Announcements:

1. Final HW due Friday of 10^{th} week
2. Professor Burke's office hours this week:
   Tu 9:30-11:30 (EH 2230)
   Th 9:30-11 (EG 2232)
   Th 2-3:30 (EH 2230)
3. Exam :
   - Will cover all of Chs. 1,2,3,4,6, 7.1-7.3, 9 (not delta-Y), 14.1 14.2 14.3 14.5 14.6 14.7
   - Not covered: 3.6, 3.8, 4.9, 6.6.
   - No calculators. We will give sin, cos, tan tables as shown on last slide of today's notes..
4. Extra credit (5pts) for students who fill out online course evals.

EECS 70A: Network Analysis

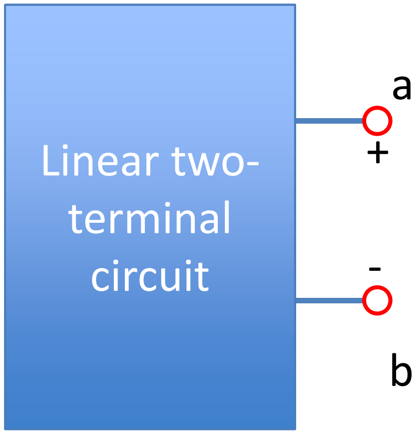
Lecture 16

Comprehensive review

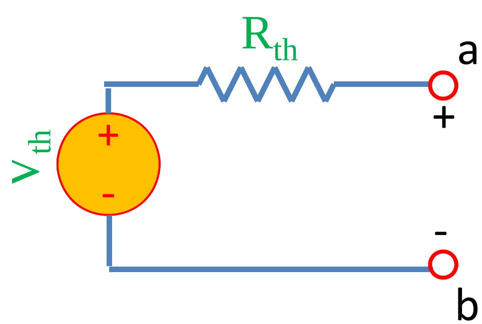
Topics covered

- KCL, KVL
- Nodal analysis
- Mesh analysis
- Thevenin/Norton theorem
- RL, RC circuits (time dependence)
- R,L,C circuits
  - Phasors
  - Impedances
  - Transfer function

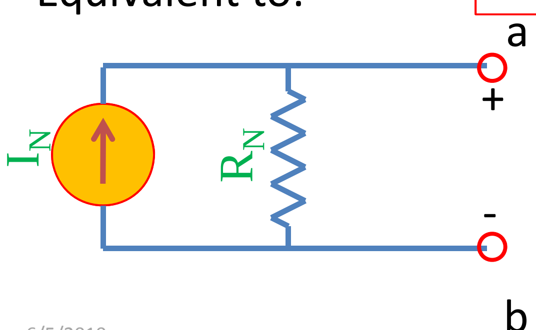
Thevenin, Norton Theorems:



Equivalent to:



Equivalent to:



Thevenin:

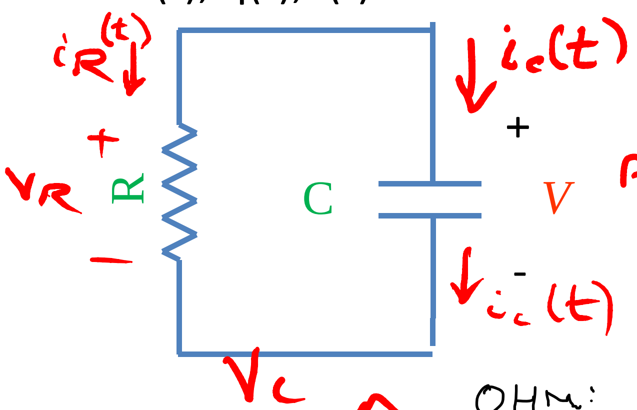
1. **Calculating V_{th}:**
   Connect nothing to a-b. Calculate voltage. This is V_{th}.
2. **Calculating R_{th}:**
   Method 1:
   Connect terminal a to b (short).
   Calculate the current from a to b. This is call $I_{short\ circuit}$.
   $R_{th}=V_{th}/I_{short\ circuit}$.
   Method 2:
   Find the input resistance looking into terminals a-b after all the independent sources have been turned off. (Voltage sources become shorts, current sources become opens.)
   Trick (if dependent sources present):
   Apply a 1 A current source to terminals a-b, find V_{ab}
   $R_{th} = V_{ab}/1A$.

Norton:

1. **Calculating R_N:**
   $R_N=R_{th}$
2. **Calculating I_N:**
   $I_N=V_{th}/R_{th}$

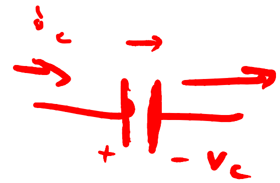
RC circuit

Find V(t), q(t), i(t)



What is $i_R(t=0)$ if $V_c(t=0) = 1$ V ?

A: $\frac{1V}{R}$



$q = C V_c$

$i_c = \frac{dq}{dt} = C \frac{dV_c}{dt}$

$$V_c(t) = V_c(t=0)\, e^{-t/RC}$$

NO TIME DEPENDENCE,

CAP = OPEN CIRCUIT

OHM:

