EECS 70A: Network Analysis

Lecture 2

Announcements

- HW to be posted online- due Wed week 2 in disc.
- Quiz to be online eee- due Mon midnt. week 2
- Office hours posted online
- Lecture notes will be posted online
  - Skeleton before lecture
  - With annotation after lecture
- Please ask questions in lecture!
- Things will speed up…

Review & agenda

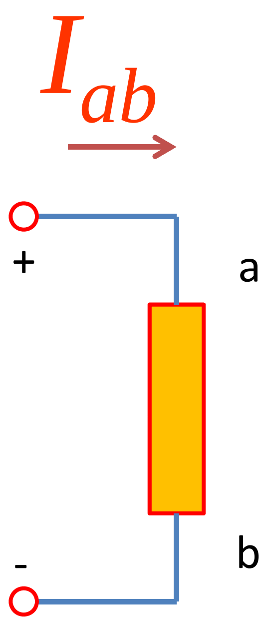
Last lecture:

- Current
- Voltage
- Power

Today:

- Examples
  - Power (sink/source)
  - Current (postive/negative)
  - Dependent sources
- Resistors
  - Series
  - Parallel

Power: Source vs. sink



$$P = I_{ab} \ x \ V_{ab}$$

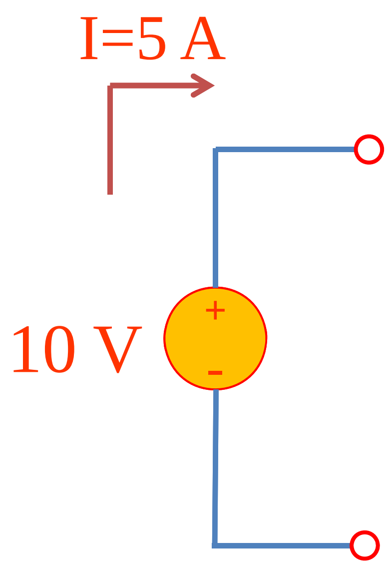
P > 0

⇒"sink": power delivered to element

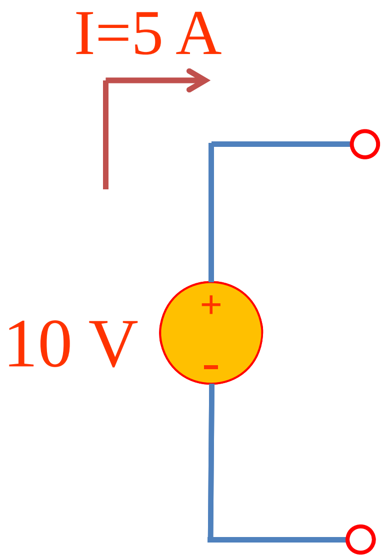
P < 0

⇒"source": power supplied by element

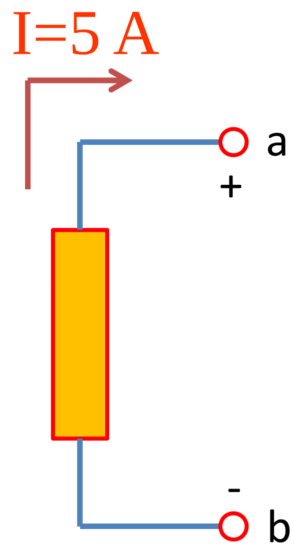
“source”



“source”



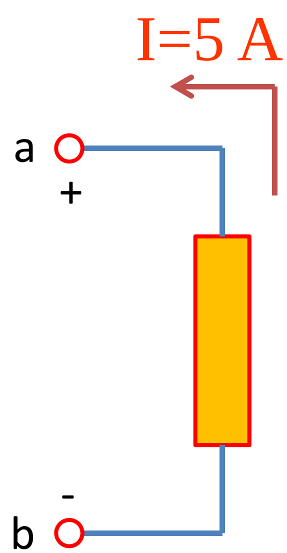
Draw or think of as:



$V_{ab} = 10 \text{ V}$

$I_{ab} = -\ 5 \text{ A}$

Same as:



$V_{ab} = 10 \text{ V}$

$I_{ab} = -\ 5 \text{ A}$

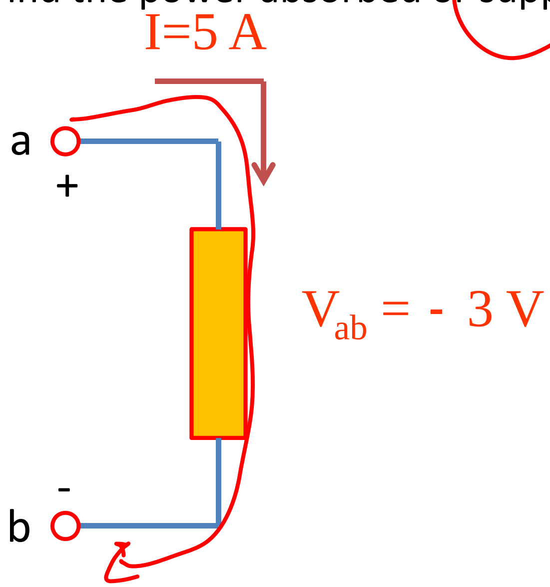
$$P = I_{ab} \times V_{ab}$$

$$= (-\ 5 \text{ A}) \times (10 \text{ V})$$

$$= -50 \text{ W}$$

Practice problems

Find the power absorbed or supplied by the element (instructor).



$P = I_{ab} \times V_{ab}$

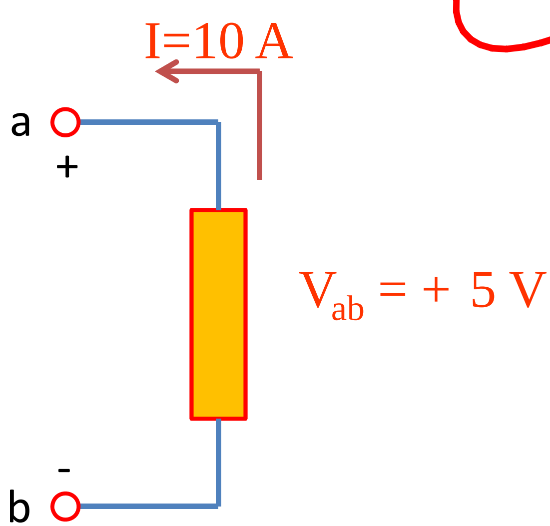
$V_{ab} = -3V$

$I_{ab} = +5A$

$P = (-3V)(+5A) = -15W$

Source

Find the power absorbed or supplied by the element (instructor).



$P = I_{ab} \times V_{ab}$

$I_{ab} = -10A$

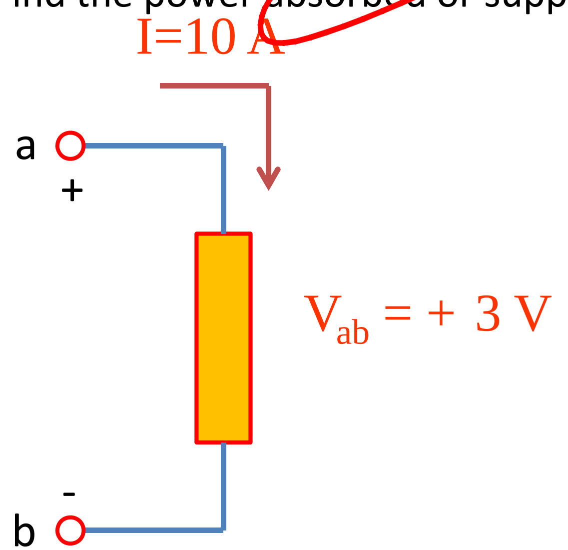
$V_{ab} = +5V$

$P = (-10A) \times (+5V) = -50W$

source

Practice problems

Find the power absorbed or supplied by the element (student).



