EECS 70A: Network Analysis

Lecture 3

Announcements

- Quiz #1 results excellent
  - A few percent of students confused about sign
- HW 1 due tomorrow in discussion
- Midterm next Thursday (ch 1-2)
  - One more HW and quiz before then
- Recorded lectures to be posted online starting today

Review & agenda

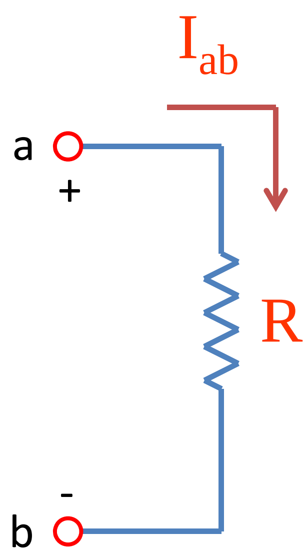
Last lecture:

- Examples
  - Power (sink/source)
  - Current (postive/negative)
  - Dependent sources
- Resistors
  - Series
  - Parallel

Today

- Examples (resistor circuits)
- Kirchoff's laws
- Example applications of Kirchoff laws

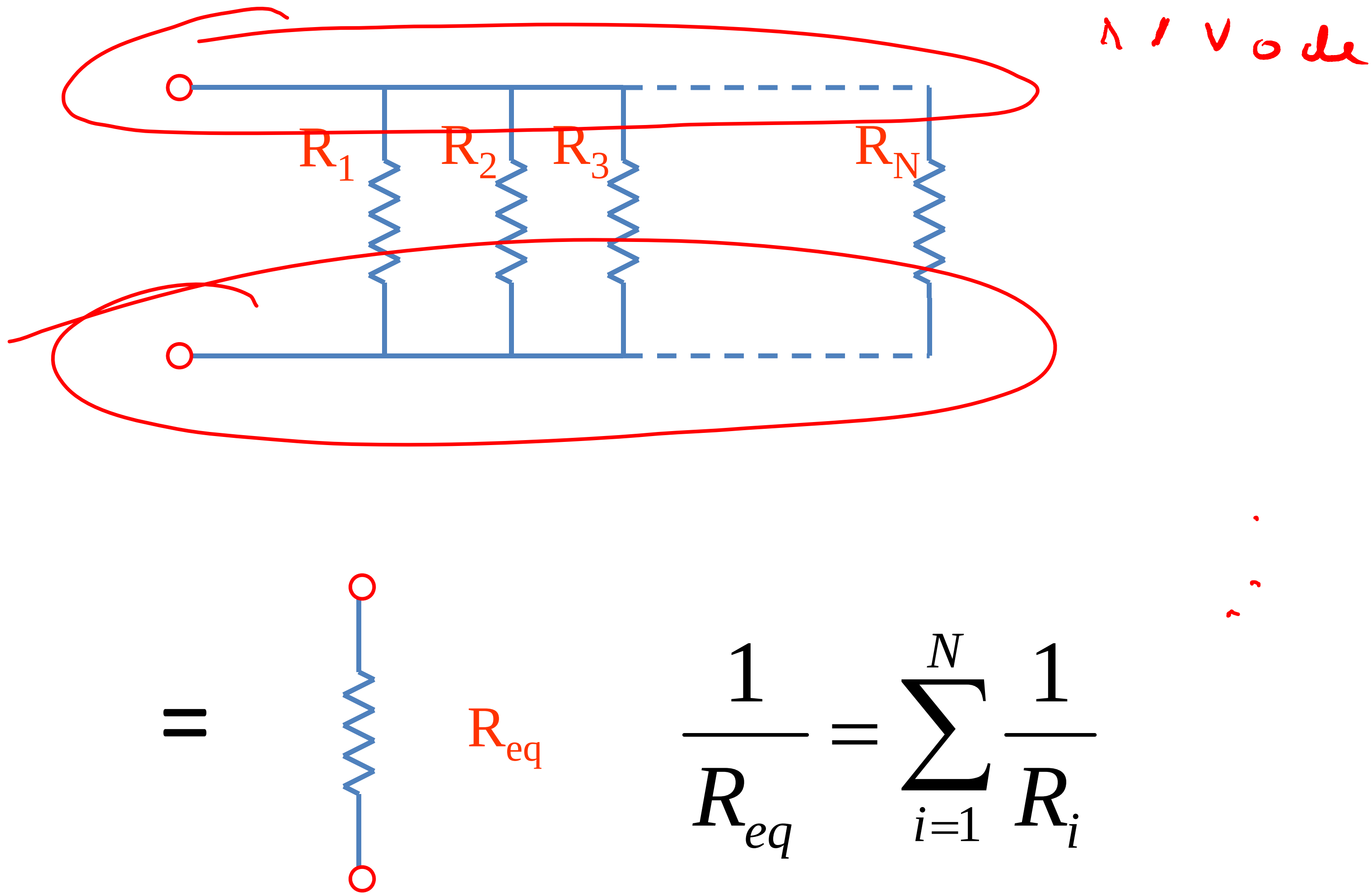
Resistors



$$V_{ab} = I_{ab} \times R$$

Resistance units: Ohms [Ω]