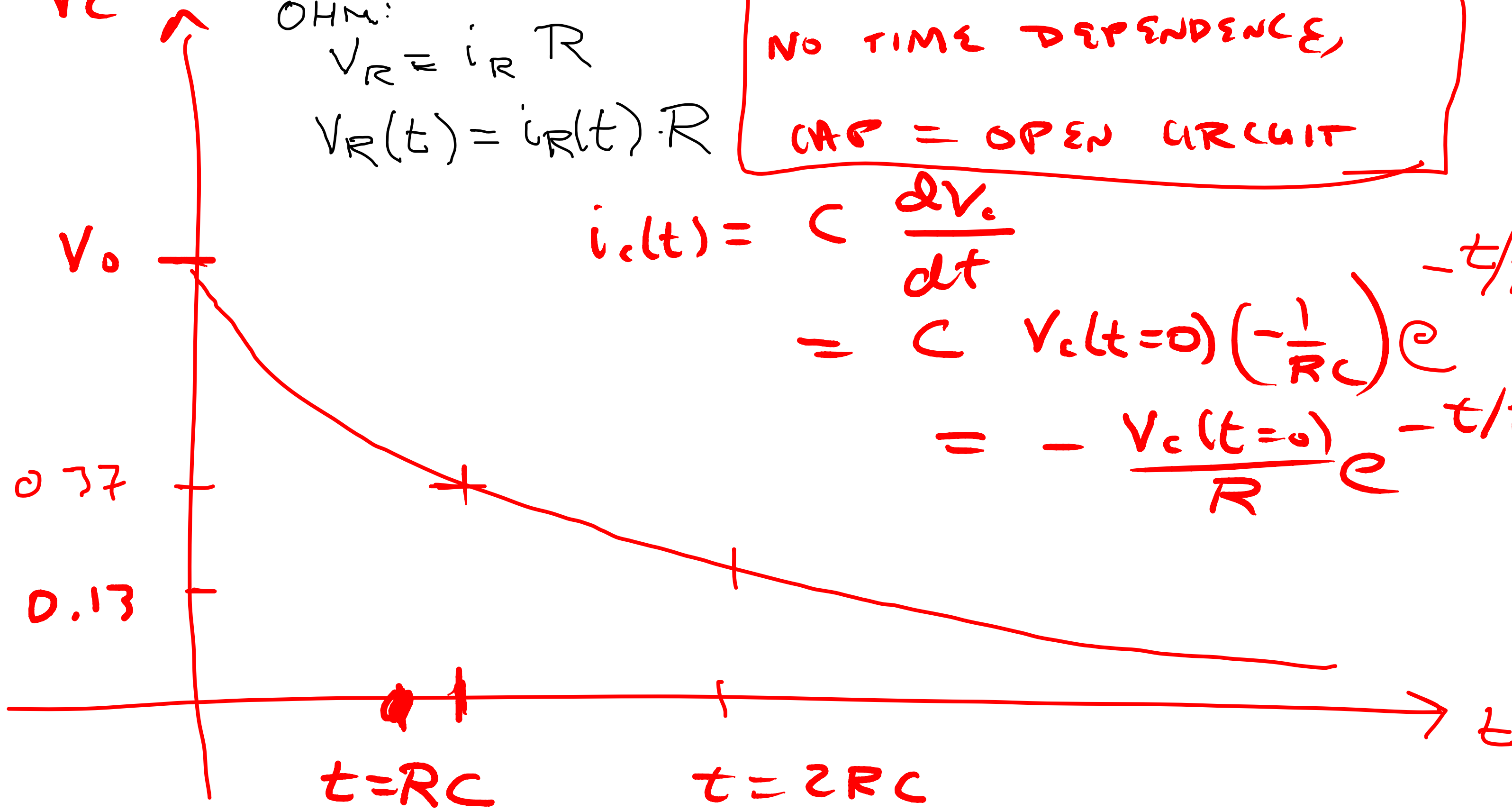
$V_R = i_R R$

$V_R(t) = i_R(t) \cdot R$

$$i_c(t) = C \frac{dV_c}{dt}$$

$$= C\, V_c(t=0) \left(-\frac{1}{RC}\right) e^{-t/RC}$$

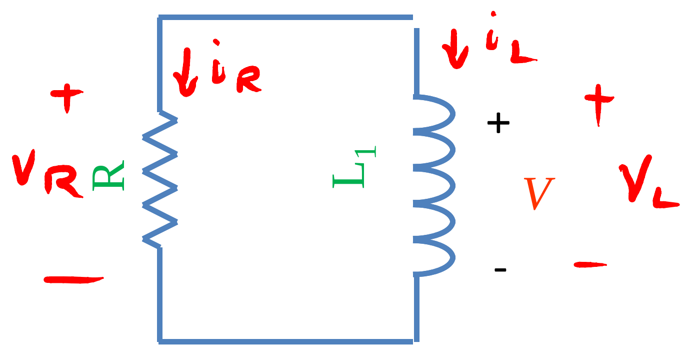
$$= -\frac{V_c(t=0)}{R} e^{-t/RC}$$



LR circuit

$V_R = i_R R$

Find V(t), i(t)