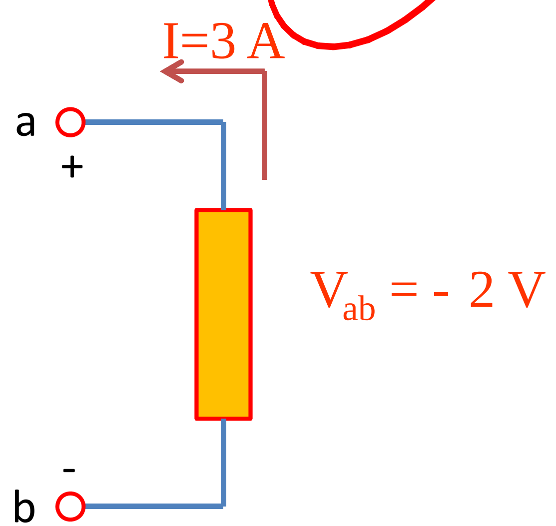
$V_{ab} = +3V$

$I_{ab} = +10A$

$P = +30W$

sink

Find the power absorbed or supplied by the element (student).



$V_{ab} = -2V$

$I_{ab} = -3A$

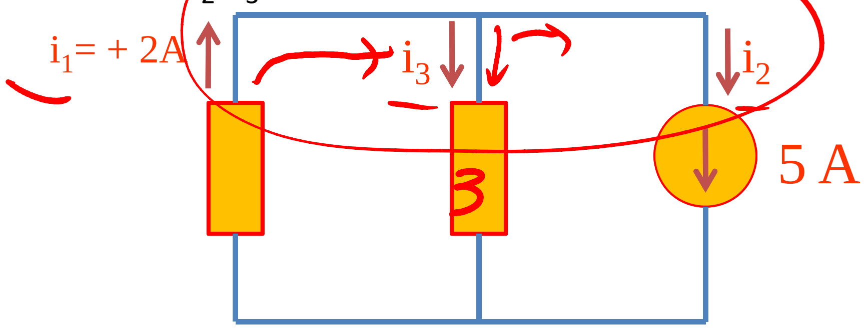
Sink

$P = (-3A)(-2V)$

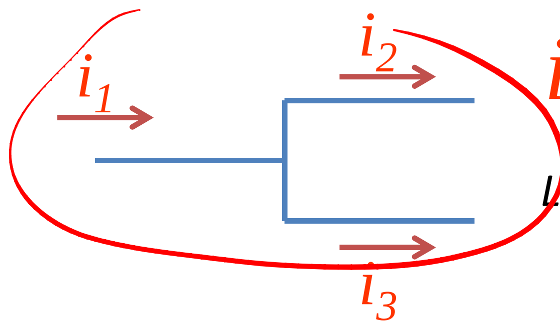
$= 6.W$

Example problem: Current (positive/negative)

Find the i_2, i_3 (instructor).



Recall:



$$i_1 = i_2 + i_3$$

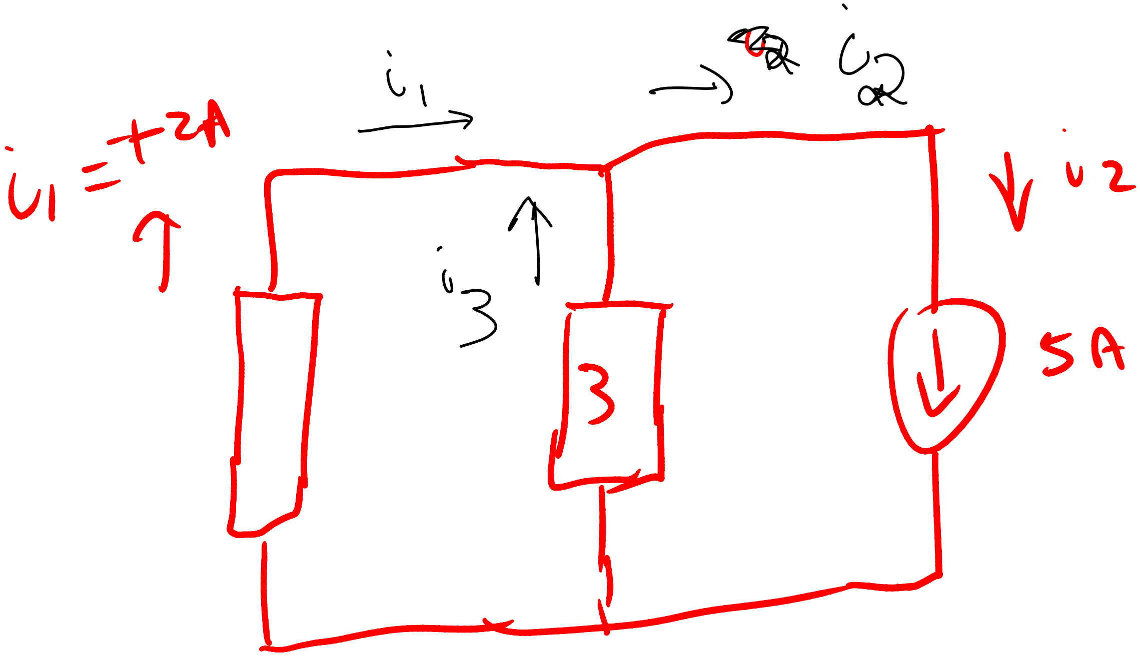
Like water in a river…

$$i_2 = +5A$$

$$i_1 = i_2 + i_3$$

$$i_3 = i_1 - i_2 = 2A - 5A = -3A$$

$\Rightarrow$ Current thru element #3 is flowing opposite the direction of arrow.

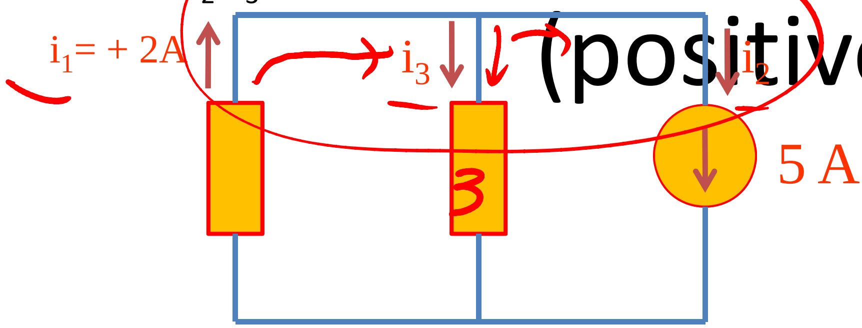


P: Find current thru el. #3

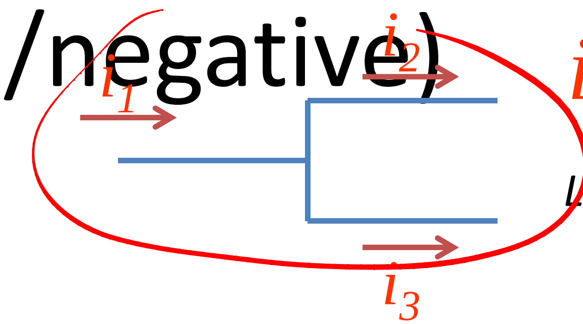
$$i_1 + i_3 = i_2 \Rightarrow i_3 = i_2 - i_1 = 5A - 2A$$
$$= 3A$$

Example problem: Current (positive/negative)

Find the i_2, i_3 (instructor).



