Announcements:

1. Final HW due Friday of 10th week
2. Professor Burke's office hours this week:
   Tu 9:30-11:30 (EH 2230)
   Th 9:30-11 (EG 2232)
   Th 2-3:30 (EH 2230)
3. Exam will cover all of Ch. 9 (not delta-Y),
   and part of Ch. 14 (14.1 14.2 14.3 14.5 14.6 14.7)

# EECS 70A: Network Analysis

## Lecture 15

# Conversion procedures

i(t) → **I**

v(t) → **V**

**I** → i(t)

**V** → v(t)

$$i(t) = I_m \cos(\omega t + \phi) \Rightarrow \quad \mathbf{I} = I_m e^{j\phi}$$

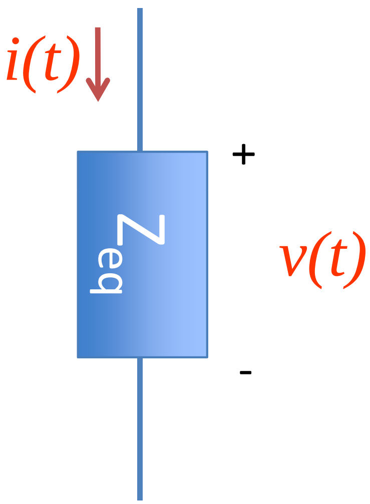
$$v(t) = V_m \cos(\omega t + \phi) \Rightarrow \quad \mathbf{V} = V_m e^{j\phi}$$

$$i(t) = \mathrm{Re}\left(\mathbf{I}\, e^{j\omega t}\right)$$

$$v(t) = \mathrm{Re}\left(\mathbf{V} e^{j\omega t}\right)$$

For the exam, you should know how to carry out these procedures.

# Conversion procedures




Given i(t) find v(t):

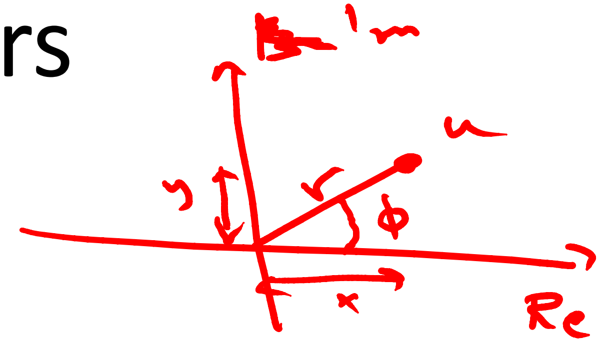
$$i(t) \rightarrow \mathbf{I} \rightarrow \mathbf{V} = \mathbf{I}\, Z_{eq} \rightarrow v(t)$$

Given v(t) find i(t):

$$v(t) \rightarrow \mathbf{V} \rightarrow \mathbf{I} = \mathbf{V} / Z_{eq} \rightarrow i(t)$$

For the exam, you should know how to carry out these procedures.

# Complex numbers




Euler's relationship:

$$e^{j\phi} = \cos\phi + j\sin\phi$$

*Memorize*

Need to be able to manipulate complex numbers, e.g. given:

$$u = \frac{A + jB}{C + jD}$$

(A,B,C,D all real)

$u = re^{j\phi}$

Class: Find $|u|$ in terms of $r, \phi$:

$|u| = r$

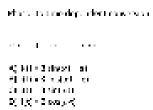
- Find Re(u), Im(u)
- Express u as x+jy, $re^{j\phi}$
- Find $\mathrm{Re}(u\, e^{j\omega t})$
- Find $|u| = \sqrt{\mathrm{Re}(u)^2 + \mathrm{Im}(u)^2}$

Did some of this in office hours..

# Quiz #6 Results

| | % | # | |
|---|---|---|---|
| A | 85.71% | 108/126 | C |
| B | 3.97% | 5/126 | I |
| C | 8.73% | 11/126 | I |
| D | 1.59% | 2/126 | I |
| <# of responses to this question> | 99.21% | 126/127 | |

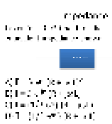
## 2. Answer the question below.