$V_L = L\frac{di_L}{dt}$

KVL ⇒ $V_L = V_R$

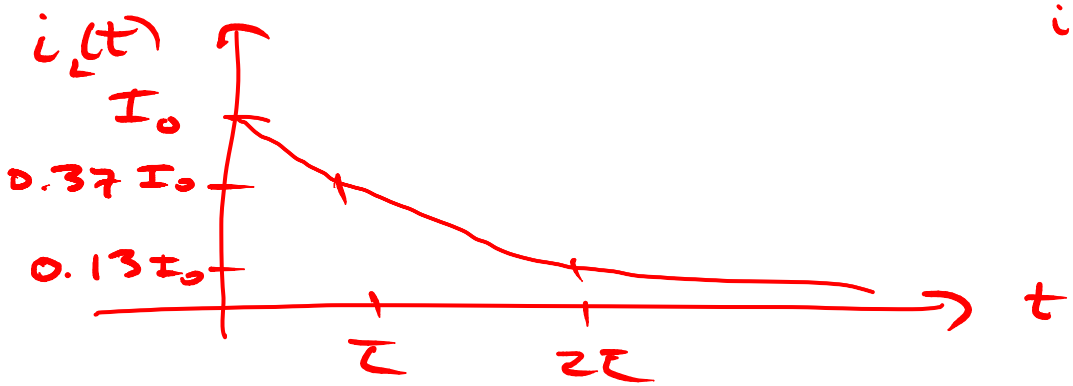
KCL ⇒ $i_R = -i_L$

$$L\frac{di_L}{dt} = i_R R = \boxed{-i_L R = L\frac{di_L}{dt}}$$

$$\Rightarrow \frac{di_L}{dt} = -\frac{1}{L/R} i_L$$

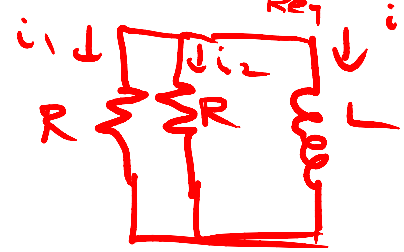
$\tau \equiv \frac{L}{R}$ time constant

$$i_L(t) = i_L(t=0)\, e^{-t/\tau}$$



$i_L(t) = i_L(t=0)\, e^{-t/\tau}$

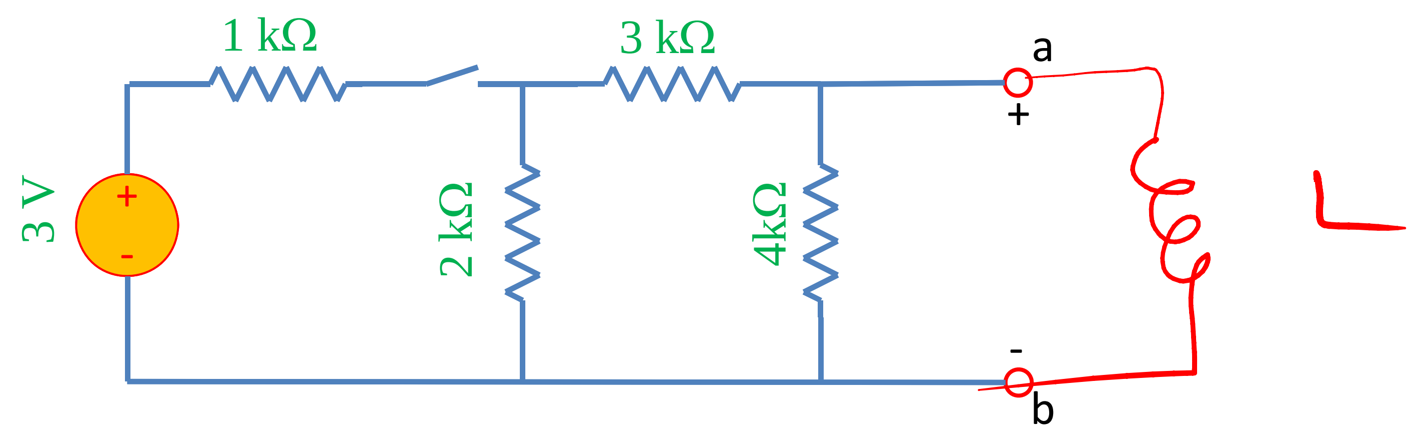
$\tau = \frac{L}{R_{eq}} = \frac{L}{R/2} = \frac{2L}{R}$



Given $i_L(t=0)$

Find $i_1(t)$, $i_2(t)$

Comprehensive Example



A) This circuit is connected to a capacitor of value 1 μF. The switch is in the closed position. After a long time, what are all the voltages and currents in this circuit?

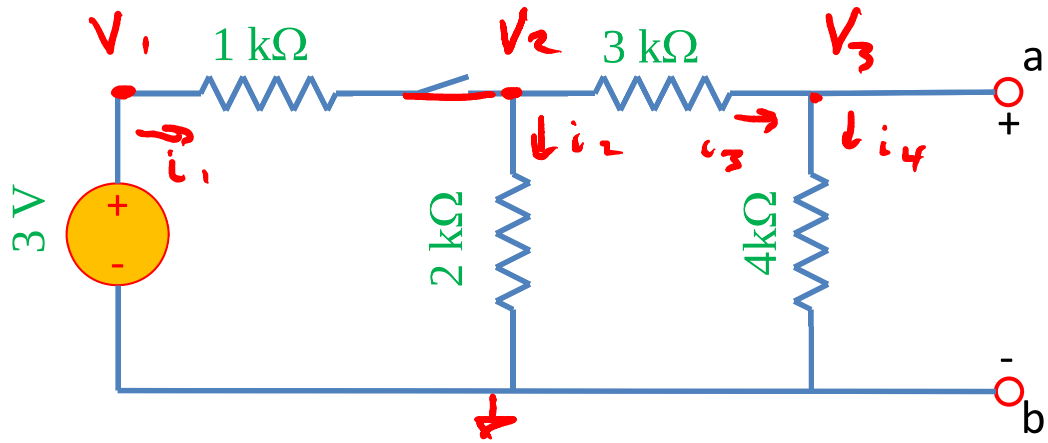
B) Next, the switch is opened. What are all the voltages and currents in this circuit as a function of time after the switch is opened?

C) This circuit is now connected to a resistor R_0. What is the power dissipated in R_0?

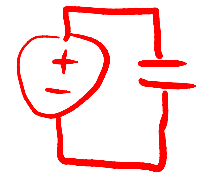
D) If you were to pick a value of R_0 to absorb as much power as possible, what would it be?

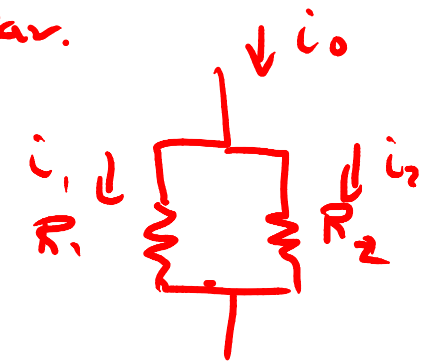
E) Exercise: Do the same as A,B with an inductor instead.

(A) Comprehensive Example



2 ways
1) Nodal
2) Ser./Par.





$$i_1 = \frac{R_2}{R_1+R_2} i_0$$

Node:

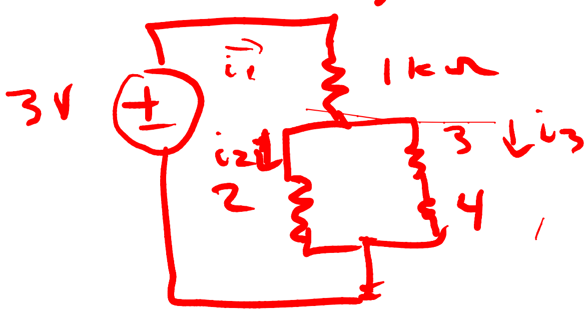
$V_1 = 3V$

$i_1 = i_2 + i_3 \Rightarrow \frac{V_2 - 3V}{1k\Omega} = \frac{V_2}{2k\Omega} + \frac{V_3 - V_2}{3k\Omega}$ (1)

$i_3 = i_4 \Rightarrow \frac{V_3 - V_2}{3k\Omega} = \frac{V_3}{4k\Omega}$ (2)

Solve V_2, V_3 then $i_1 - i_4$

"Easier way"