Recall:



$$i_1 = i_2 + i_3$$

Like water in a river…

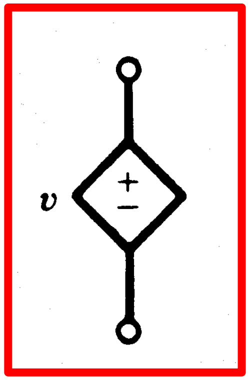
$i_2 = +5A$

$i_1 = i_2 + i_3$

$i_3 = i_1 - i_2 = 2A - 5A = -3A$

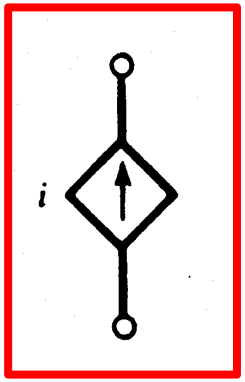
$\Rightarrow$ Current thru element #3 is flowing opposite the direction of arrow.

Dependent sources



VCVS: Voltage controlled voltage source

CCVS: Current controlled voltage source

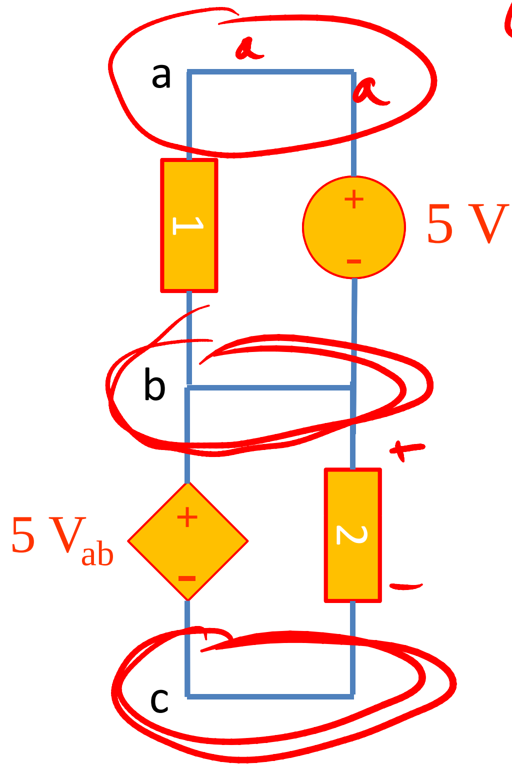


VCCS: Voltage controlled current source

CCCS: Current controlled current source

Voltage controlled voltage source (VCVS)

Find the voltage drop across element 2 (instructor).



$\hookrightarrow = V_{bc}$

= voltage drop across dependent source

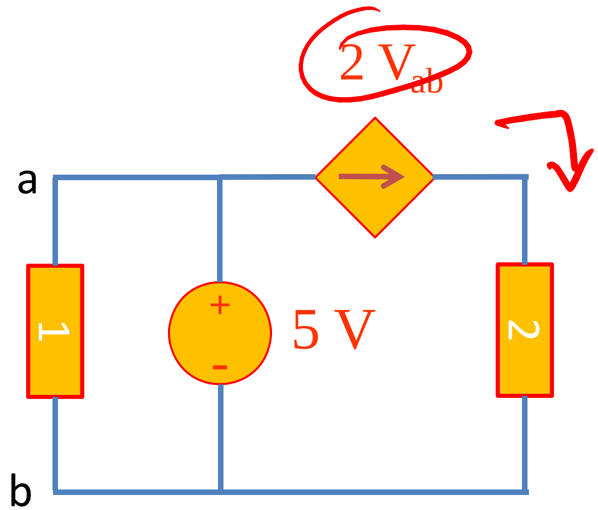
$= 5 \times V_{ab}$

$= 5 \times (5V \qquad)$

$= 25\ V$

Voltage controlled current source (VCCS)

Find the current through element 2 (student).



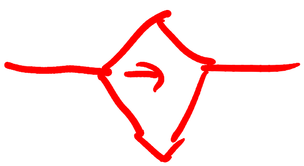
+10A

$V_{ab} = +5V$

$\Rightarrow 2 \times V_{ab} = 10A$

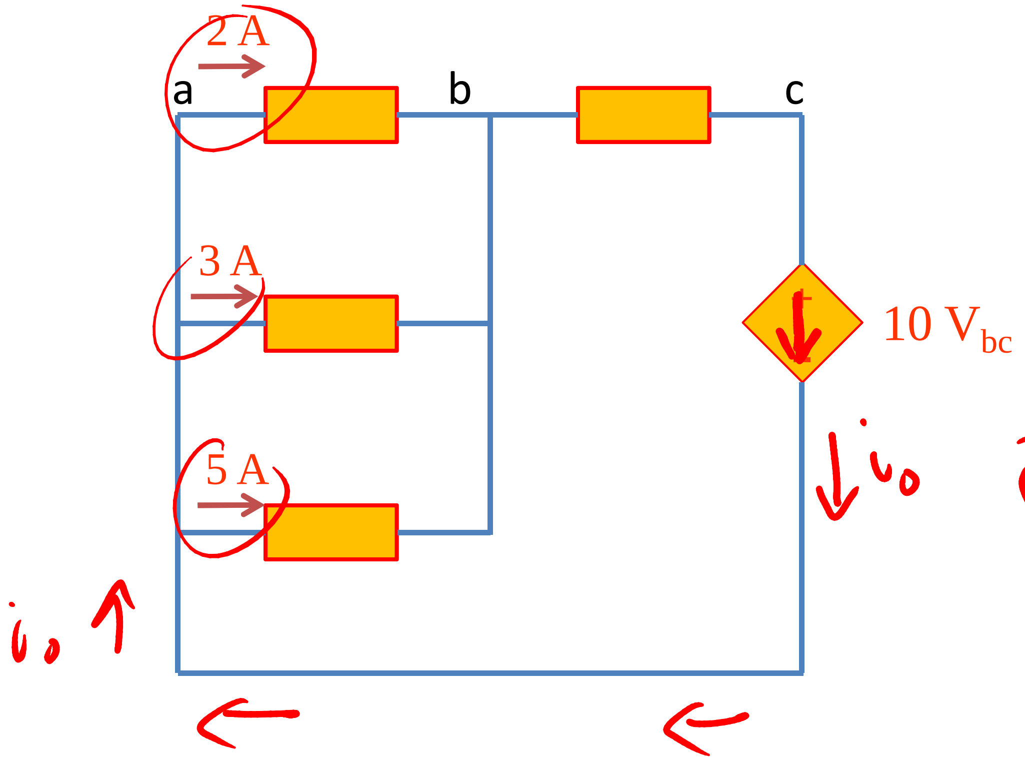
$2\left[\frac{A}{V}\right] V_{ab}$

$2\left[\frac{A}{V}\right] \times 5[V] = 10A$



Example problem.

Find the V_{bc} (instructor).