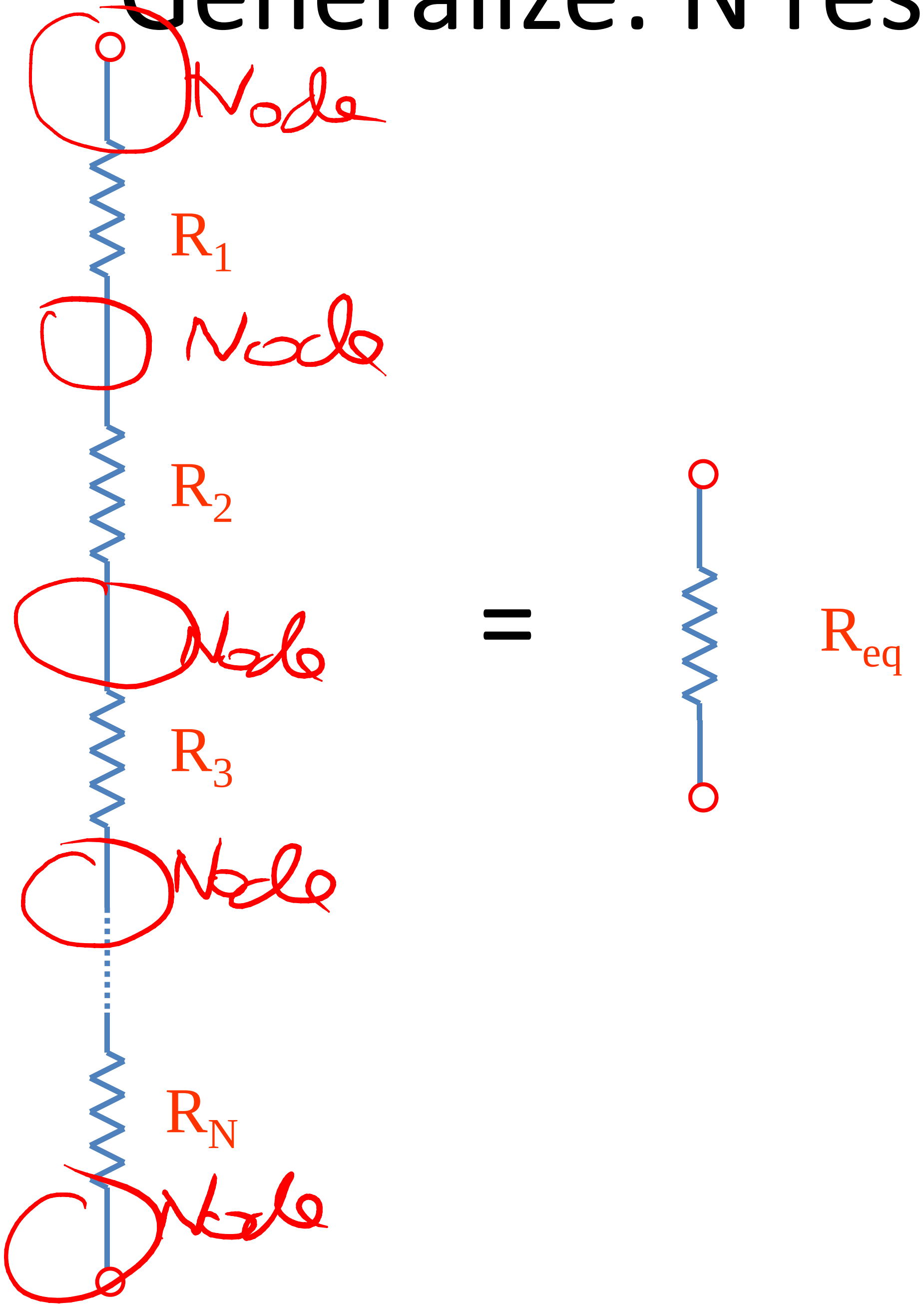
Generalize: N resistors in parallel



$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

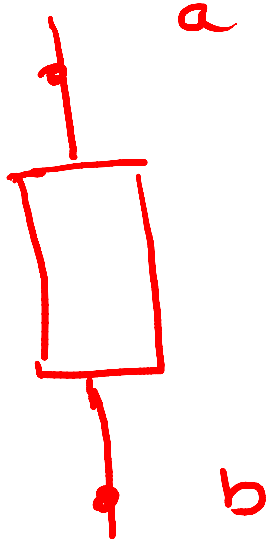
$R_1 \parallel R_2$ is notation for "R_1 in parallel with R_2"