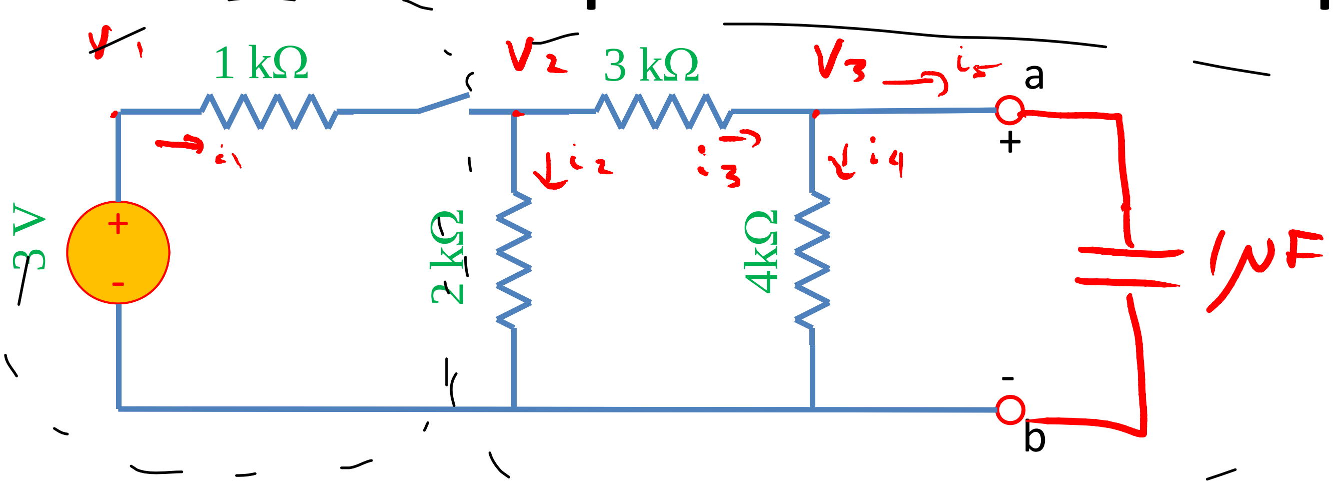


$$i_1 = \frac{3V}{1 + 2 \,||\, (3+4)\ k\Omega} = \frac{3}{1 + 2 \,||\, 7} \text{ mA}$$

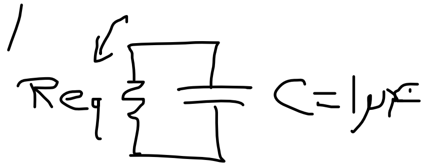
$$= \frac{3}{1 + \frac{2 \cdot 7}{2+7}} = \ldots$$

$$i_2 = i_1 \frac{3+4}{2+(3+4)} \qquad V_3 = \frac{24}{23} V$$

(B) Comprehensive Example



$i_5(t) = \frac{V_c(t)}{R_{eq}}$

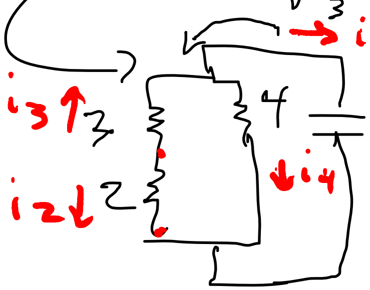


$\tau = R_{eq} C$

$R_{eq} =$

OPEN $t=0$ $\quad V_3(t=0) = \frac{24}{23}\,V$

$V_3(t) = V_3(t=0)\, e^{-t/\tau} = V_c(t)$

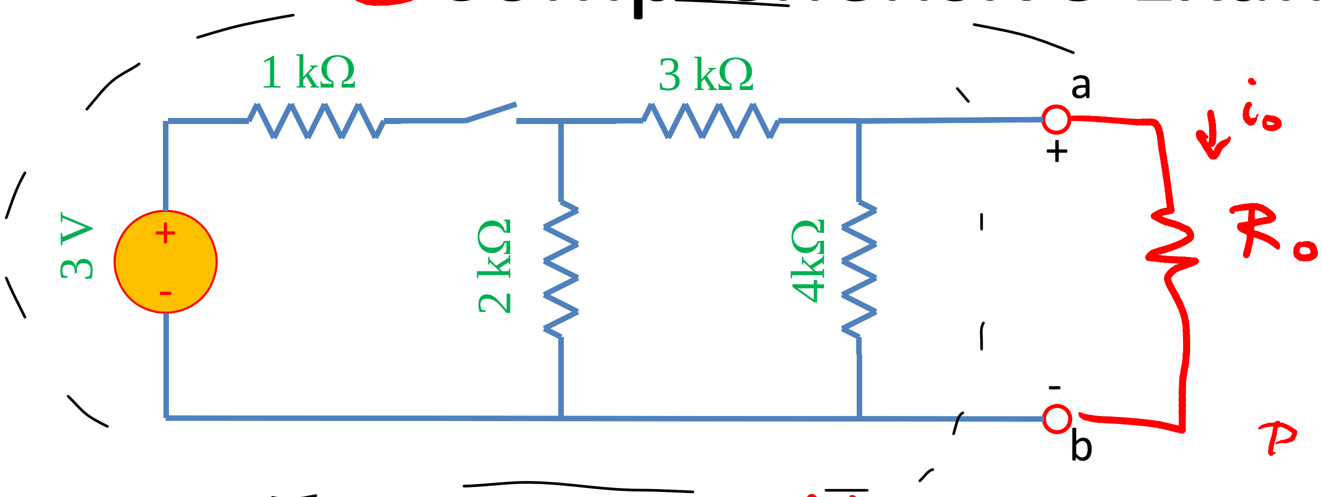


$R_{eq} = \frac{20}{9} = 4 \parallel (3+2) = 4 \parallel 5 = \frac{4 \cdot 5}{4+5}$