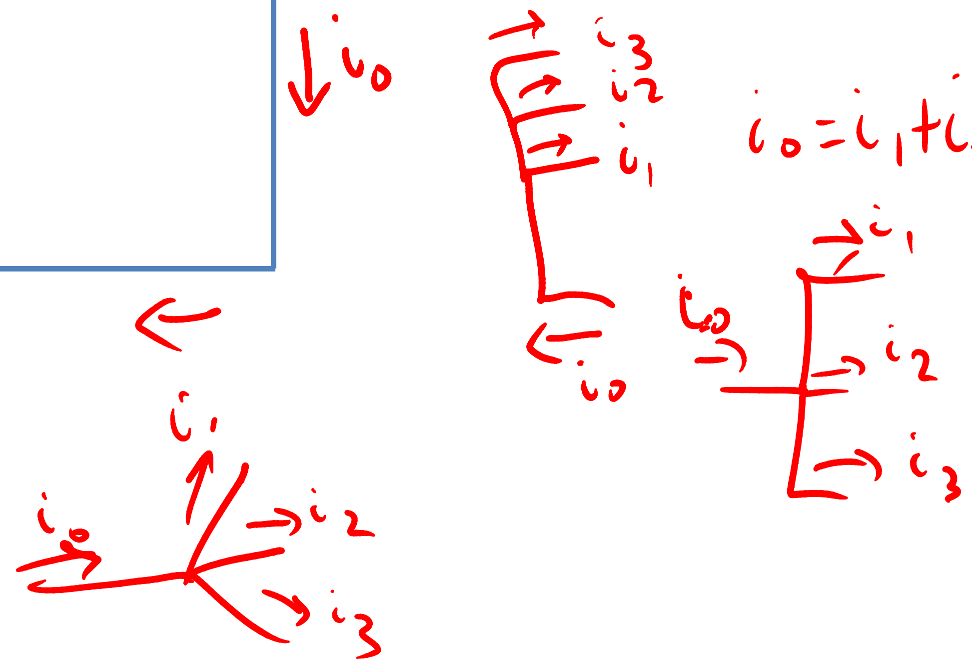


VCCS

$i_0 = 10A$

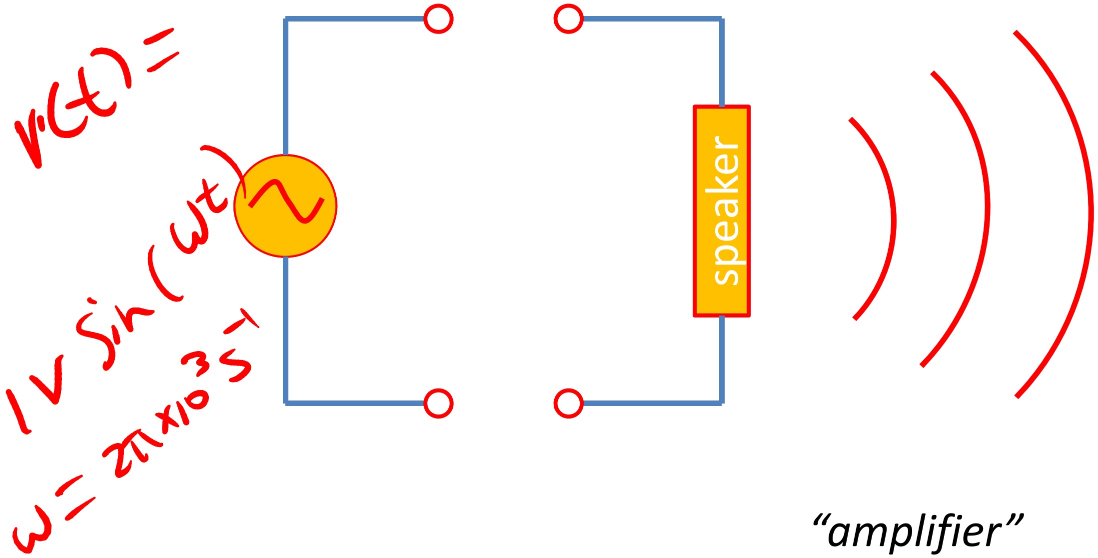
$V_{bc} = 1V$

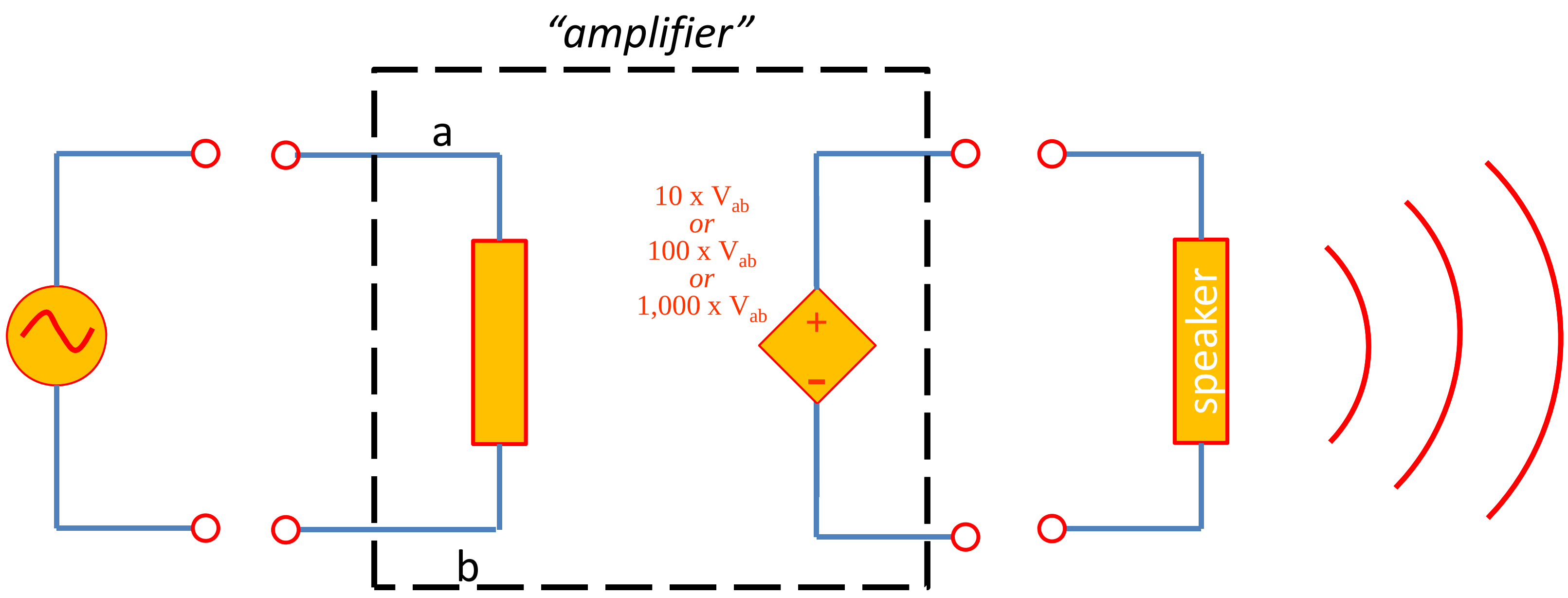
$10\left[\frac{A}{V}\right] V_{bc}$



$i_0 = i_1 + i_2 + i_3$

Practical example (demo).