Type: Multiple Choice | View Students — Points: 10

| Answer Option | % | # | |
|---|---|---|---|
| A | 1.57% | 2/127 | I |
| B | 93.7% | 119/127 | C |
| C | 0.79% | 1/127 | I |
| D | 3.94% | 5/127 | I |
| <# of responses to this question> | 100% | 127/127 | |

## 3. Answer the question below.



Type: Multiple Choice | View Students — Points: 10

| Answer Option | % | # | |
|---|---|---|---|
| A | 95.28% | 121/127 | C |
| B | 1.57% | 2/127 | I |
| C | 2.36% | 3/127 | I |
| D | 0.79% | 1/127 | I |

# "Transfer Function" $H(\omega)$


Could define $\frac{V_{out}(t)}{V_{in}(t)}$ Too complicated for arbitrary circuits and arbitrary waveforms.

Instead define $\frac{V_{out}}{V_{in}}$ assume periodic functions i.e.

$$V_{in}(t) = V_{in,m} \cos(\omega t + \phi_{in})$$

$$V_{out}(t) = V_{out,m} \cos(\omega t + \phi_{out})$$

Transfer Function

$$H(\omega) \equiv \frac{V_{out}}{V_{in}}$$

Note: $\frac{V_{out}(t)}{V_{in}(t)}$ and $\frac{V_{out}}{V_{in}}$ will depend on $\omega$.

# Significance of H(ω)

REAL

$V_{in}(t) = V_{in,m} \cos(\omega t + \phi_{in})$ $\qquad \tilde{V}_{in} = V_{in,m} e^{j\phi_{in}}$

$V_{out}(t) = V_{out,m} \cos(\omega t + \phi_{out})$ $\qquad \tilde{V}_{out} = V_{out,m} e^{j\phi_{out}}$

$$H(\omega) \equiv \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{V_{out,m} e^{j\phi_{out}}}{V_{in,m} e^{j\phi_{in}}}$$

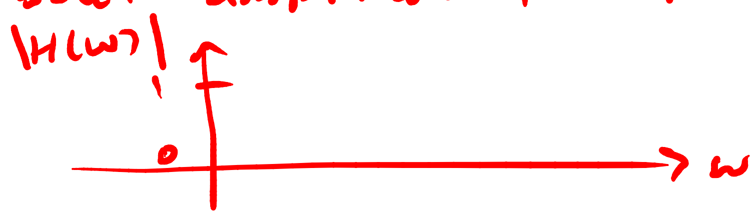
$$= \frac{V_{out,m}}{V_{in,m}} \frac{e^{j\phi_{out}}}{e^{j\phi_{in}}} = \frac{V_{out,m}}{V_{in,m}} e^{j(\phi_{out} - \phi_{in})}$$

$$|H(\omega)| = \frac{V_{out,m}}{V_{in,m}}$$

tells us important info. about amplitude of output signal.