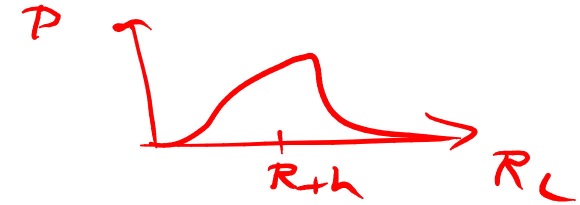
$i_2(t) = -i_3(t)$ ~~i_4, i_2~~ divic

$i_5(t)$ divided into i_2, i_4

Comprehensive Example



$$P = i_o \cdot V_{ab} = i_o^2 R_o = \frac{V_{ab}^2}{R_o}$$

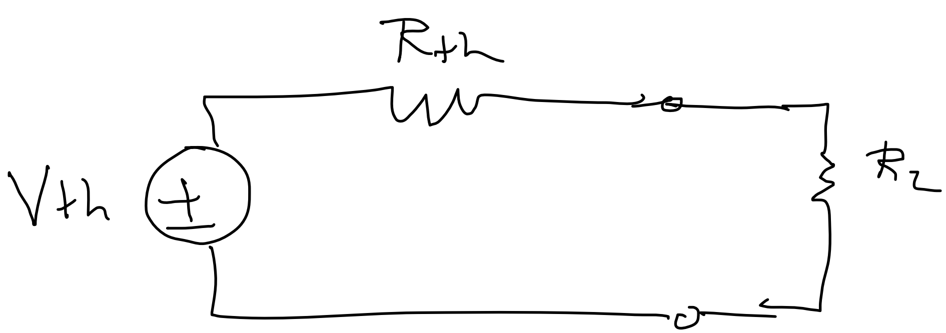


Nodal

Mesh

Ser/Par

Nort/Th



$$P = \frac{V^2}{R_L} = \frac{1}{R_L}\left[V_{th}\frac{R_L}{R_L+R_{th}}\right]^2$$

Conversion procedures

i(t) → **I**

v(t) → **V**

I → i(t)

V → v(t)

$$i(t) = I_m \cos(\omega t + \phi) \Rightarrow \quad \mathbf{I} = I_m e^{j\phi}$$

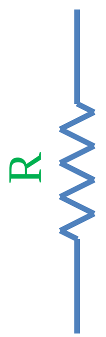
$$v(t) = V_m \cos(\omega t + \phi) \Rightarrow \quad \mathbf{V} = V_m e^{j\phi}$$

$$i(t) = \mathrm{Re}\left(\mathbf{I}\, e^{j\omega t}\right)$$

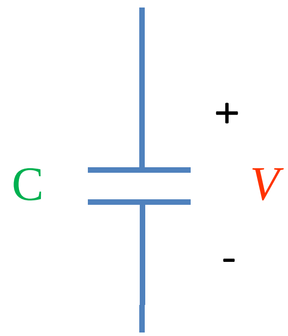
$$v(t) = \mathrm{Re}\left(\mathbf{V} e^{j\omega t}\right)$$

For the exam, you should know how to carry out these procedures.

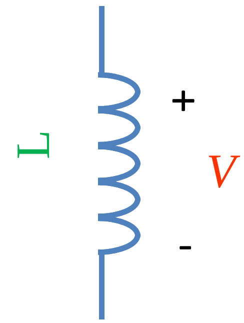
Circuits



$\mathbf{V} = \mathbf{I}\, R$



$\mathbf{V} = \mathbf{I}/j\omega C$



$\mathbf{V} = j\omega L\, \mathbf{I}$

"Impedance"

$Z = R$ $\qquad$ $Z = 1/j\omega C$ $\qquad$ $Z = j\omega L$

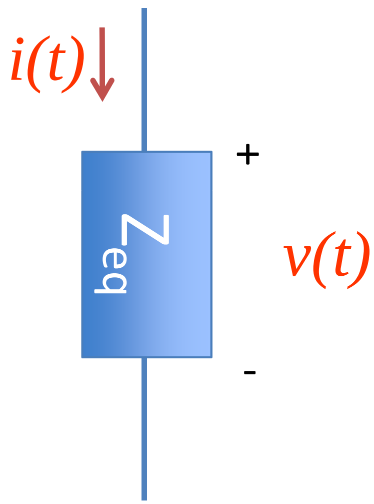
KCL, KVL hold for relationship between **V**, **I**.

Series/Parallel Impedances

$$Z_{eq} = Z_1 + Z_2 + Z_3$$

$$Z_{eq}^{-1} = Z_1^{-1} + Z_2^{-1} + Z_3^{-1}$$

Conversion procedures



Given i(t) find v(t):

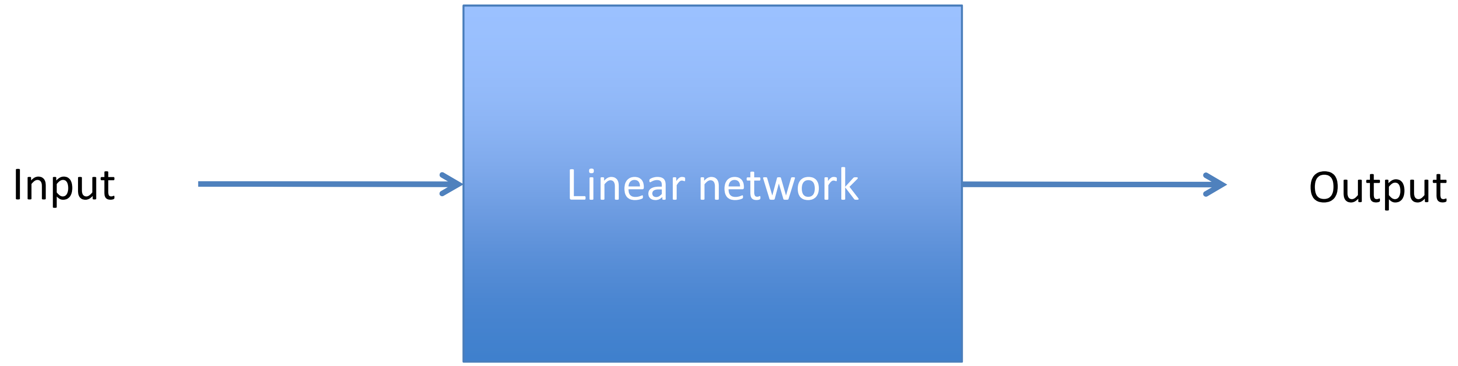
$$i(t) \rightarrow \mathbf{I} \rightarrow \mathbf{V} = \mathbf{I}\, Z_{eq} \rightarrow v(t)$$

Given v(t) find i(t):

$$v(t) \rightarrow \mathbf{V} \rightarrow \mathbf{I} = \mathbf{V} / Z_{eq} \rightarrow i(t)$$

For the exam, you should know how to carry out these procedures.