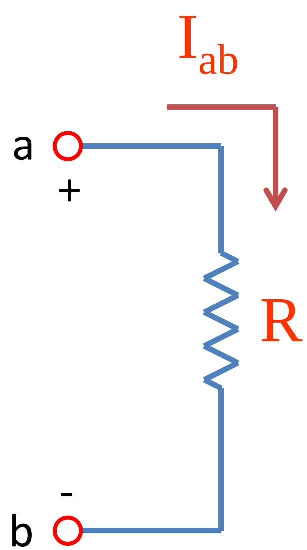


MKSA units-cheat sheet

Quantity	Name	Symbol	Expression in terms of other units	Expression in terms of MKSA base units
Frequency	Hertz	Hz		s^{-1}
Force	Newton	N		$m\text{-}kg/s^2$
Pressure	Pascal	Pa	N/m^2	$kg/m\text{-}s^2$
Energy	Joule	J	N-m	$kg\text{-}m^2/s^2$
Power	Watt	W	J/s	$kg\text{-}m^2/s^3$
Charge	Coulomb	C		A-s
Voltage	Volt	V	W/A	$kg\text{-}m^2/A\text{-}s^3$
Capacitance	Farad	F	C/V	$A^2\text{-}s^4/kg\text{-}m^2$
Resistance	Ohm	Ω	V/A	$kg\text{-}m^2/A^2\text{-}s^3$
Conductance	Siemens	S	A/V	$A^2\text{-}s^3/kg\text{-}m^2$
Inductance	Henry	H		$kg\text{-}m^2/A^2\text{-}s^2$

Questions?

Resistors



$$V_{ab} = I_{ab} \times R$$

Resistance units: Ohms [Ω]

Example circuit:
(Voltage source in series with resistor.)

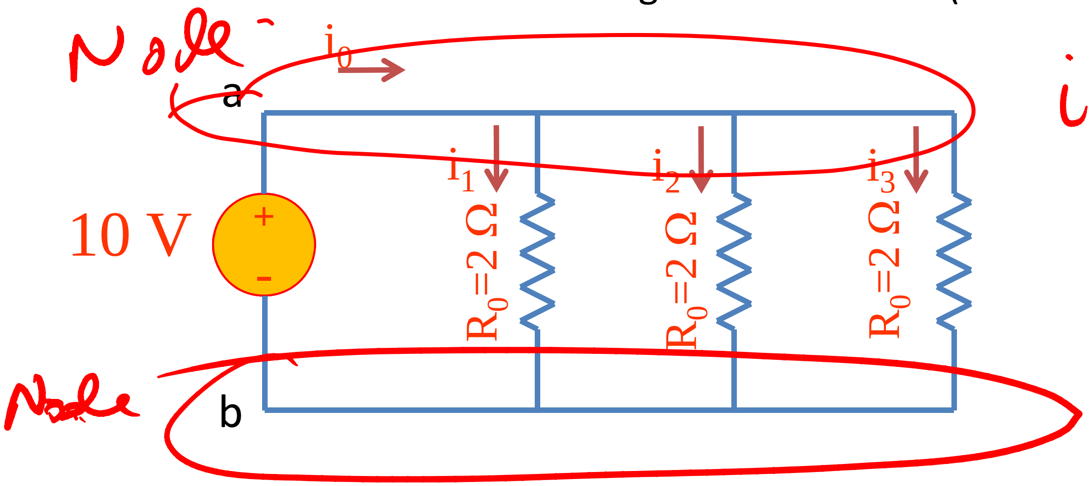
$V_{ab} = 10V$

$R = 1k\Omega$

$\Rightarrow I_{ab} = \frac{10V}{1k\Omega} = 0.01 \frac{V}{\Omega} = 0.01A$

Example problem: Resistors in parallel.

Solve for all the currents and voltages in this circuit. (instructor).



$$i_0 = i_1 + i_2 + i_3$$

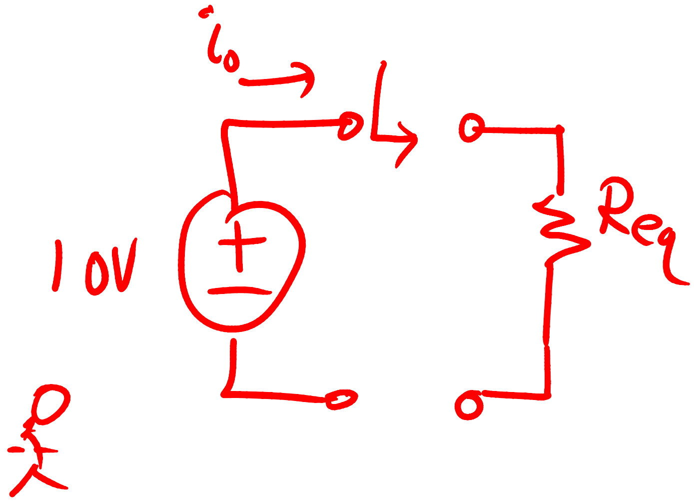
$$i_1 = \frac{10V}{2\Omega} = 5A$$

$$i_2 = \frac{10V}{2\Omega} = 5A$$

$$i_3 = \frac{10V}{2\Omega} = 5A$$

$$i_0 = 15A$$

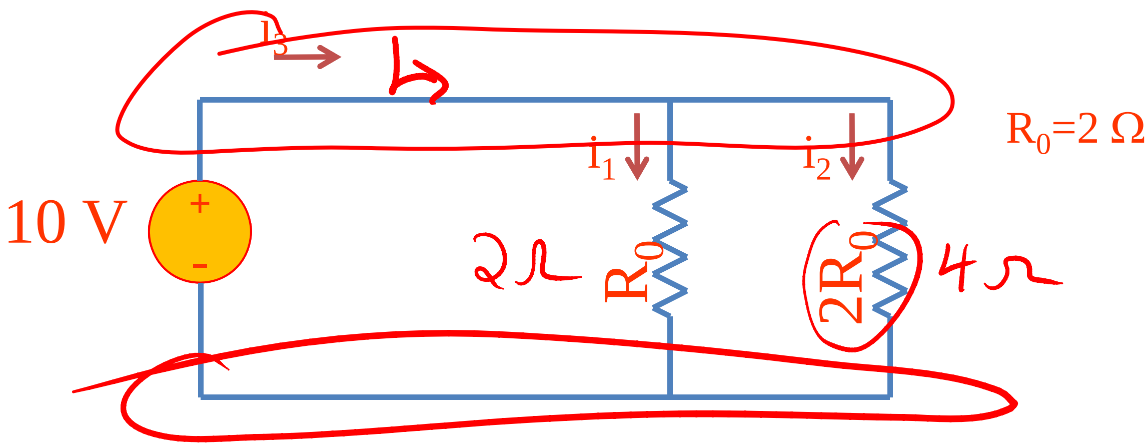
Q: $i_1 = i_2 = i_3$ "by symmetry"



$$R_{eq} = \frac{10V}{i_0} = \frac{10V}{15A} = \frac{2}{3}\Omega$$

Example problem: Resistors in parallel.

