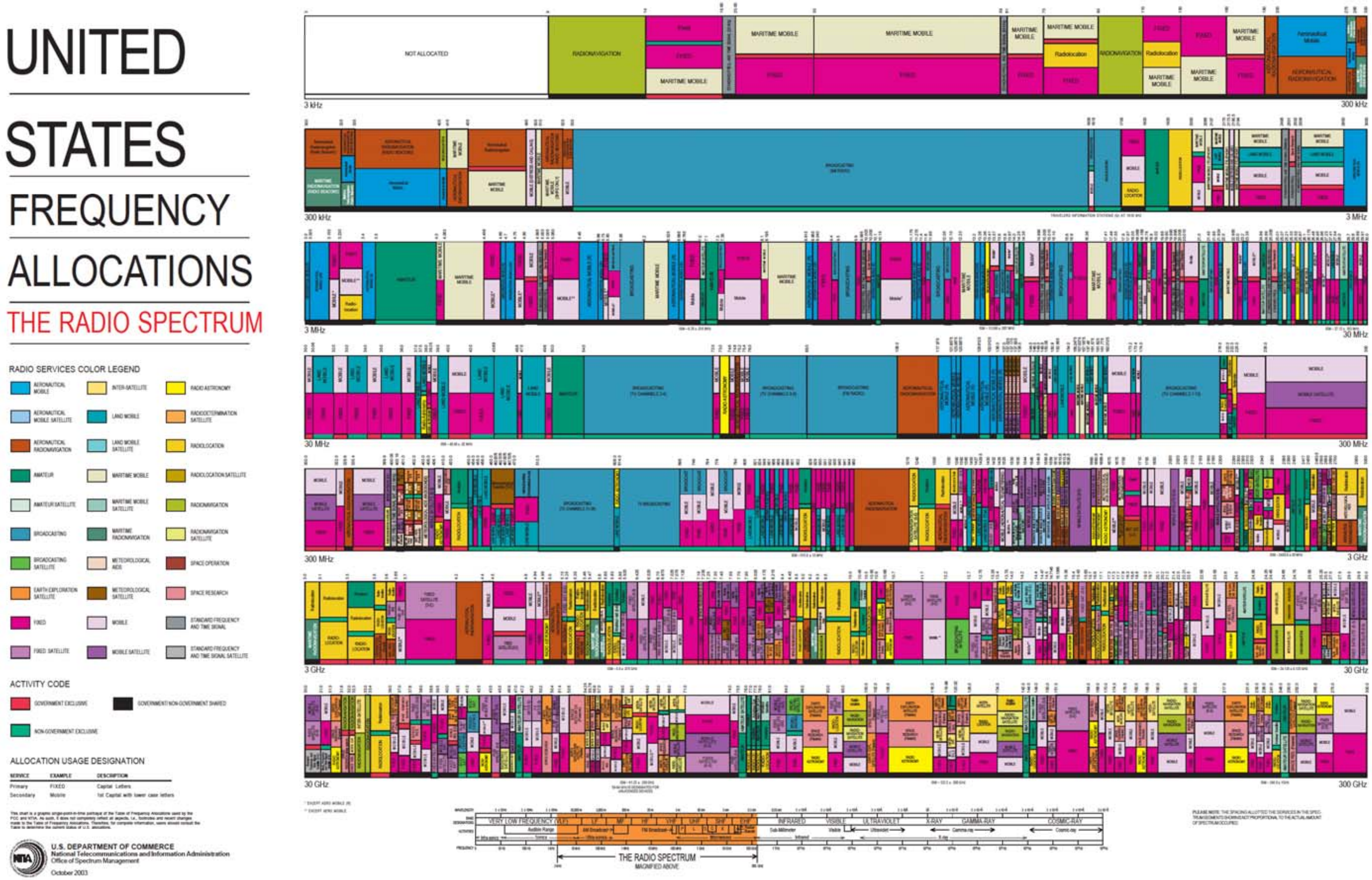
All use sine waves (phasors) as way to describe signals and circuits.

$z = re^{i\theta}$

Modulation schemes



Frequency Allocations



http://www.ntia.doc.gov/osmhome/allochrt.PDF

Phasors

Euler: $\cos\theta + j\sin\theta = e^{j\theta}$

$$V(t) = V_m \cos(\omega t + \phi) = \text{Re}\left(V_m e^{j(\omega t+\phi)}\right)$$

$$= \text{Re}\left(V_m e^{j\phi} e^{j\omega t}\right)$$

"Voltage Phasor" $\mathbf{V}$

(Complex #)

$$i(t) = I_m \cos(\omega t + \phi) = \text{Re}\left(I_m e^{j(\omega t+\phi)}\right)$$