Aside: E real, φ real

$$\left| E e^{j\phi} \right| = E$$



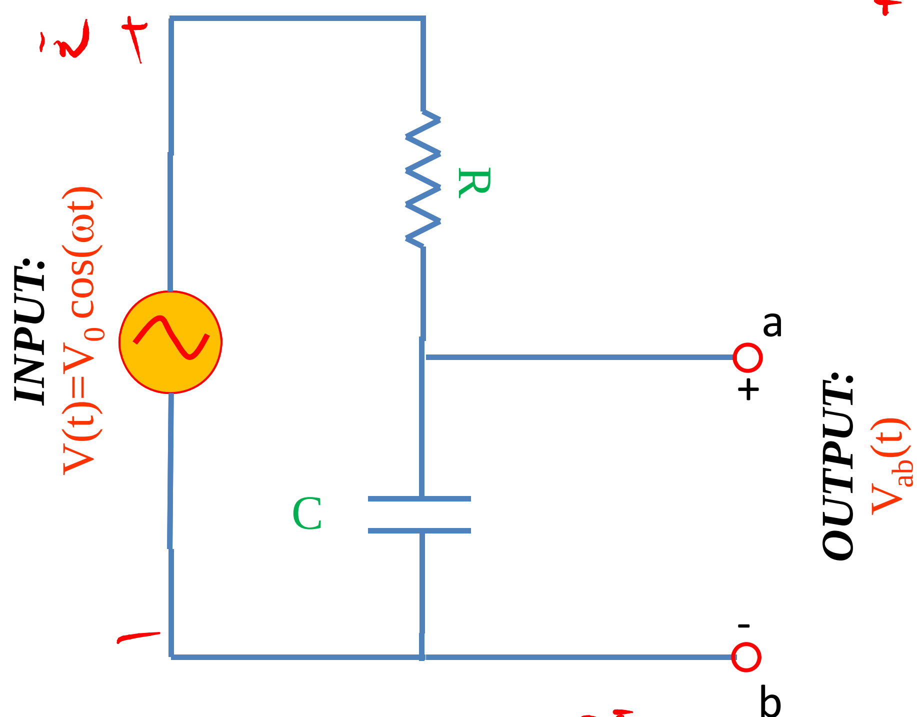

"Bode plot." log-log typical

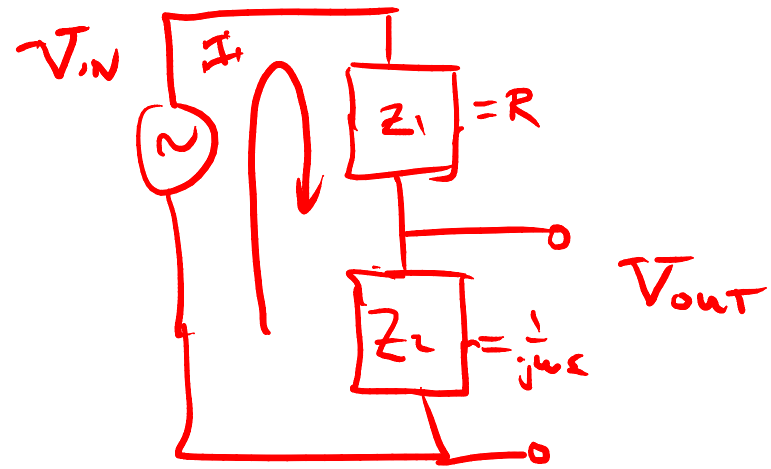
$|H(\omega)|$ dB $\equiv 20 \log |H(\omega)|$

~~dtesh~~ Decibel

# RC transfer function

FIND $H(\omega)$




$$I = \frac{V_{IN}}{Z_1 + Z_2}$$

$$V_{OUT} = I\, Z_2 = \frac{V_{IN}}{Z_1 + Z_2} Z_2$$

$$\Rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{Z_2}{Z_1 + Z_2} = \boxed{\frac{1}{1 + j\omega RC} = H(\omega)}$$