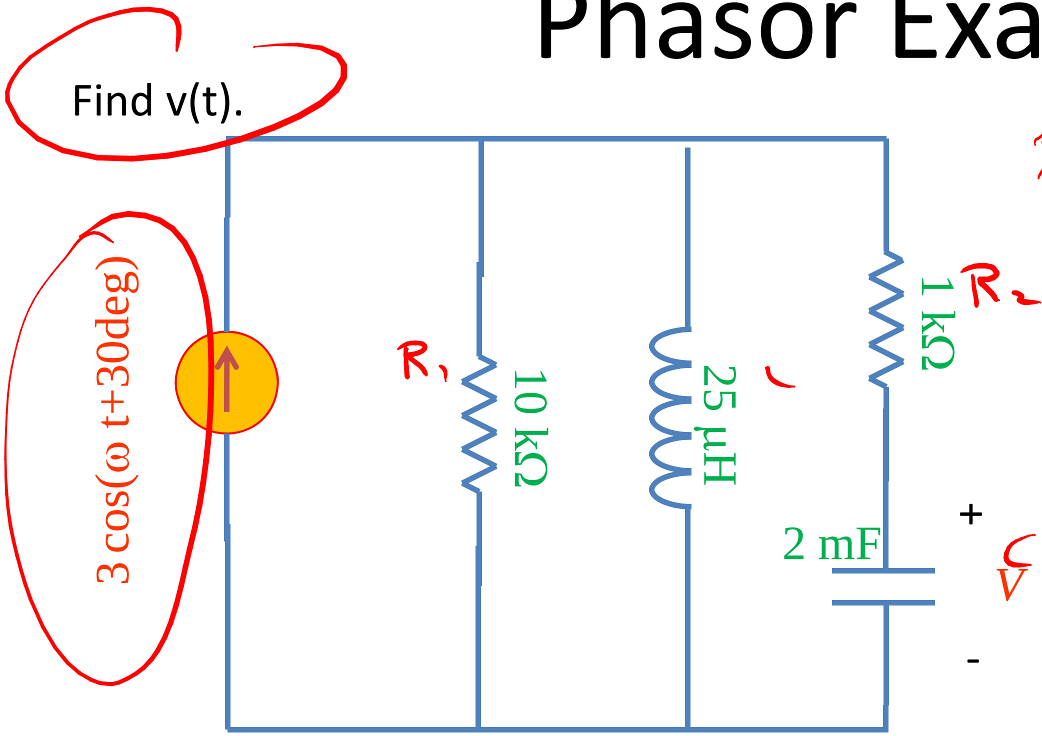
“Transfer Function”

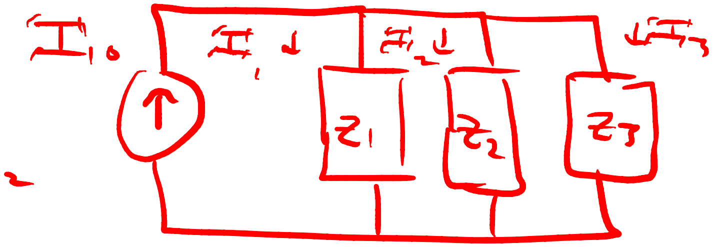


$$H(\omega) = \mathbf{V_{out}/V_{in}}$$

Phasor Example 1

Find v(t).



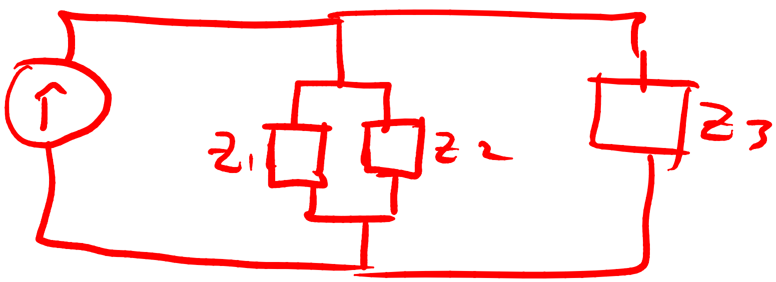


$$Z_1 = R_1$$

$$Z_2 = L j\omega$$

$$Z_3 = R_2 + \frac{1}{j\omega C}$$

$$I_0 = 3e^{j30^\circ} = 3e^{j\pi/6}$$



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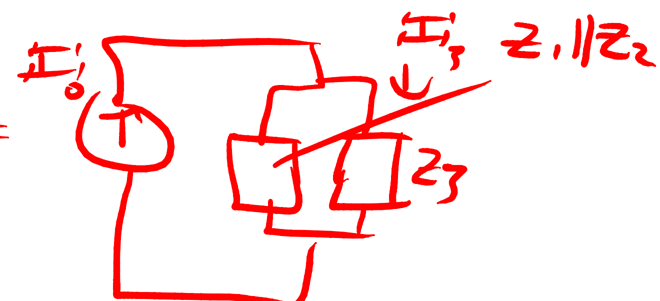


$$I_3 = I_0 \frac{(Z_1 // Z_2)}{Z_3 + (Z_1 // Z_2)}$$

$$V_c = I_3 \frac{1}{j\omega C}$$

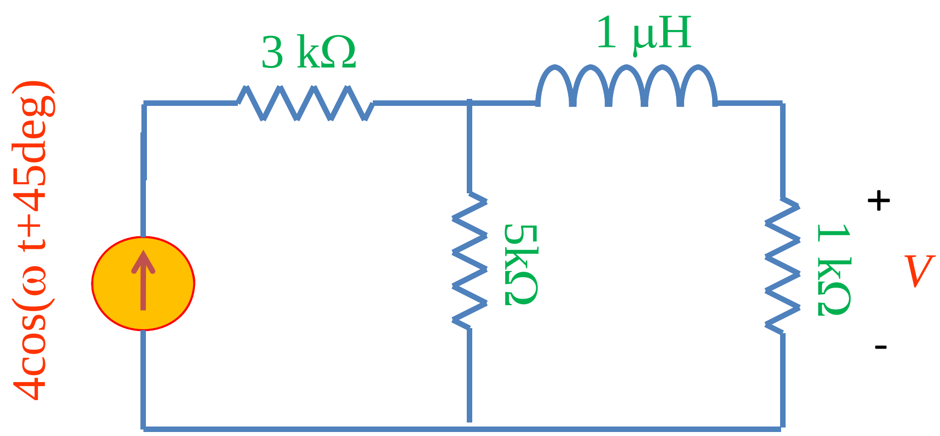
$$v(t) = \mathrm{Re}(V_c e^{j\omega t})$$

answer

$$V(t) = \mathrm{Re}\left[\frac{1}{j\omega C} \frac{\dfrac{R_1 j\omega L}{R_1 + j\omega L}}{\dfrac{R_1 j\omega L}{R_1 + j\omega L} + R_2 + \dfrac{1}{j\omega C}} 3e^{j\pi/6} e^{j\omega t} \right]$$