Solve for all the currents and voltages in this circuit. (students).



a) Find i_1 $= \frac{10V}{2\Omega} = \ldots 5A$

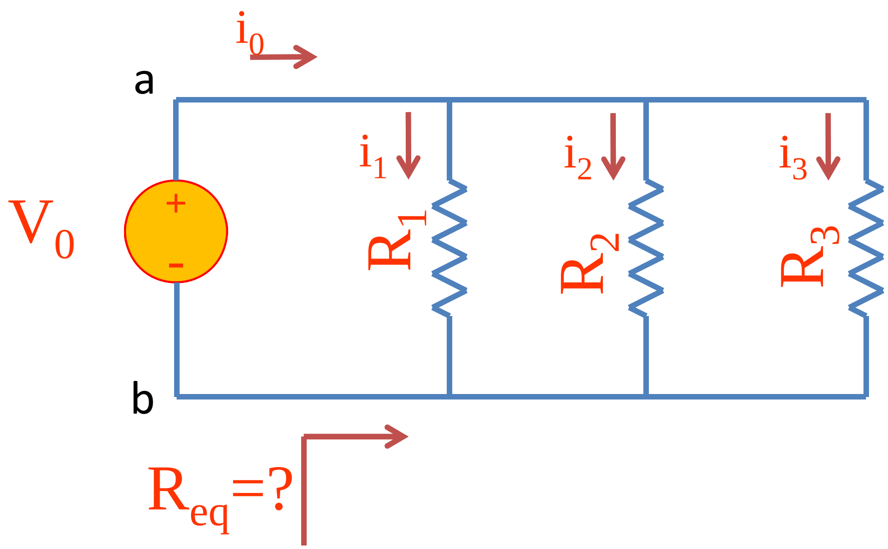
b) Find i_2 $= \frac{10V}{4\Omega} = \frac{5}{2}A$

c) Find i_3 $= \left(5 + \frac{5}{2}\right)A = 7.5A$

d) Find R_{eq} $= \frac{10V}{7.5A} = \frac{10}{7.5}\Omega$

Generalize: N resistors in parallel.

Solve for R_{eq}. (instructor).



a) $i_1 = V_0/R_1$

b) $i_2 = V_0/R_2$

c) $i_3 = V_0/R_3$

d) $i_0 = i_1 + i_2 + i_3$

e) $\frac{V_0}{i_0} \equiv R_{eq}$

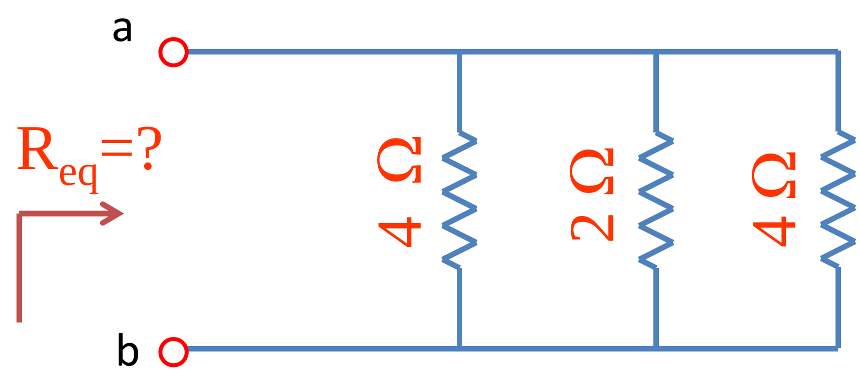
$$R_{eq} = \frac{V_0}{\frac{V_0}{R_1} + \frac{V_0}{R_2} + \frac{V_0}{R_3}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots}$$

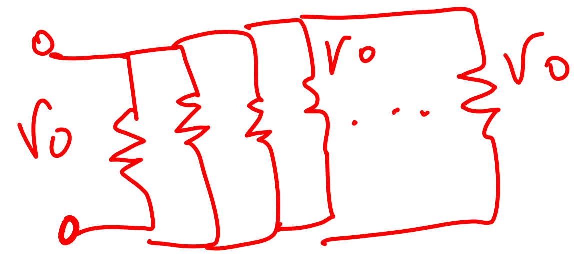
$R_1 \| R_2 \| R_3$ Notation

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \rightarrow \sum_{i=1}^{N} \frac{1}{R_i}$$

Example problem:

Solve for R_{eq}. (sudents).

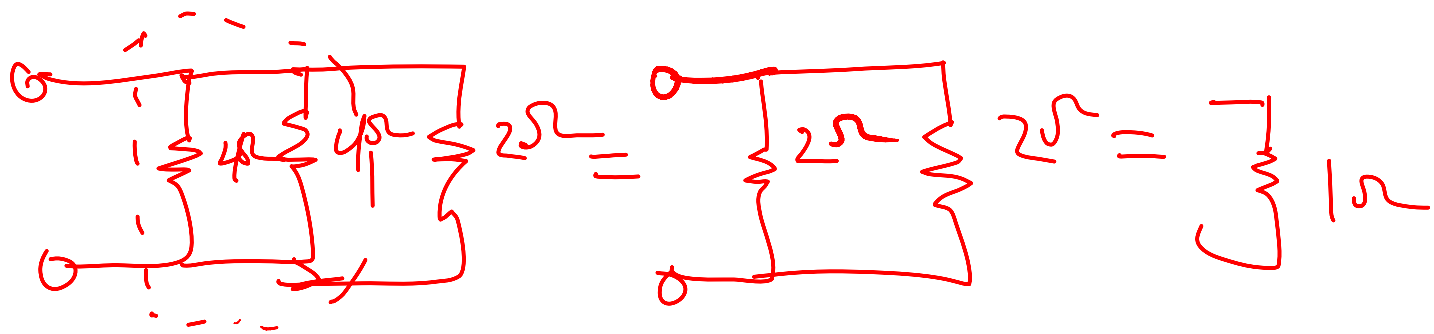




$1\,\Omega$

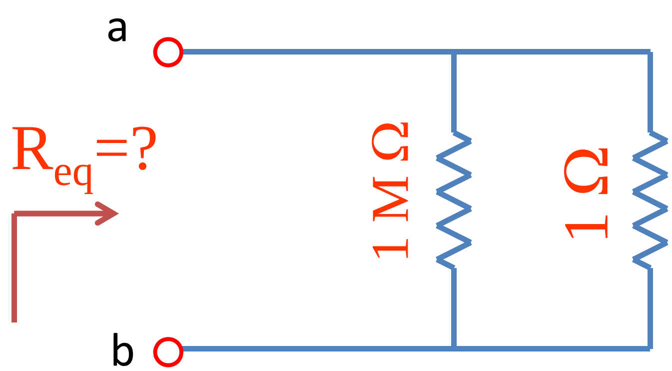
$R_{eq} = \frac{r_o}{N}$

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1\,\Omega$$



Important practical example:

Solve for R_{eq}. (instructor).



1Ω

$\frac{R_1}{R_2}$ small

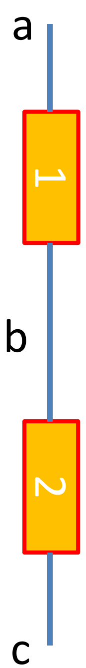
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \frac{\frac{1}{R_2}}{\frac{1}{R_2}}$$

$$= R_1 \frac{1}{1 + R_1/R_2} \Rightarrow R_1 \qquad \frac{1}{1+x} = 1 - x + \ldots$$

$$\simeq R_1 \left(1 - \frac{R_1}{R_2}\right) = 1\Omega \left(1 - \frac{1\Omega}{10^6\Omega}\right)$$

Questions?