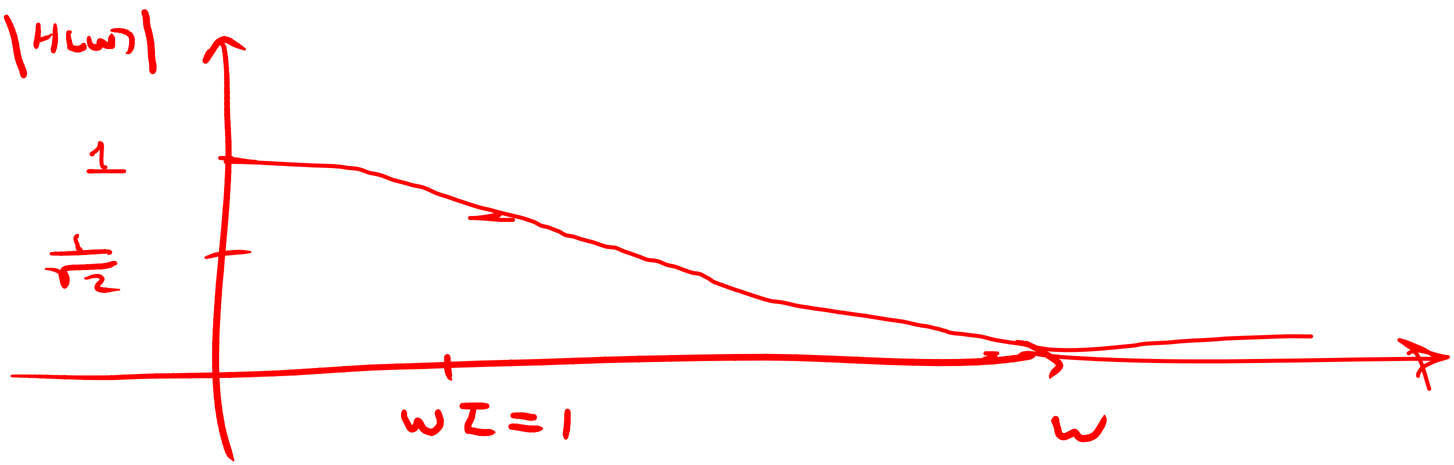
# Magnitude of H

$\tau \equiv RC$

$$|H(\omega)| = \left|\frac{1}{1+jRC\omega}\right| = \left|\frac{1}{1+j\omega\tau}\right|$$

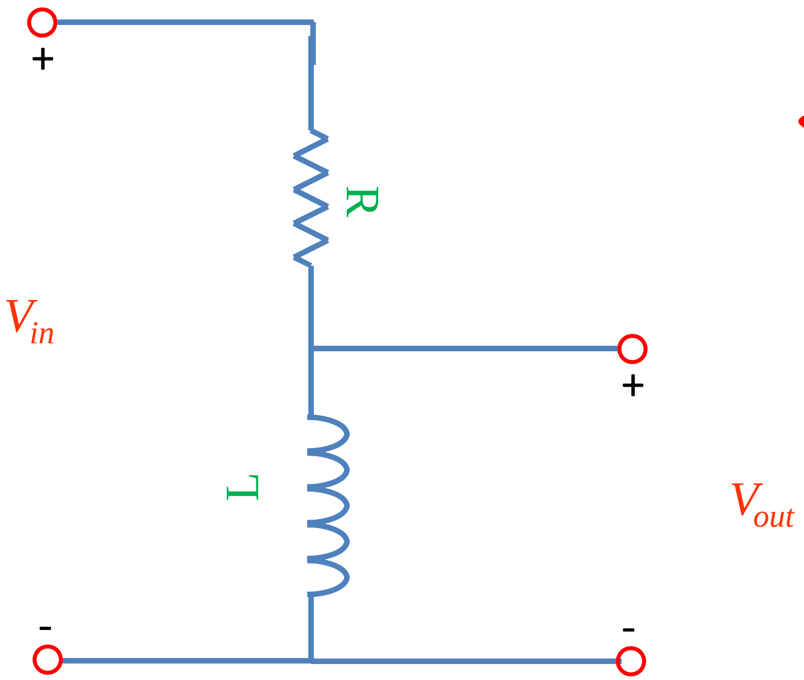
$$= \frac{1}{\sqrt{1+(\omega\tau)^2}}$$

LPF



# Example problem

Find H(ω) for this circuit, then sketch the magnitude of H(ω) vs ω: (students)