$$= \text{Re}\left(I_m e^{j\phi} e^{j\omega t}\right)$$

"Current Phasor" $\mathbf{I}$

Circuits

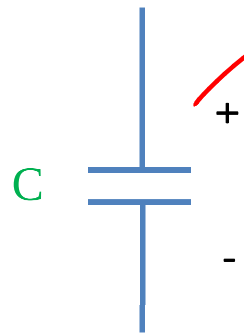
$v(t) = R\,i(t)$

$q(t) = C\,v(t)$

$i(t) = C \frac{dv(t)}{dt}$

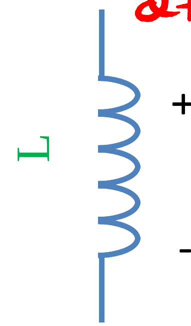
$\mathbf{V} = \mathbf{I}\,R$

$V = \mathbf{I}\,R$



$\mathbf{V} = \mathbf{I}/j\omega C$

$V = \mathbf{I}\left(\frac{1}{j\omega C}\right)$



$\mathbf{V} = j\omega L\,\mathbf{I}$

"Impedance"

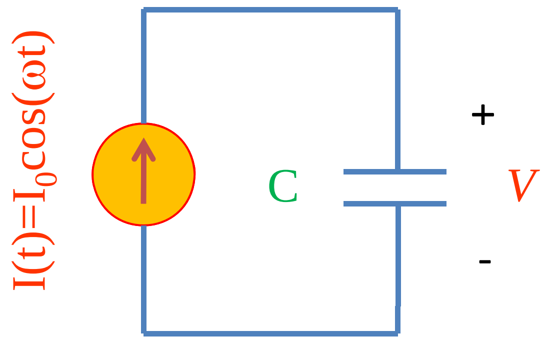
$Z = R$ $Z = 1/j\omega C$ $Z = j\omega L$

Can think of this as a "generalized Ohm's law for ac circuits"

KCL, KVL hold for relationship between **V**, **I**.

Phasor to voltage conversion

Find V(t), q(t)



Problem gives us:

$I(t)=I_0\cos(\omega t)$

Compare to definition of current phasor:

$$i(t) = I_m \cos\left(\omega t + \phi\right) = \mathrm{Re}\left(I_m e^{j\phi} e^{j\omega t}\right) = \mathrm{Re}\left(\mathbf{I} e^{j\omega t}\right)$$

$$\Rightarrow \mathbf{I} = I_0$$

Find voltage phasor using generalized Ohm's law:

$$\mathbf{V}=\mathbf{I}/j\omega C$$

Find v(t) from voltage phasor:

$$v(t) = \mathrm{Re}\left[\mathbf{V} e^{j\omega t}\right] = \mathrm{Re}\left[\left(\frac{\mathbf{I}}{j\omega C}\right) e^{j\omega t}\right] = \mathrm{Re}\left[\left(\frac{I_0}{j\omega C}\right) e^{j\omega t}\right]$$

$$= \frac{I_0}{\omega C}\mathrm{Re}\left[\left(\frac{1}{j}\right) e^{j\omega t}\right] = \frac{I_0}{\omega C}\mathrm{Re}\left[-j e^{j\omega t}\right] = \frac{I_0}{\omega C}\mathrm{Re}\left[-j\left(\cos(\omega t) + j\sin(\omega t)\right)\right]$$

$$= \frac{I_0}{\omega C}\mathrm{Re}\left[-j\cos(\omega t) + (-j)\,j\sin(\omega t)\right] = \frac{I_0}{\omega C}\mathrm{Re}\left[\sin(\omega t) - j\cos(\omega t)\right] = \frac{I_0}{\omega C}\sin(\omega t)$$

Phase vs. impedance (Z)

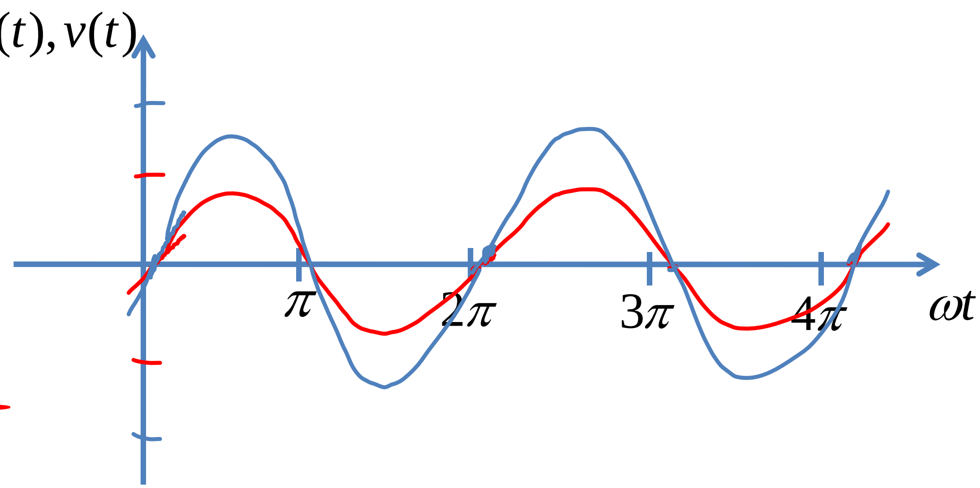
In general:

Z real i.e. Z = x + jy

$\neq 0$ $= 0$

=> i(t), v(t) "in phase"

PROOF: TRIVIAL



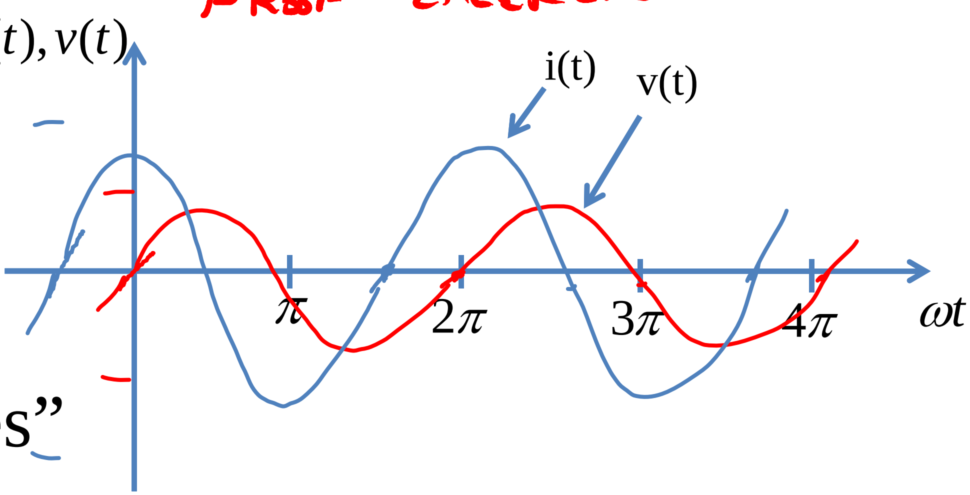
In general:

Z imag i.e. Z = x + jy

$= 0$ $\neq 0$

=> i(t), v(t) "out of phase
by 90 degrees"

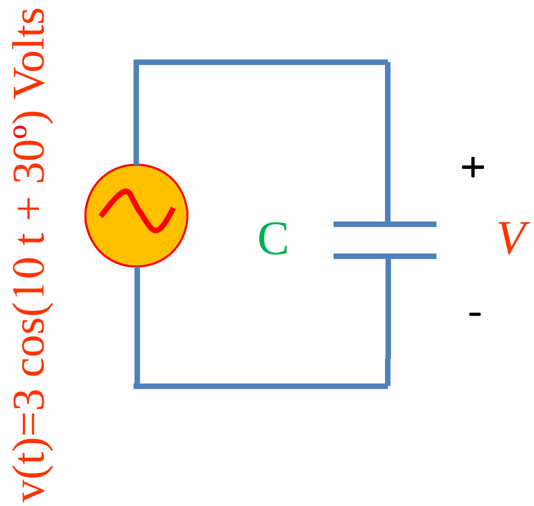
PROOF: EXCERCISE



HW problem: Find relationship between phase shift and impedance (Z).

Example phasor problem

Find i(t) (students)



Problem gives us:

v(t)=3 cos(10 t + 30º) Volts

Compare to definition of voltage phasor:

$$v(t) = V_m \cos(\omega t + \phi) = \mathrm{Re}\left(V_m e^{j\phi} e^{j\omega t}\right) = \mathrm{Re}\left[\mathbf{V} e^{j\omega t}\right]$$

TRICK: $j = e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$ (0, 1) $= j$

$\Rightarrow \mathbf{V} = ?$ $(3)e^{j(30^\circ)}$ (phasor; RADIANS; time dependence)

Find current phasor using generalized Ohm's law:

$\mathbf{I} = j\omega C\,\mathbf{V} = \ldots$ $(\quad)e^{j(\quad)}$

$= 10jC\,3e^{j30^\circ}$

Find i(t) from current phasor:

$i(t) = \mathrm{Re}\left(\mathbf{I} e^{j\omega t}\right) = \ldots$

$= \mathrm{Re}\left[10jC\,3e^{j30^\circ}e^{j\omega t}\right] =$

$= 10\times3\times C\,\mathrm{Re}\left[j\,e^{j30^\circ}e^{j\omega t}\right]$

$= 10\times3\times C\,\mathrm{Re}\left[e^{j\frac{\pi}{2}}e^{j30^\circ}e^{j\omega t}\right]$

$= 10\times3\times C\,\mathrm{Re}\left[e^{j\frac{\pi}{2}}e^{j\frac{\pi}{6}}e^{j\omega t}\right] = 30C\,\mathrm{Re}\left[e^{j\left(\frac{2\pi}{3}+\omega t\right)}\right]$