Generalize: N resistors in series

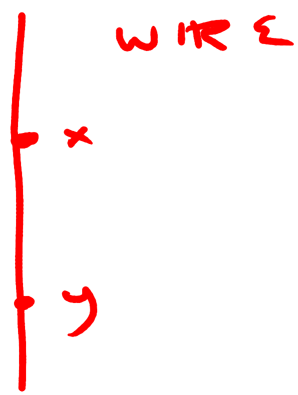


$$R_{eq} = \sum_{i=1}^{N} R_i$$

Node



$$V_{ab} \equiv \int_a^b E\,dx$$

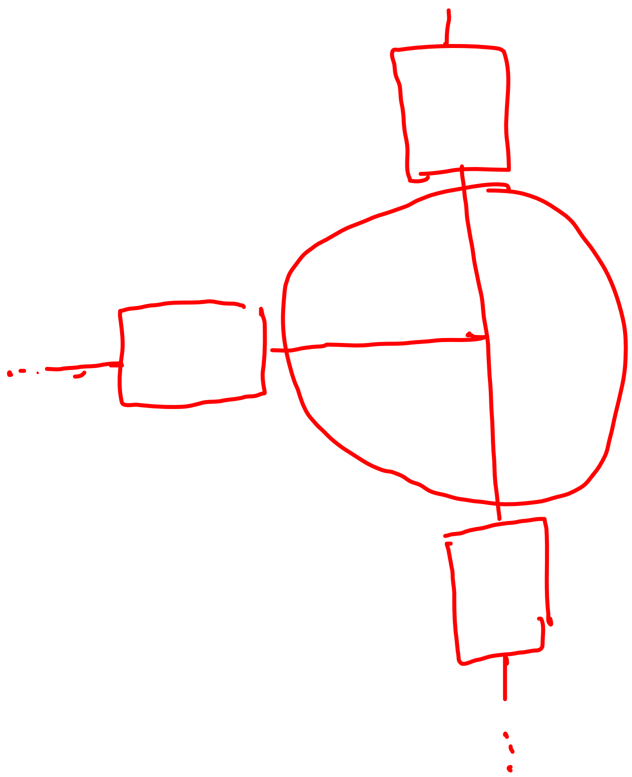


$$V_{xy} \equiv \int_x^y E\,dx$$

In ideal metal $E = 0$

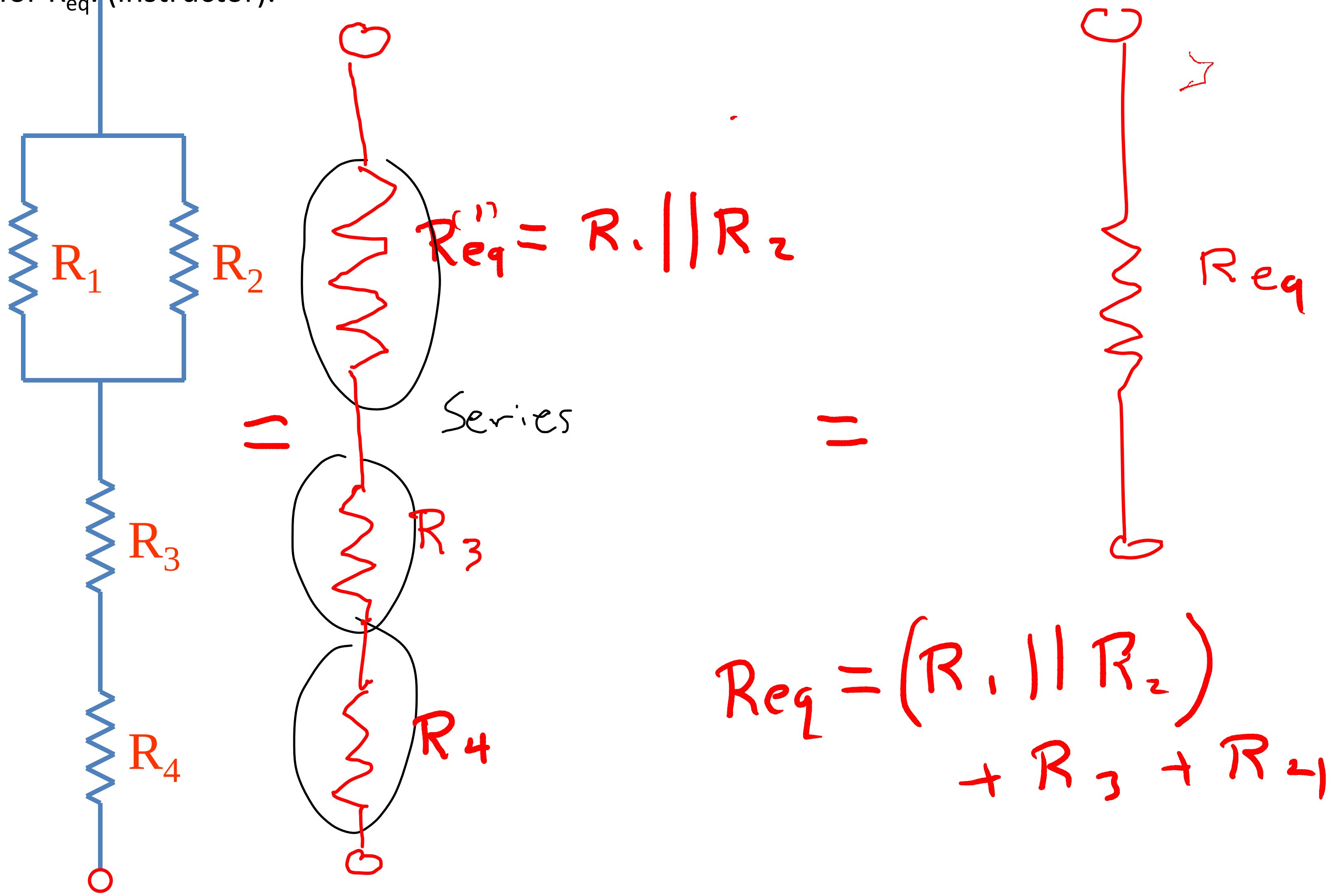
$\Rightarrow V_{xy} = 0$

Node:



Example problems

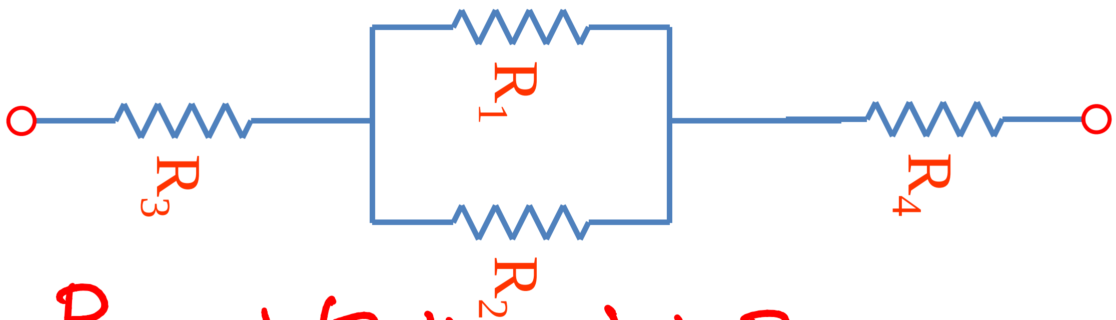
Solve for R_{eq}. (instructor).