Voltage addition in circuits



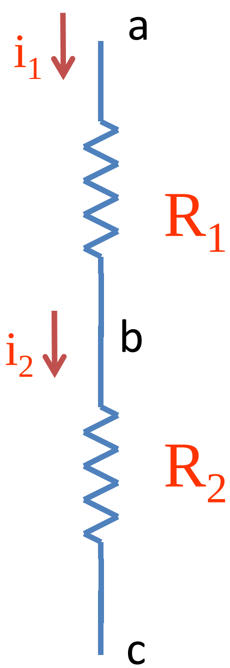
$$V_{ab} \equiv \int_a^b E dx$$

$$V_{bc} \equiv \int_b^c E dx$$

$$\Rightarrow V_{ac} \equiv \int_a^c E dx = \int_a^b E dx + \int_b^c E dx = V_{ab} + V_{bc}$$

$$V_{ac} = V_{ab} + V_{bc}$$

2 resistors in series



$$i_1 = i_2$$

$$i_1 = \frac{V_{ab}}{R_1} \qquad i_2 = \frac{V_{bc}}{R_2}$$

$$\frac{V_{ab}}{R_1} = \frac{V_{bc}}{R_2} \Rightarrow V_{bc} = \frac{R_2}{R_1} V_{ab}$$

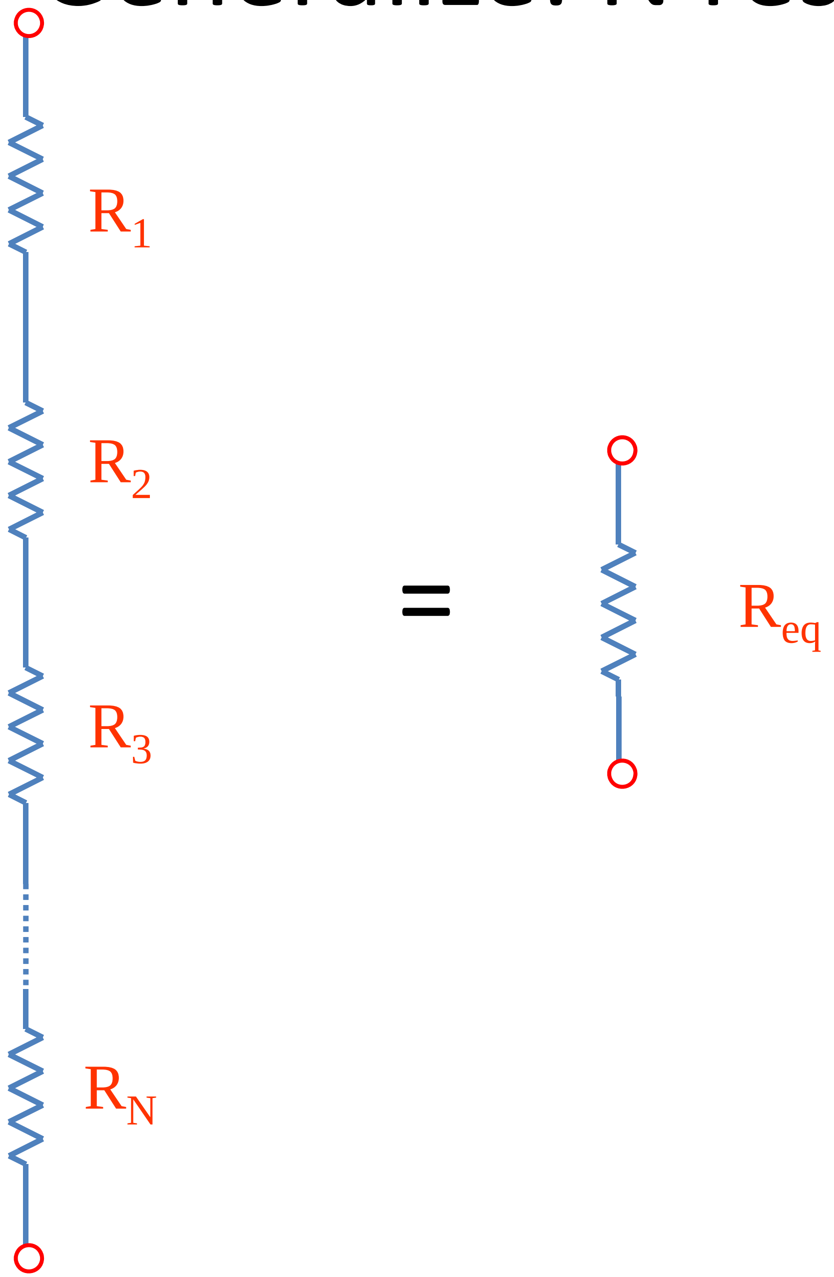
$$V_{ac} = V_{ab} + V_{bc} = V_{ab}\left(1 + \frac{R_2}{R_1}\right)$$

$$= i R_1 \left(1 + \frac{R_1}{R_2}\right) = i(R_1 + R_2) = V_{ac}$$

R_{eq}

$$R_{eq} = R_1 + R_2$$

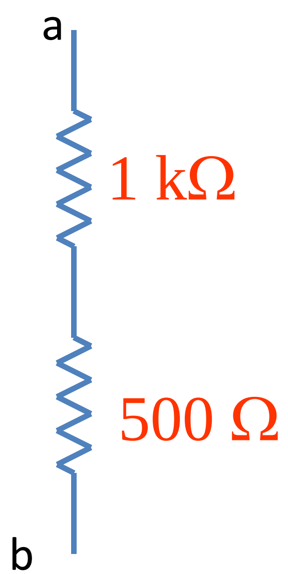
Generalize: N resistors in series



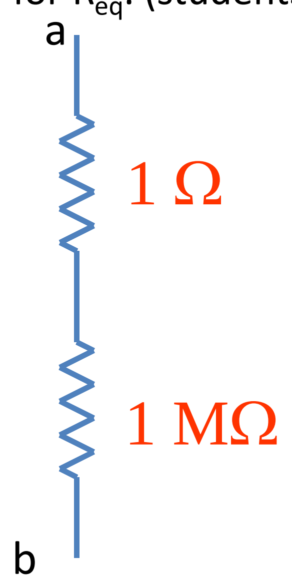
$$R_{eq} = \sum_{i=1}^{N} R_i$$

Example problems

Solve for R_{eq}. (instructor).



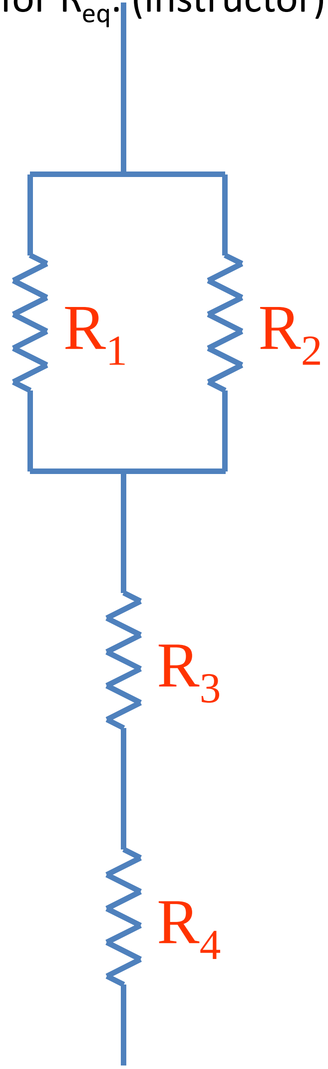
Solve for R_{eq}. (students).



Questions?