$= 30C\cos\left[\omega t + \frac{2\pi}{3}\right]$ ✓

TODAY!!! PHASOR CONSTANT IE DOES NOT DEPEND ON TIME

Trick #1

Euler

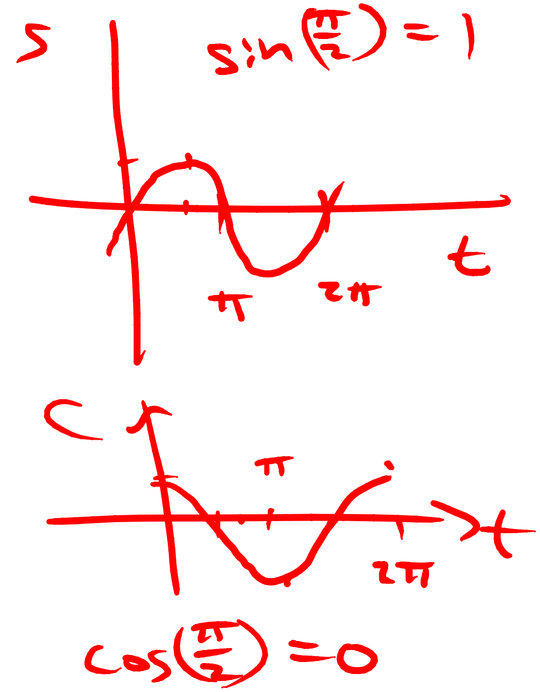
$$e^{j\theta} = \cos\theta + j\sin\theta$$

"Consider" $\theta = \frac{\pi}{2}$

$$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$$

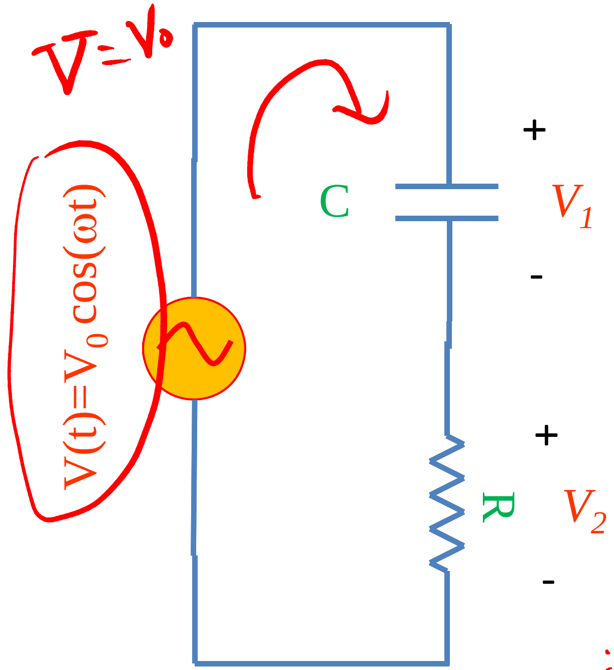
$$= 0 + j(1) = j$$

$$\Rightarrow j = e^{j\frac{\pi}{2}}$$

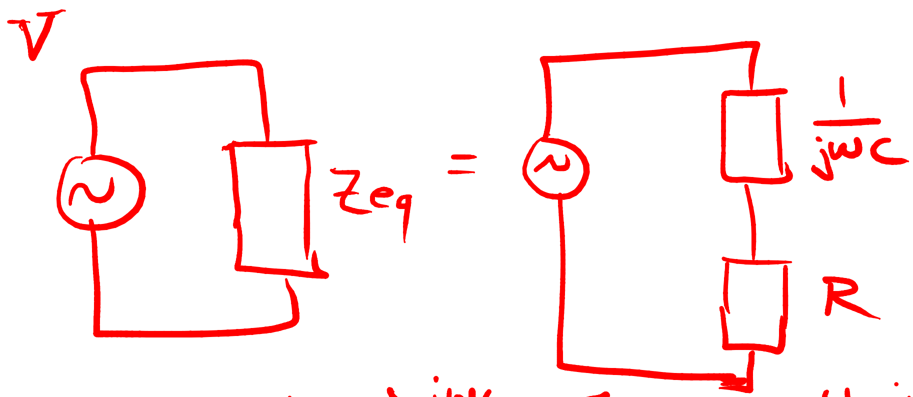


Example problem #3

Find i(t), $V_1(t)$, $V_2(t)$ for this circuit: (instructor)