Example problems

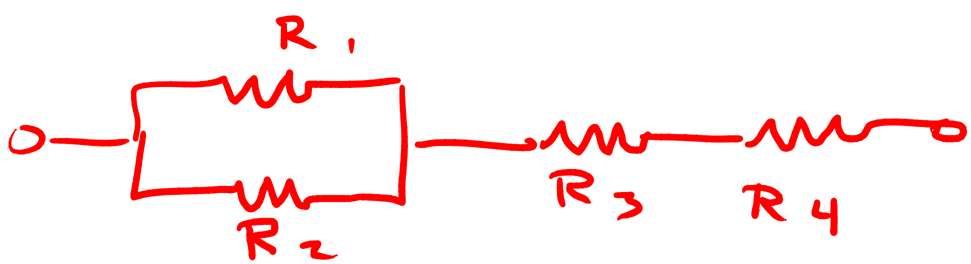
Solve for R_{eq}. (students).

pen test

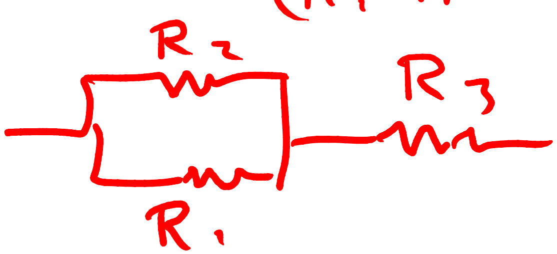


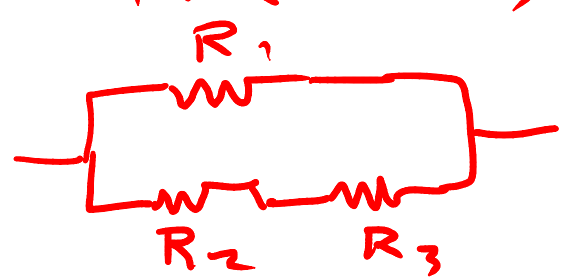
$$R_{eq} = R_3 + (R_1 \| R_2) + R_4$$

$$= (R_1 \| R_2) + R_3 + R_4$$



Careful $(R_1 \| R_2) + R_3 \neq R_1 \| (R_2 + R_3)$

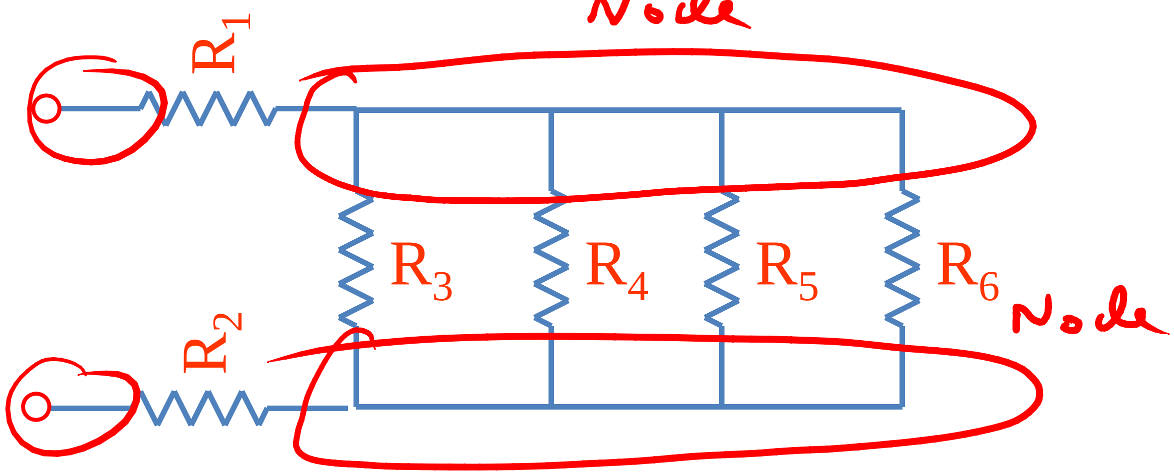


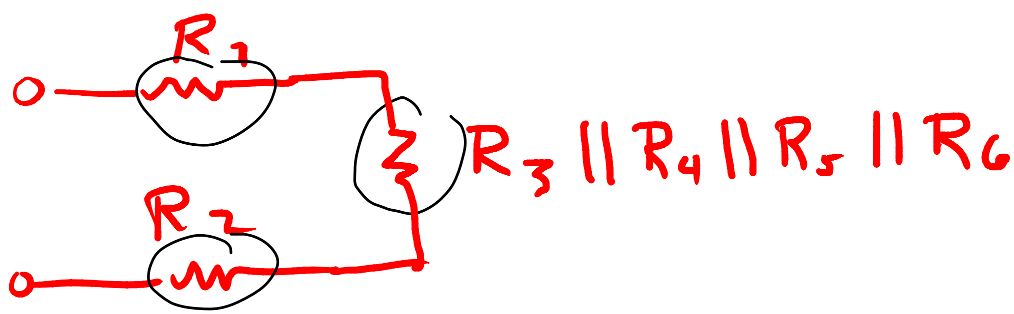


Example problems

Solve for R_{eq}. (instructor).