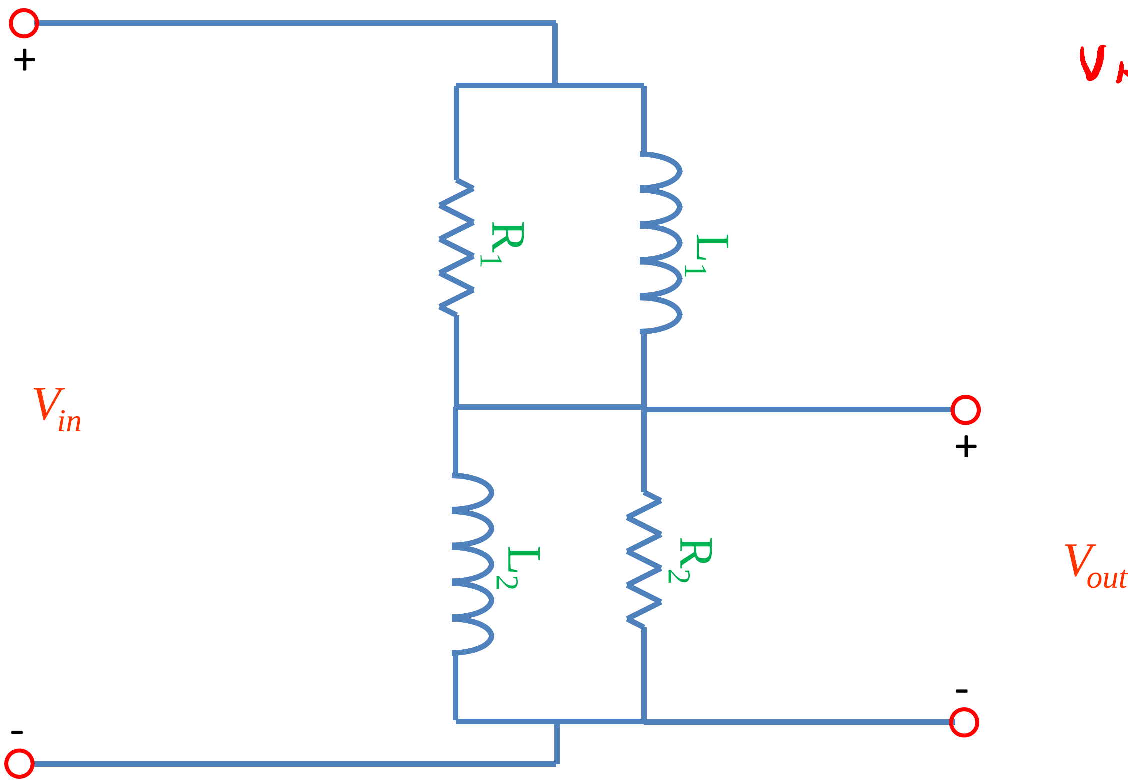
$$= \ldots$$

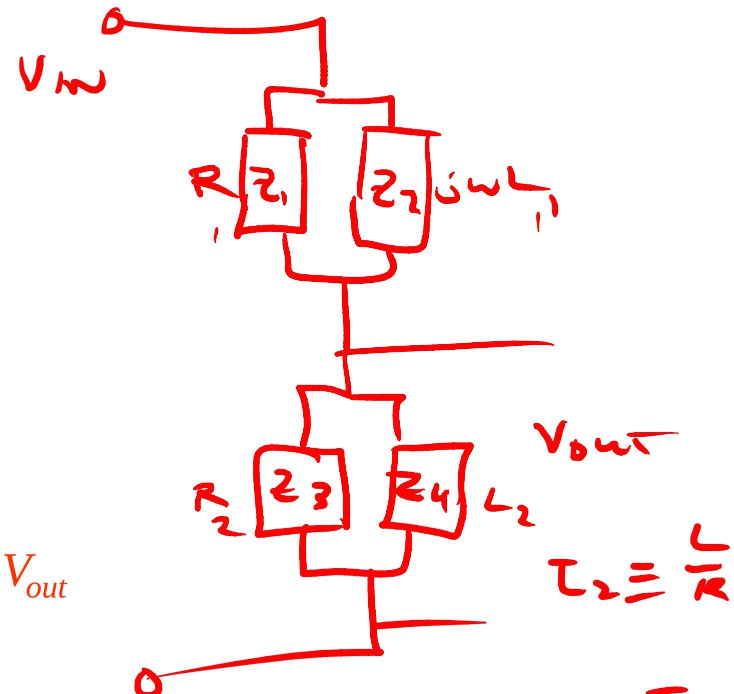
Phasor Example 2



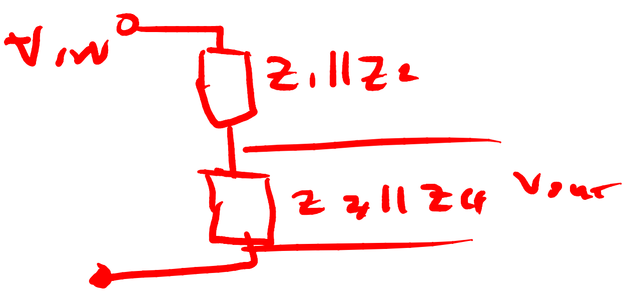
Find v(t).