$V = V_0$



$$Z_{eq} = \left(R + \frac{1}{j\omega C}\right)\frac{j\omega C}{j\omega C} = \frac{1 + j\omega RC}{j\omega C}$$

$$V = Z_{eq} I \Rightarrow I = \frac{V}{Z_{eq}}$$

$$i(t) = \mathrm{Re}\left[I e^{j\omega t}\right] = \mathrm{Re}\left[\frac{V}{Z_{eq}} e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[\frac{V_0}{R + (j\omega C)^{-1}} e^{j\omega t}\right] = \mathrm{Re}\left[\frac{1}{R}\frac{V_0\, j\omega RC}{1 + j\omega RC} e^{j\omega t}\right]$$

$$= \frac{V_0}{R}\frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}\cos\left(\omega t + \tan^{-1}\frac{1}{\omega\tau}\right)$$

$\tau = RC$

$e^{j\omega t} = \cos\omega t + j\sin\omega t$

CASE 1: $\omega = \infty$

CASE 2: $\omega \to 0$

$$i(t) = \mathrm{Re}\left[\frac{1}{R}\frac{V_0 j\omega RC}{1+j\omega RC} e^{j\omega t}\right] = \frac{V_0}{R}\mathrm{Re}\left[\frac{j\omega\tau}{1+j\omega\tau} e^{j\omega\tau}\right]$$

want $\frac{j\omega\tau}{1+j\omega\tau}$ as $re^{j\phi}$

$$\frac{1}{(1+j\omega\tau)}\frac{1-j\omega\tau}{(1-j\omega\tau)} = \frac{1-j\omega\tau}{1+(\omega\tau)^2}$$

$$\frac{j\omega\tau}{1+j\omega\tau} = j\omega\tau\frac{(1-j\omega\tau)}{1+(\omega\tau)^2} = \frac{j\omega\tau - (\omega\tau)^2}{1+(\omega\tau)^2}$$

$$= \omega\tau\,\frac{\omega\tau + j}{1+(\omega\tau)^2}$$

$$i(t) = \frac{V_0}{R}\mathrm{Re}\left[\omega\tau\frac{\omega\tau+j}{1+(\omega\tau)^2} e^{j\omega t}\right]$$

$$= \frac{V_0}{R}\frac{\omega\tau}{1+(\omega\tau)^2}\mathrm{Re}\left[(\omega\tau+j)e^{j\omega t}\right]$$

$$= \frac{V_0}{R}\frac{\omega\tau}{1+(\omega\tau)^2}\mathrm{Re}\left[\sqrt{(\omega\tau)^2+1}\, e^{j\phi}e^{j\omega t}\right]$$

$$\omega\tau + j = |\omega\tau + j|\, e^{j\phi} = \sqrt{(\omega\tau)^2+1}\; e^{j\phi}$$

$$\phi \equiv \tan^{-1} \frac{\mathrm{Im}(\omega\tau + j)}{\mathrm{Re}(\omega\tau + j)} \equiv \tan^{-1}\left(\frac{1}{\omega\tau}\right)$$

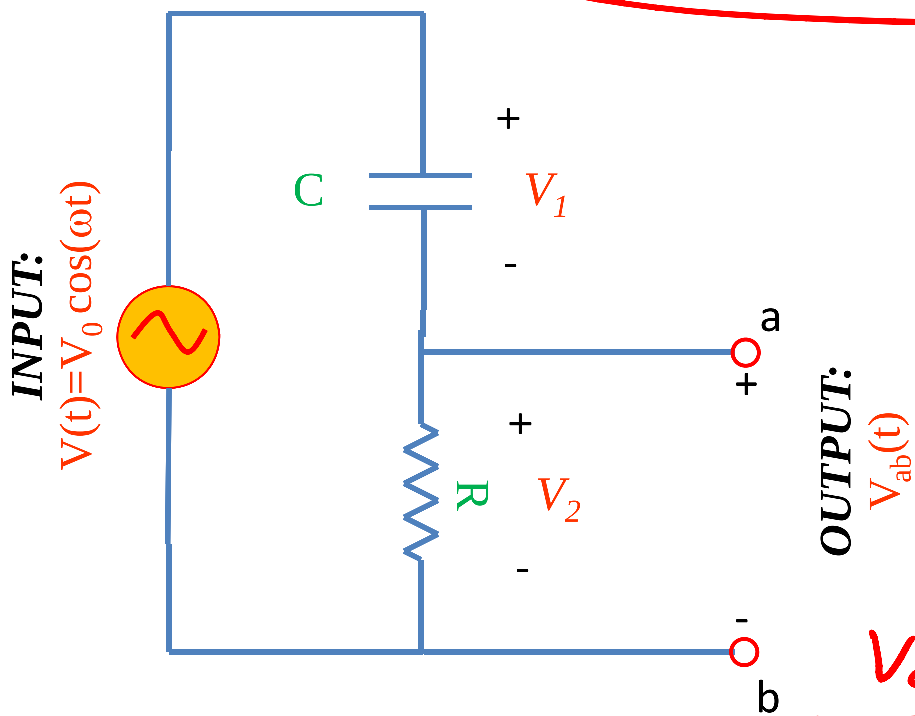
$$|\omega\tau + j| = \sqrt{(\omega\tau)^2 + 1}$$