How many nodes?





$$R_{eq} = R_1 + (R_3 || R_4 || R_5 || R_6) + R_2$$

$$= R_1 + R_2 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}}$$

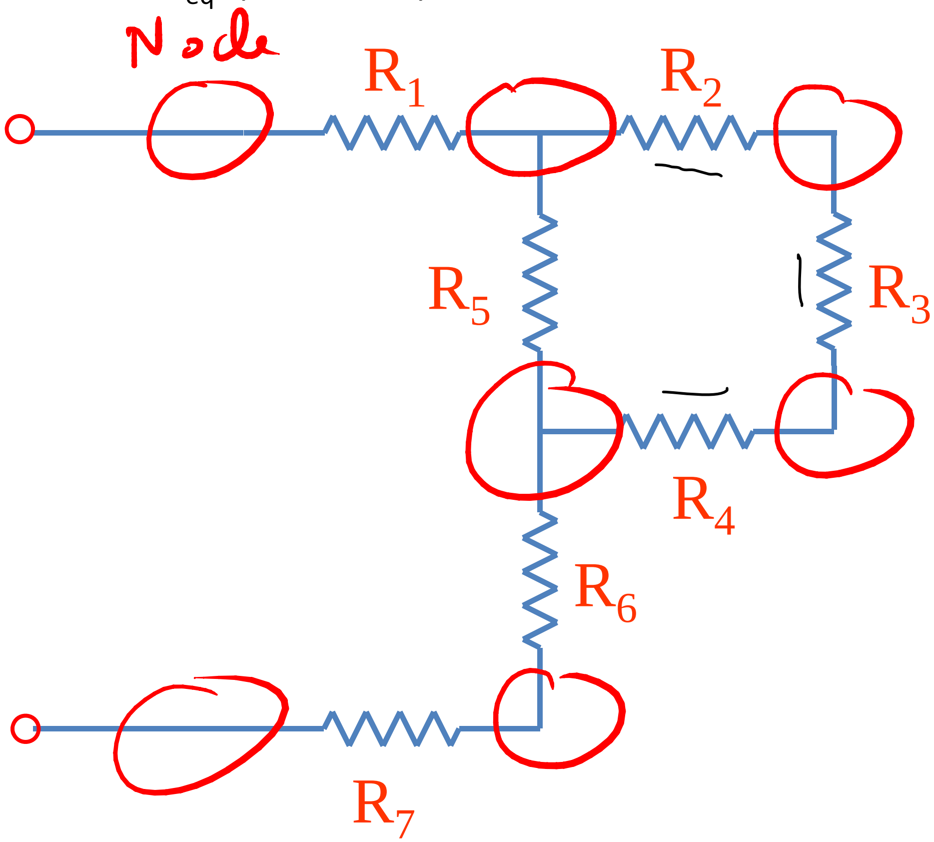
$R_3 - R_6$ $\parallel$

$$\frac{1}{R_{eq}^*} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}$$

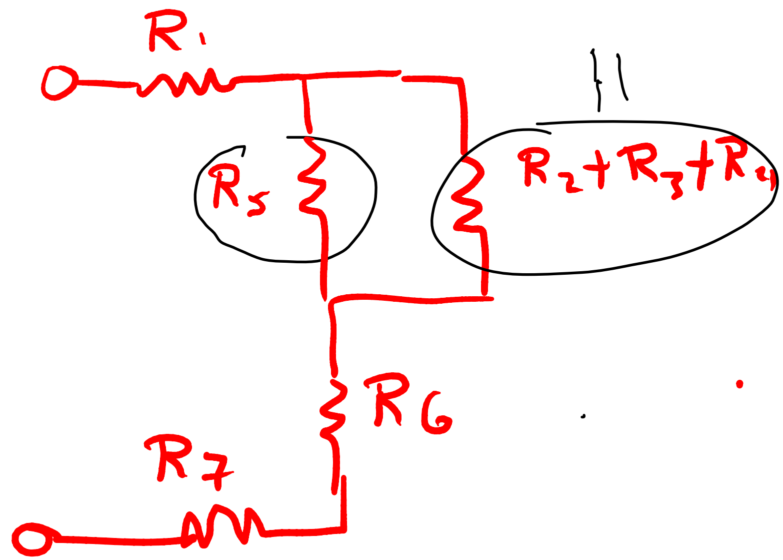
$$R_{eq}^* = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}}$$

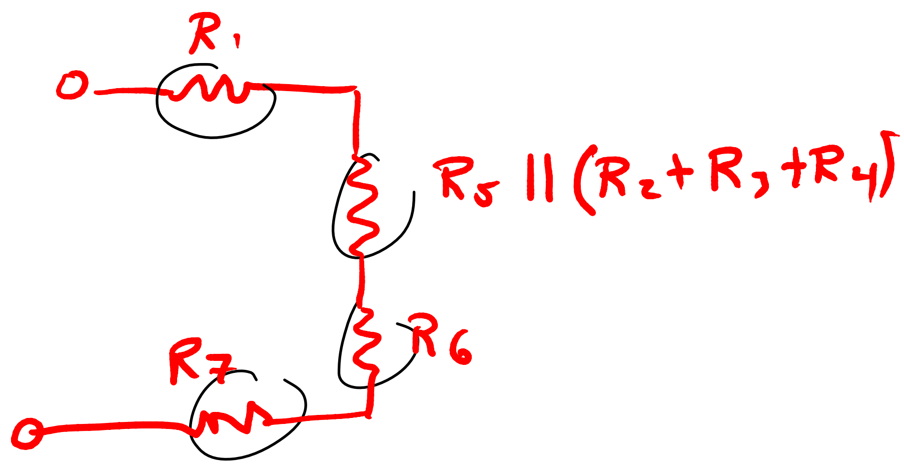
Example problems

Solve for R_{eq}. (instructor).