Example Transfer function





Calculate H(ω) for this circuit. Sketch the magnitude of H(ω) vs. ω.



$$\frac{V_{out}}{V_{in}} = \frac{(Z_3 \| Z_4)}{(Z_3 \| Z_4) + (Z_1 \| Z_2)} = H(\omega) = \frac{\frac{j\omega L_2}{1 + j\omega\tau_2}}{Z_3 \| Z_4 + \frac{j\omega L_1}{1 + j\omega \tau_1}}$$

$R \parallel L$

$$\frac{j\omega L}{1+j\omega\tau} \qquad \tau \equiv \frac{L}{R}$$

$$H(\omega) = \frac{\dfrac{j\omega L_2}{1+j\omega\tau_2}}{\dfrac{j\omega L_2}{1+j\omega\tau_2} + \dfrac{j\omega L_1}{1+j\omega\tau_1}}$$

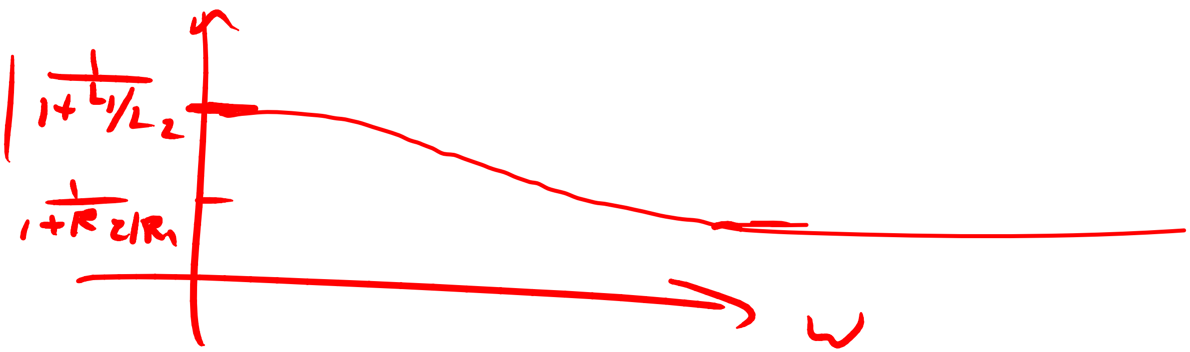
$$\tau_1 = \frac{L_1}{R_1} \qquad \tau_2 = \frac{L_2}{R_2}$$

$$\tau_2 = L_2/R_2$$

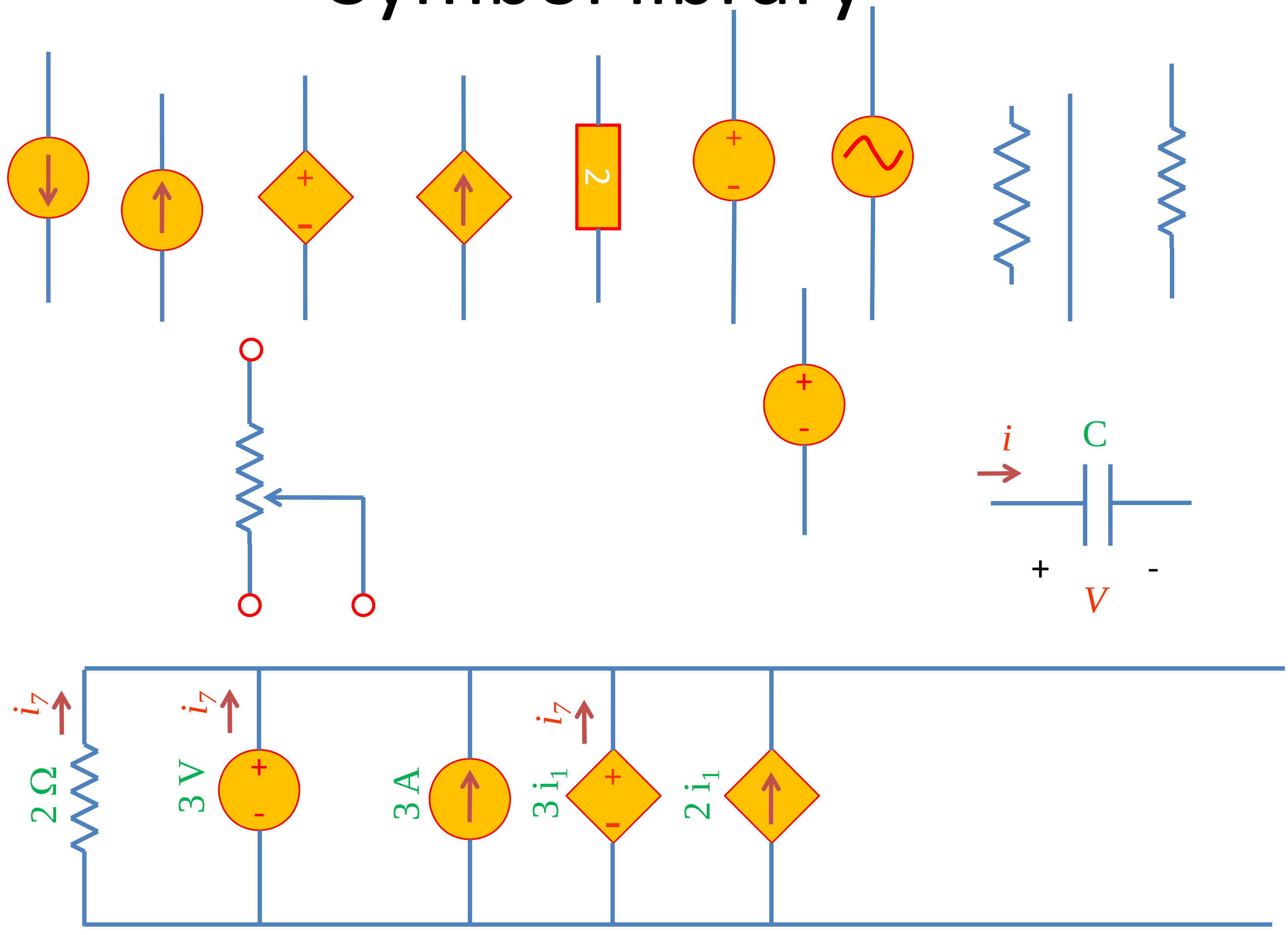
$$\tau_1 = L_1/R_1$$

$$= \frac{1}{1 + \dfrac{1+j\omega\tau_2}{1+j\omega\tau_1}\dfrac{L_1}{L_2}}$$

$|H(\omega)|$



Symbol library



Exam cheat sheet

This will be provided with the exam.

radians :	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$	0
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$	-1
tan	$\frac{\sqrt{0}}{\sqrt{4}}$	$\frac{\sqrt{1}}{\sqrt{3}}$	$\frac{\sqrt{2}}{\sqrt{2}}$	$\frac{\sqrt{3}}{\sqrt{1}}$	DNE	0

where $\sqrt{\cdot}$ always denotes the positive square root, and DNE means does not exist.