$$i(t) = \frac{V_0}{R} \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}\, \mathrm{Re}\left[e^{j\phi} e^{j\omega t}\right]$$

$$= \frac{V_0}{R} \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} \cos(\omega t + \phi)$$

$$\phi \equiv \tan^{-1}\left(\frac{1}{\omega\tau}\right)$$

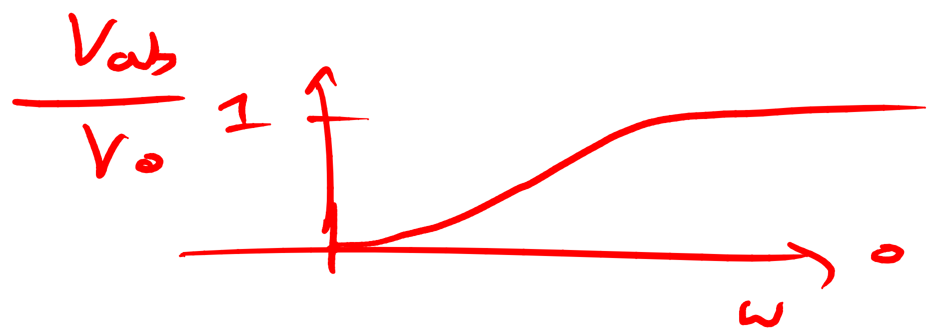
High pass filter



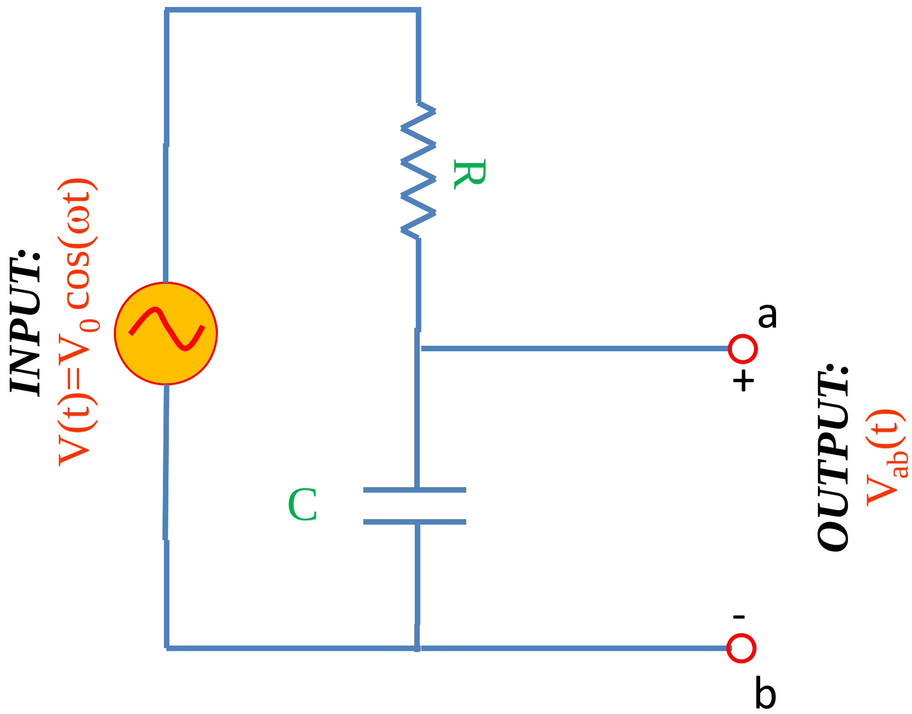
$$V_{ab}(t) = R\, i(t)$$

$$= R \frac{V_0}{R} \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} \cos(\omega t+\phi)$$

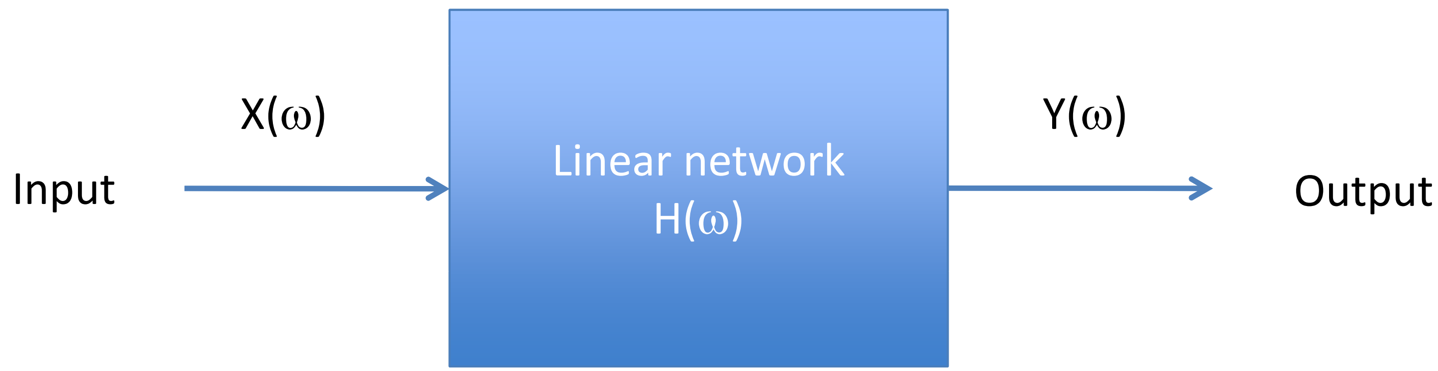
$$\frac{V_{ab}}{V_0} = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} \cos(\omega t+\phi)$$