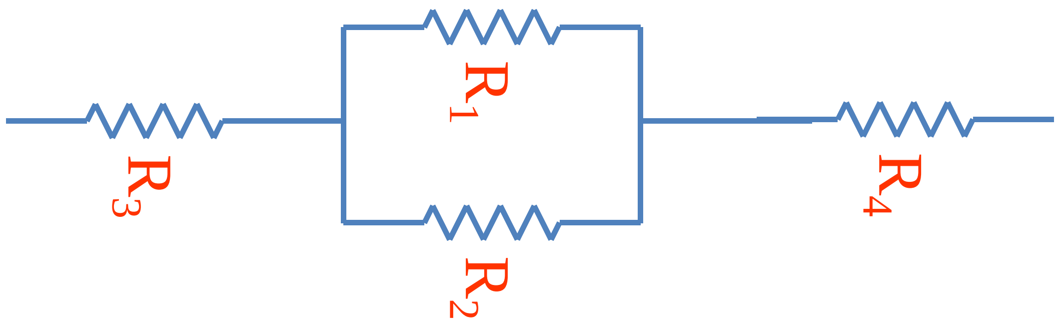
Example problems

Solve for R_{eq}. (instructor).



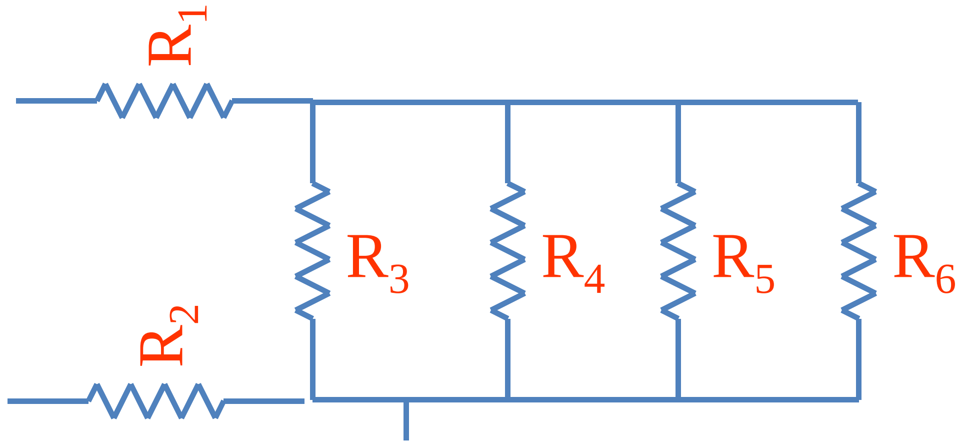
Example problems

Solve for R_{eq}. (students).



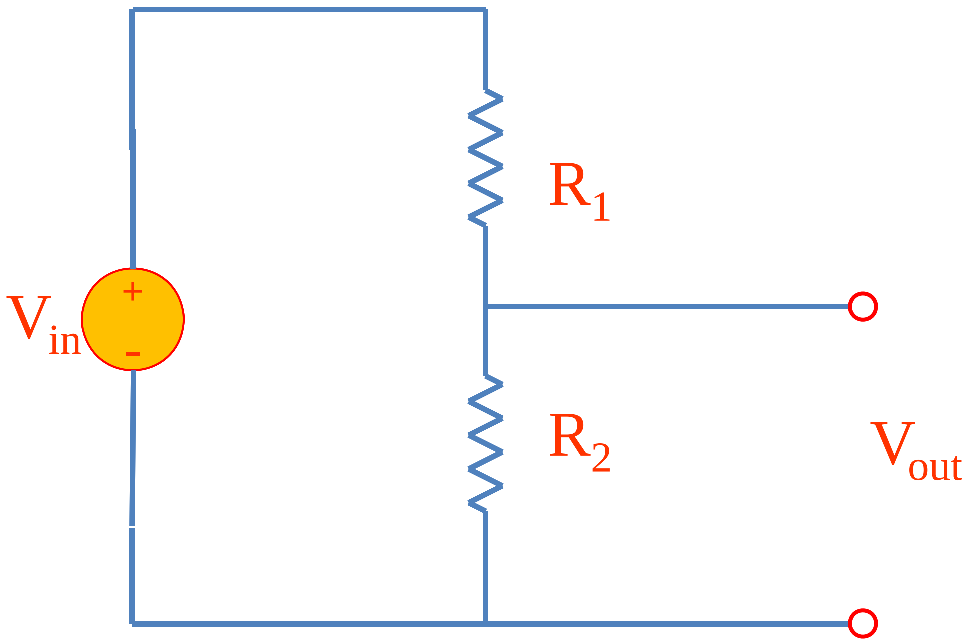
Example problems

Solve for R_{eq}. (instructor).



Example problems

Voltage divider

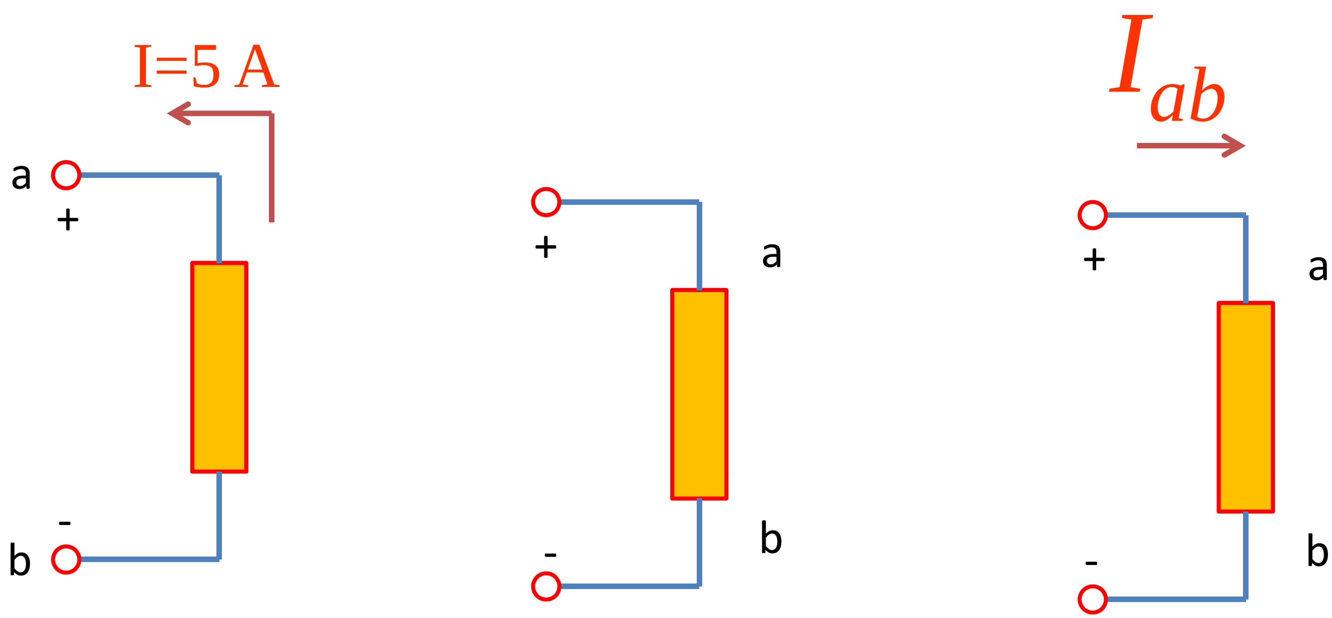


$$V_{out} = \frac{R_1}{R_1 + R_2} V_{in}$$

Why important?
Concept of source/load. (Thevenin…)

Source/load concept

Symbol library



Symbol library