STEP 1: DRAW AS IMPEDANCES

STEP 2: $\frac{V_{out}}{V_{in}} = H(\omega)$ RECALL VOLTAGE DIVIDER

STEP 3: $|H(\omega)|$

HPF

high pass Filter


$\omega\tau = 1$

$\tau = \frac{L}{R}$

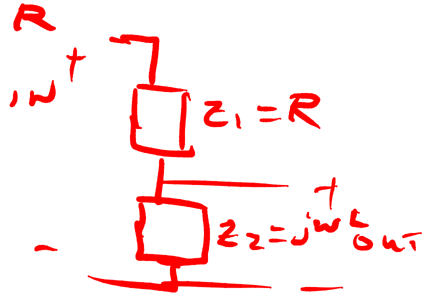
$\lim_{\omega \to 0} H(\omega) = 0$

$\lim_{\omega \to \infty} H(\omega) = 1$

$$H(\omega) = \frac{j\omega L}{R + j\omega L} \frac{\frac{1}{R}}{\frac{1}{R}}$$

$$= \frac{j\omega \frac{L}{R}}{1 + j\omega \frac{L}{R}}$$

$$= \frac{j\omega\tau}{1 + j\omega\tau}$$

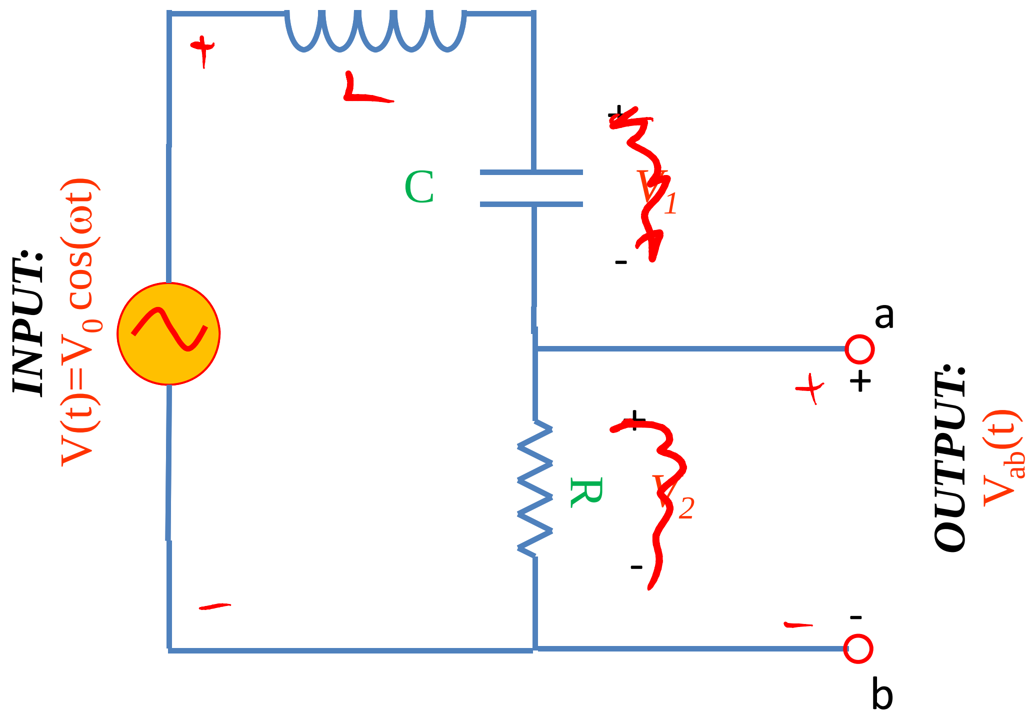


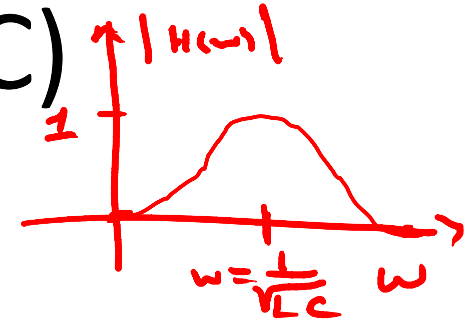

$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

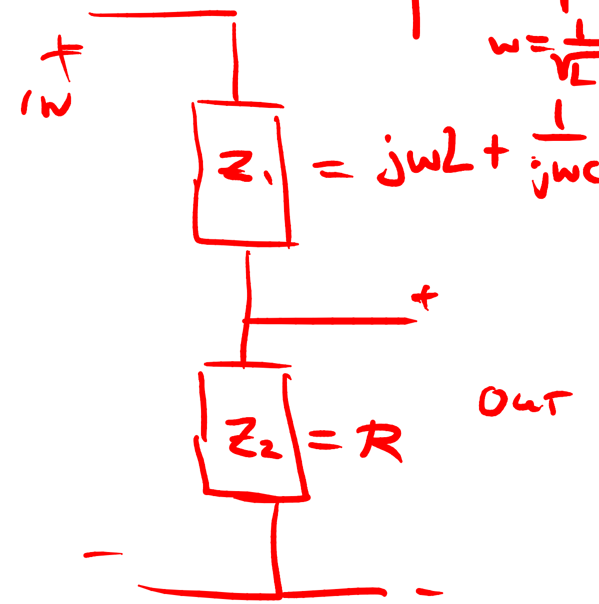
$$= \frac{j\omega L}{R + j\omega L}$$

# Band pass filter (RLC)

FIND $H(\omega)$, SKETCH $|H(\omega)|$ vs $\omega$

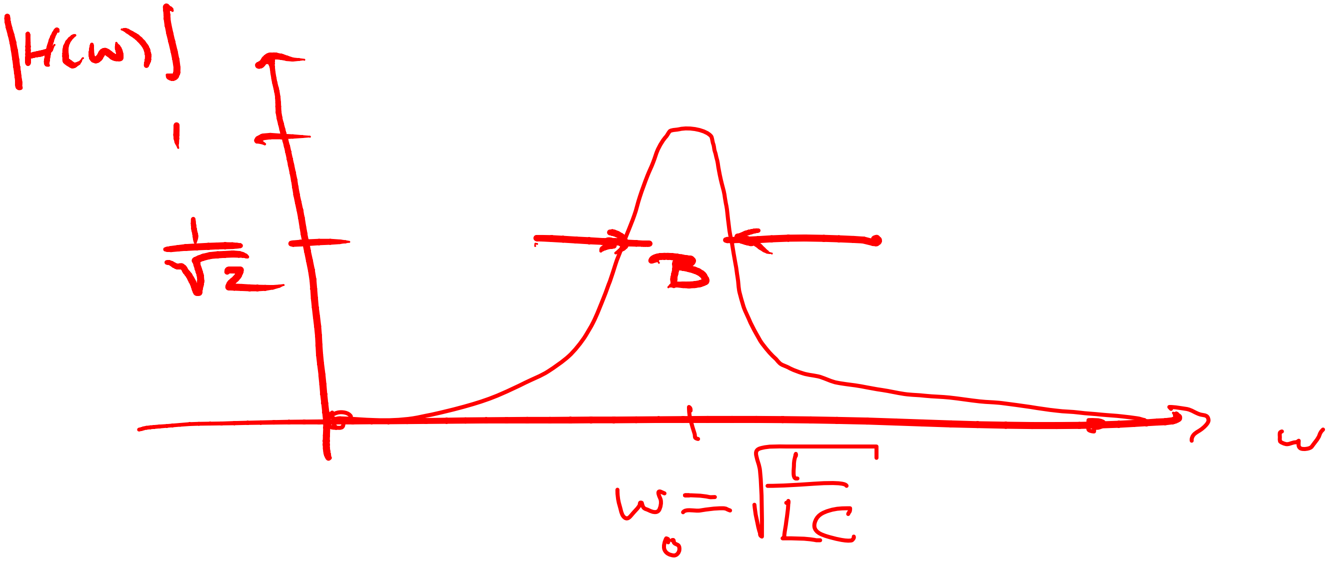



$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1+Z_2} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R}{R + j\frac{L}{\omega}\left(\omega^2 - \frac{1}{LC}\right)}$$