Low pass filter

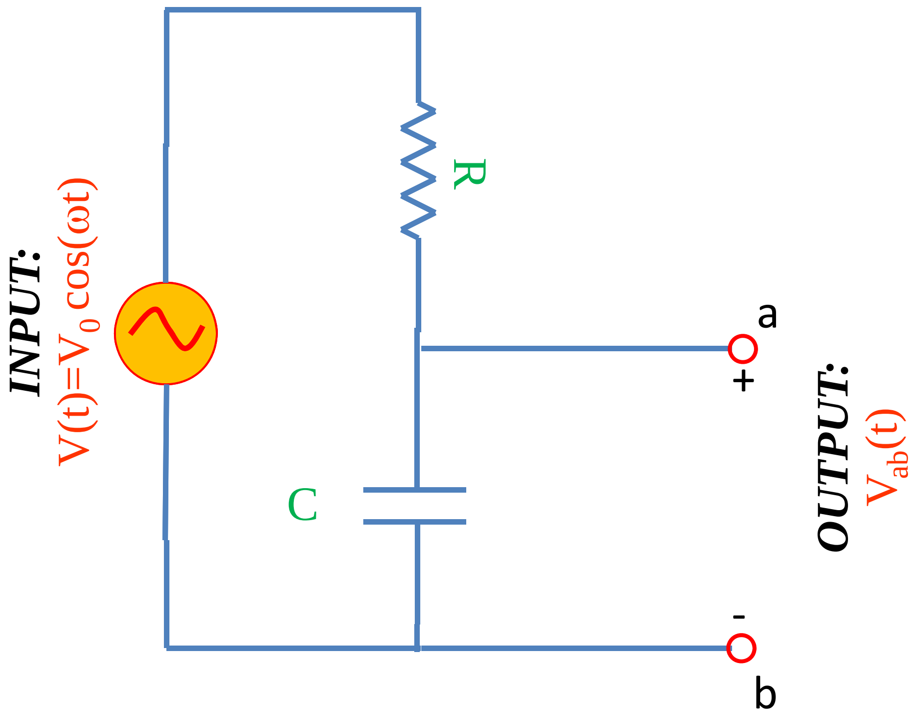


“Transfer function”