Emphasis: Nodes



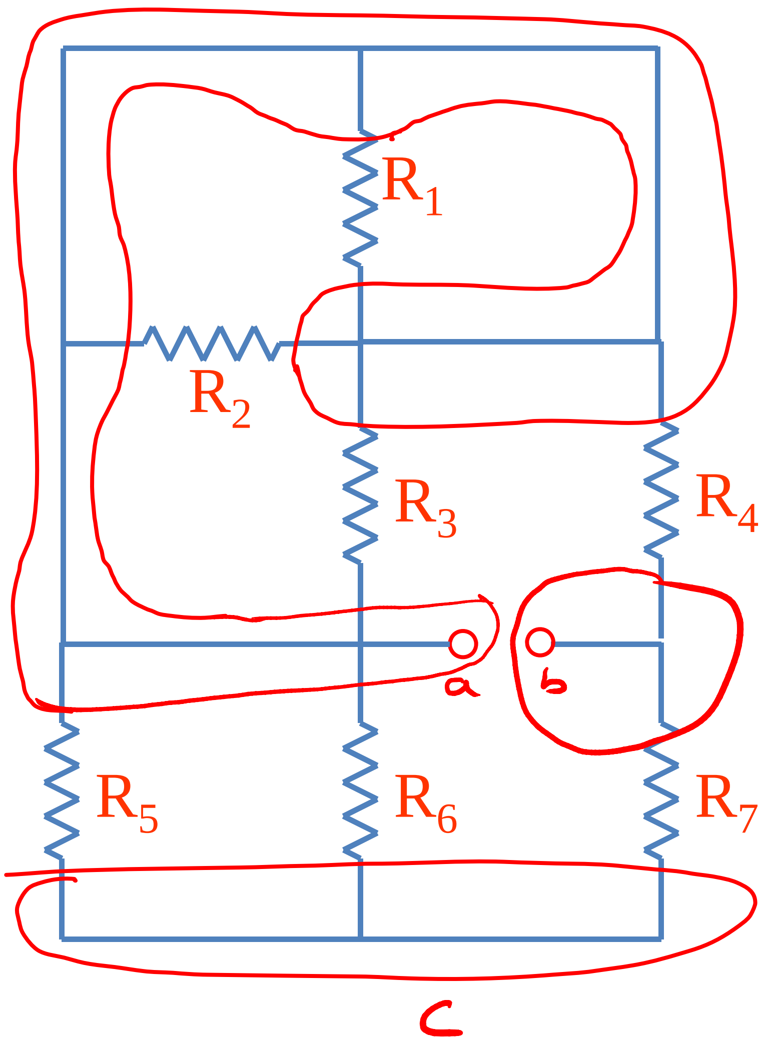


$$= R_{eq} = R_1 + R_6 + R_7 + [R_5 \,||\, (R_2 + R_3 + R_4)]$$

Example problems

Solve for R_{eq}. (instructor).