$$B = \frac{R}{L}$$

$$\frac{w_0}{B} \equiv Q \quad \text{quality factor}$$

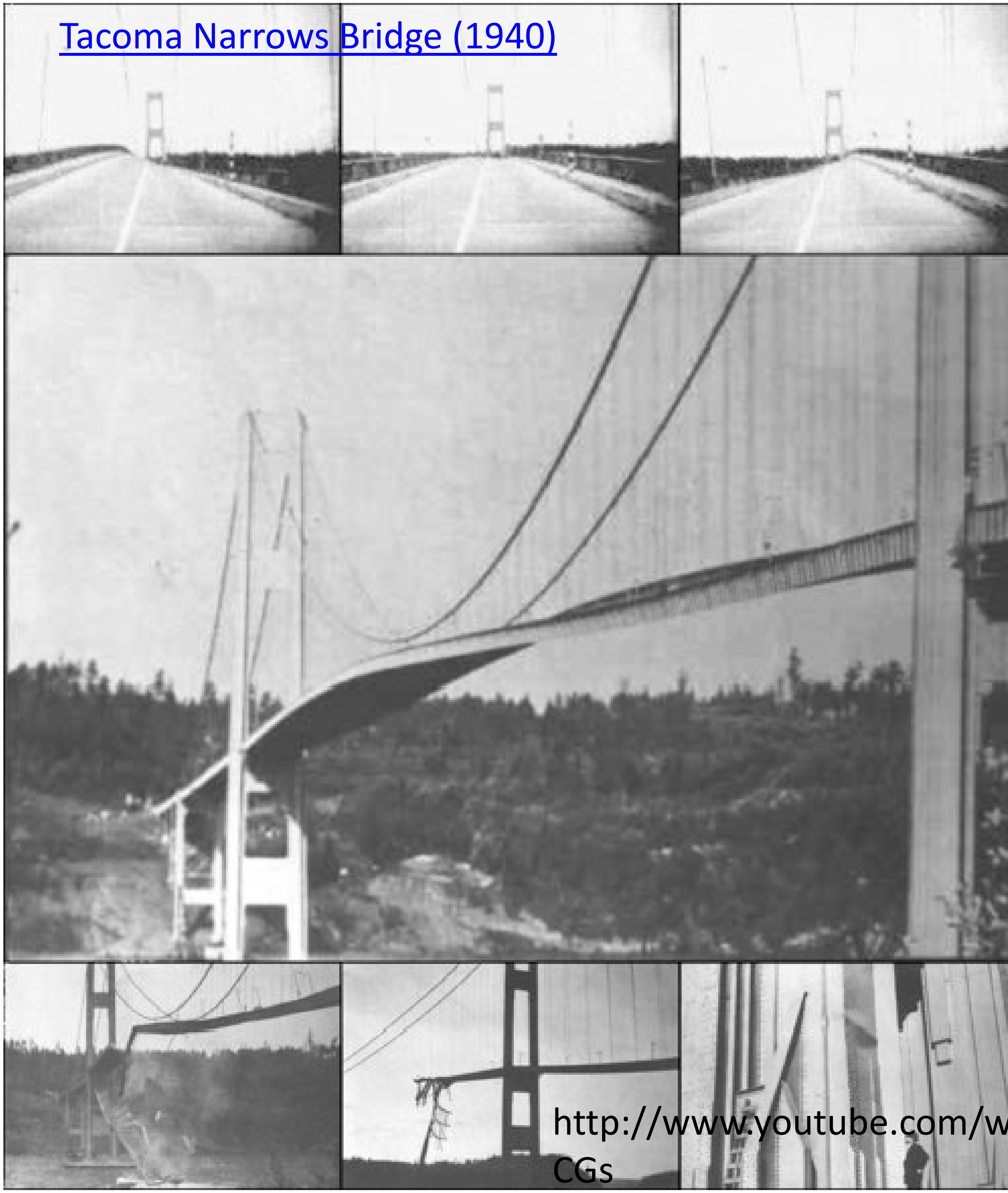
$$= \frac{w_0 L}{R} \quad \text{or} \quad \frac{1}{w_0 RC}$$

VERY IMPORTANT IN COMMUNICATIONS


L
D
E
R
FASTEN
SEAT BELTS
becker Europa
treble
tuning
M
6
9
12
15
AM
U
88
94
100
106
FM
AM
AM
FM
FM
FM
bass
http://www.peachparts.com/shopforum/showthread.php?t=256624

# Resonance

[Tacoma Narrows Bridge (1940)](http://www.youtube.com/watch?v=3mclp9QmCGs)



Nov 7, 1940

http://www.youtube.com/watch?v=3mclp9QmCGs

# Tesla