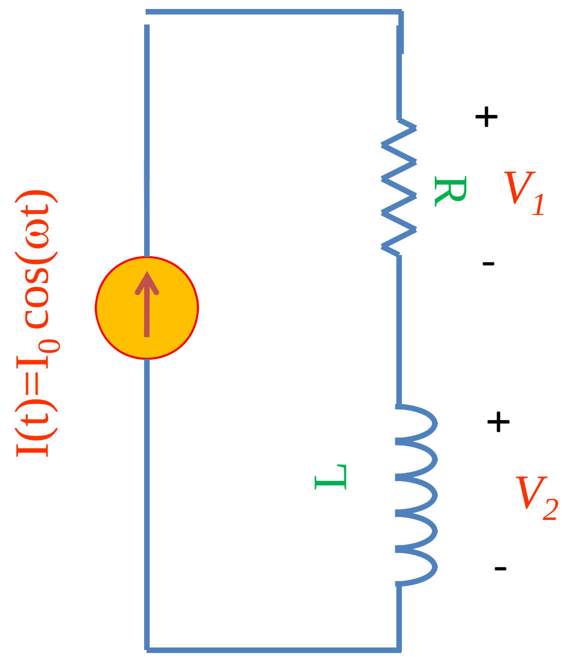
demo

RC transfer function

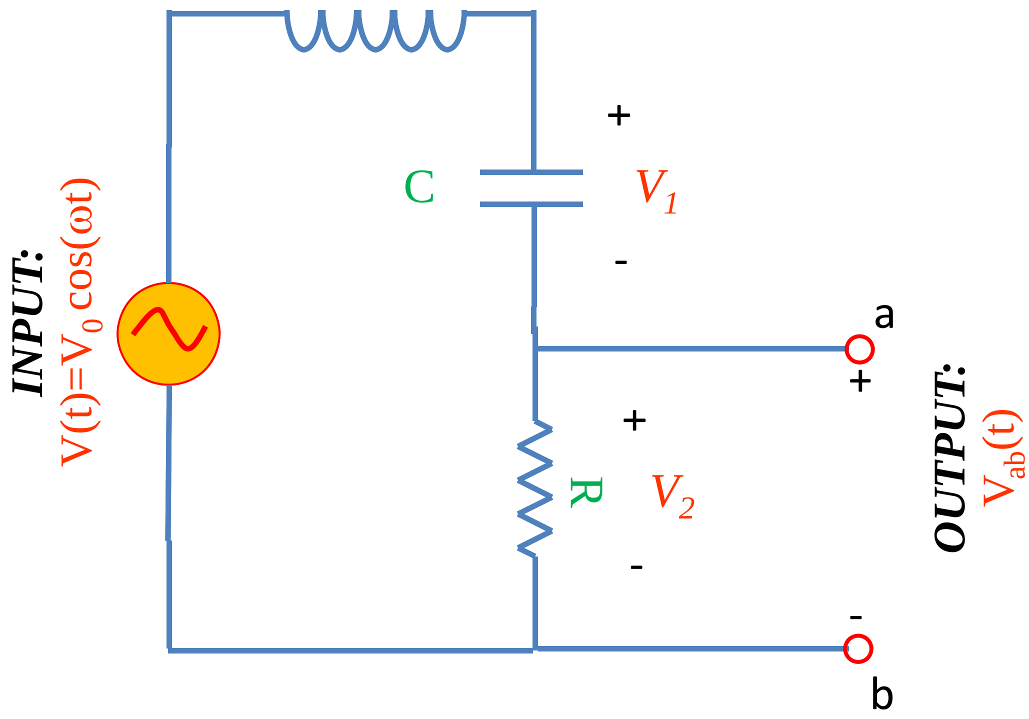


Example problem #4

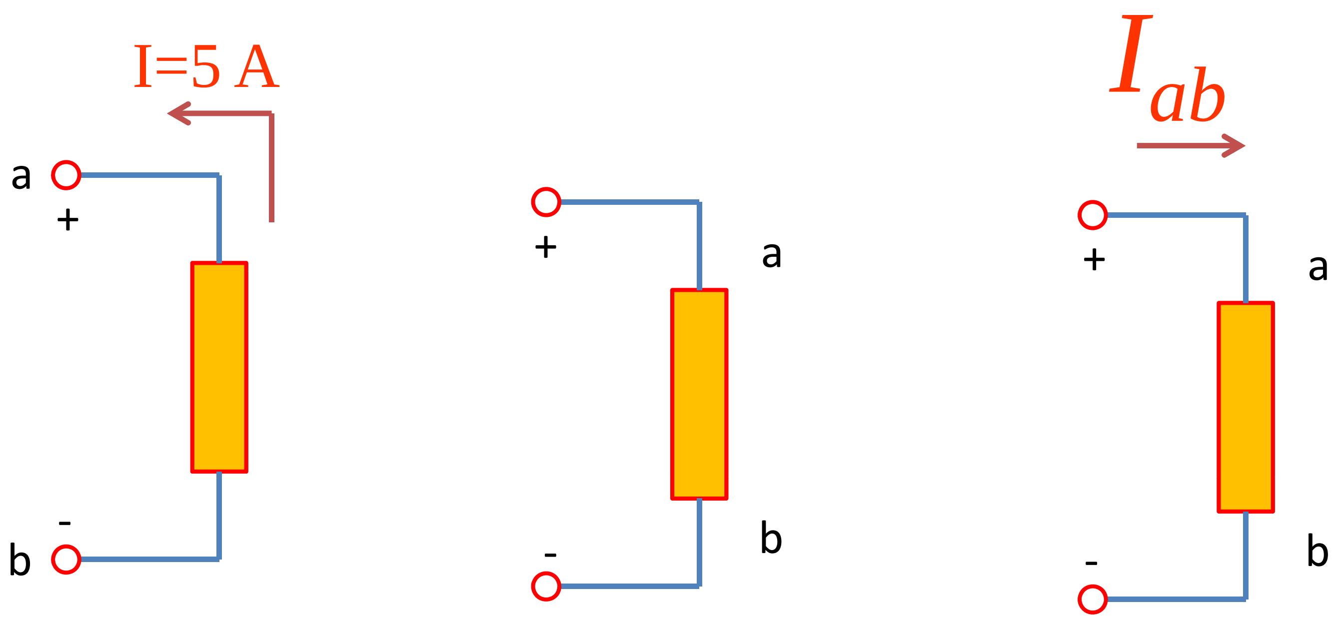
Find i(t), V_1(t), V_2(t) for this circuit: (students)



Band pass filter (RLC)



Symbol library



Symbol library