Emphasis: Nodes

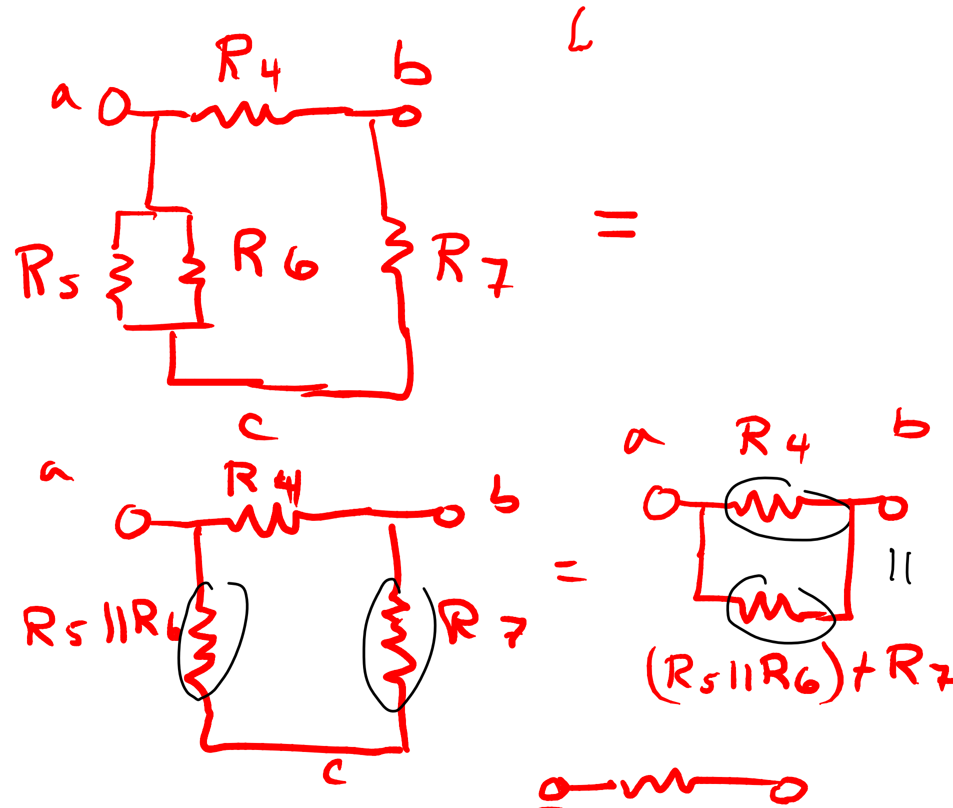


3 nodes

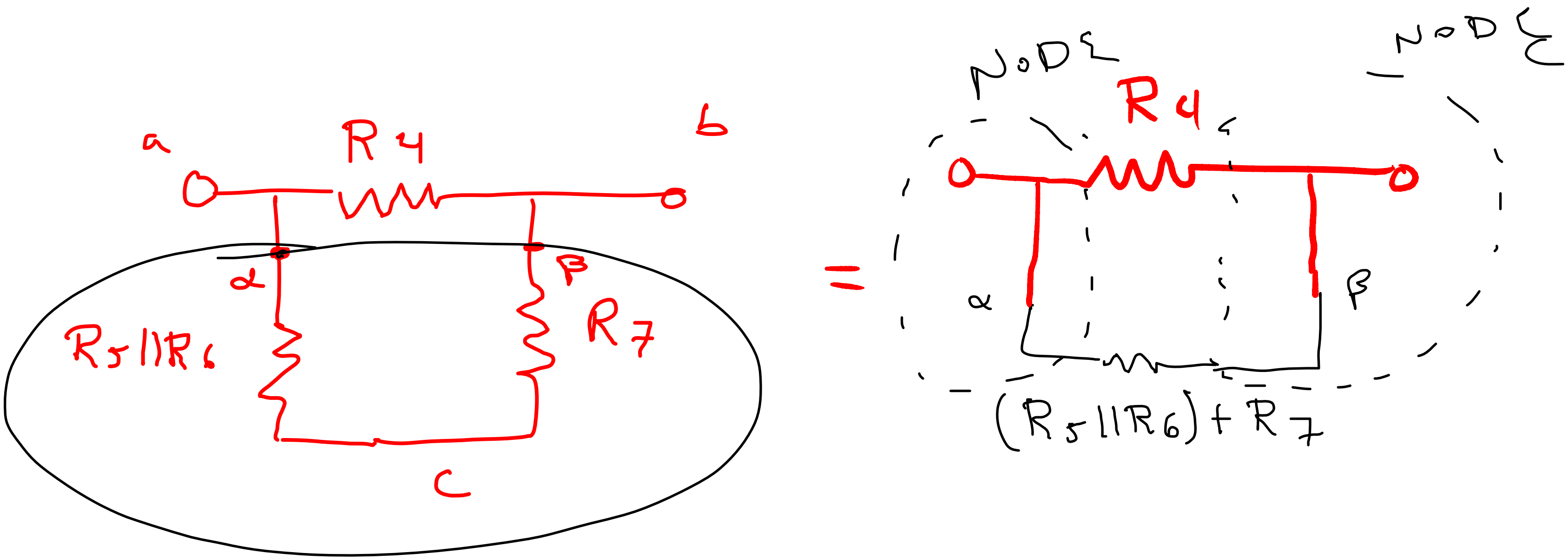
R_1 shorted

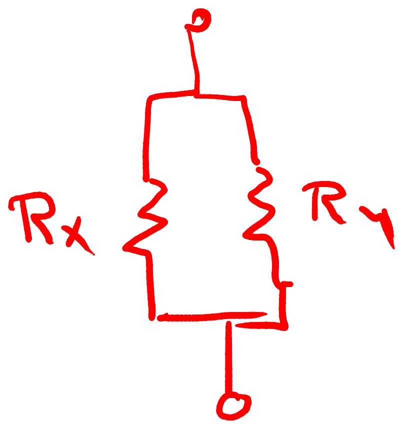
R_2 shorted

R_3 shorted



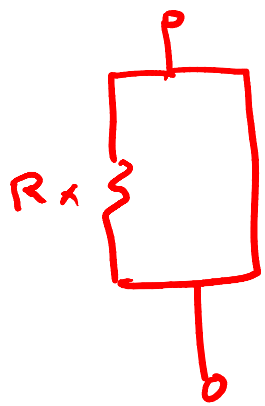
$$R_{eq} = R_4 \parallel \left[(R_5 \parallel R_6) + R_7 \right]$$


a
R4
b
α
β
R5||R6
R7
c
=
NODE
NODE
R4
α
β
(R5||R6)+R7



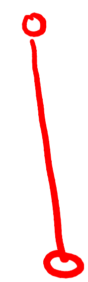
$$R_{eq} = R_x \parallel R_y$$

$$\frac{1}{R_{eq}} = \frac{1}{R_x} + \frac{1}{R_y}$$



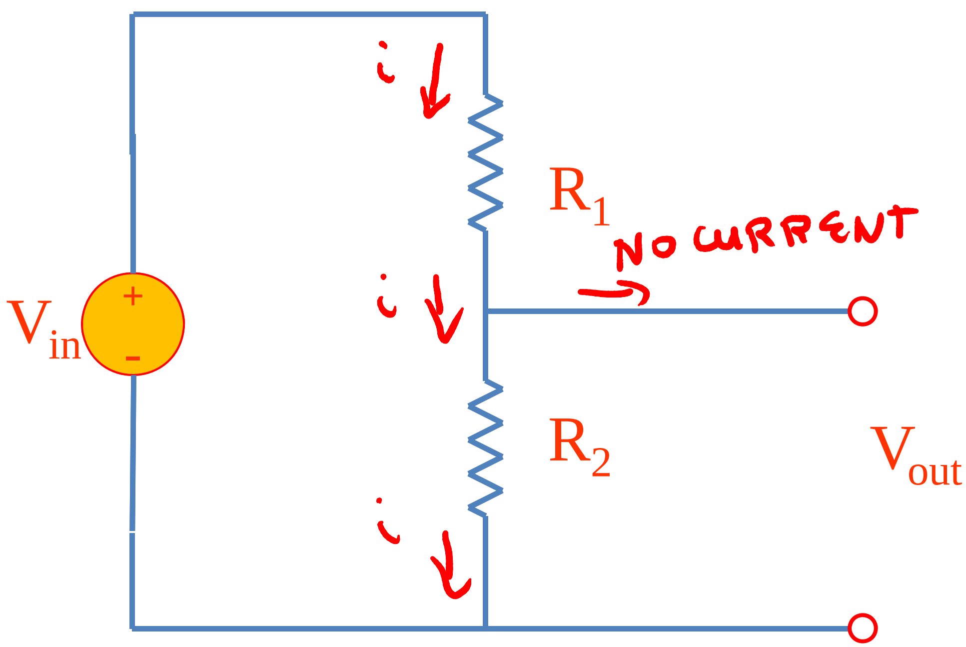
"$R_y = 0$"

$$\frac{1}{R_{eq}} = \frac{1}{R_x} + \frac{1}{\text{"}0\text{"}} =$$



$$\Rightarrow R_{eq} = 0$$

Voltage divider



Derivation:

$$iR_2 + iR_1 = V_{IN}$$

$$i = \frac{V_{IN}}{R_1 + R_2}$$

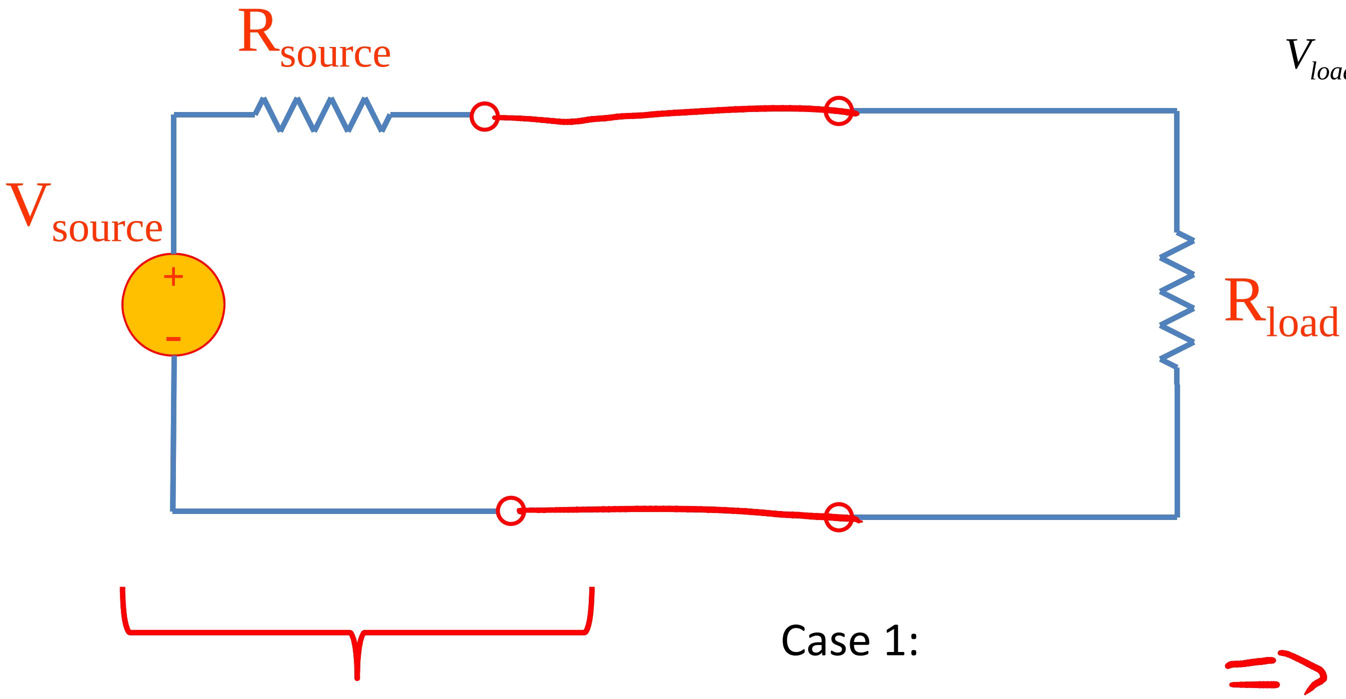
$$V_{out} = iR_2$$

$$= \frac{R_2}{R + R_2} V_{IN}$$

Why important?
Concept of source/load. (Thevenin…)

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$$

Source/load



$$V_{load} = \frac{R_{load}}{R_{load} + R_{source}} V_{source}$$

Derivation:

Thevenin Thm:
<u>*Any*</u> circuit can be represented by this equivalent circuit.

Case 1:

$$R_{load} >> R_{source}$$

$\Rightarrow V_{load} \approx V_{source}$

Case 2:

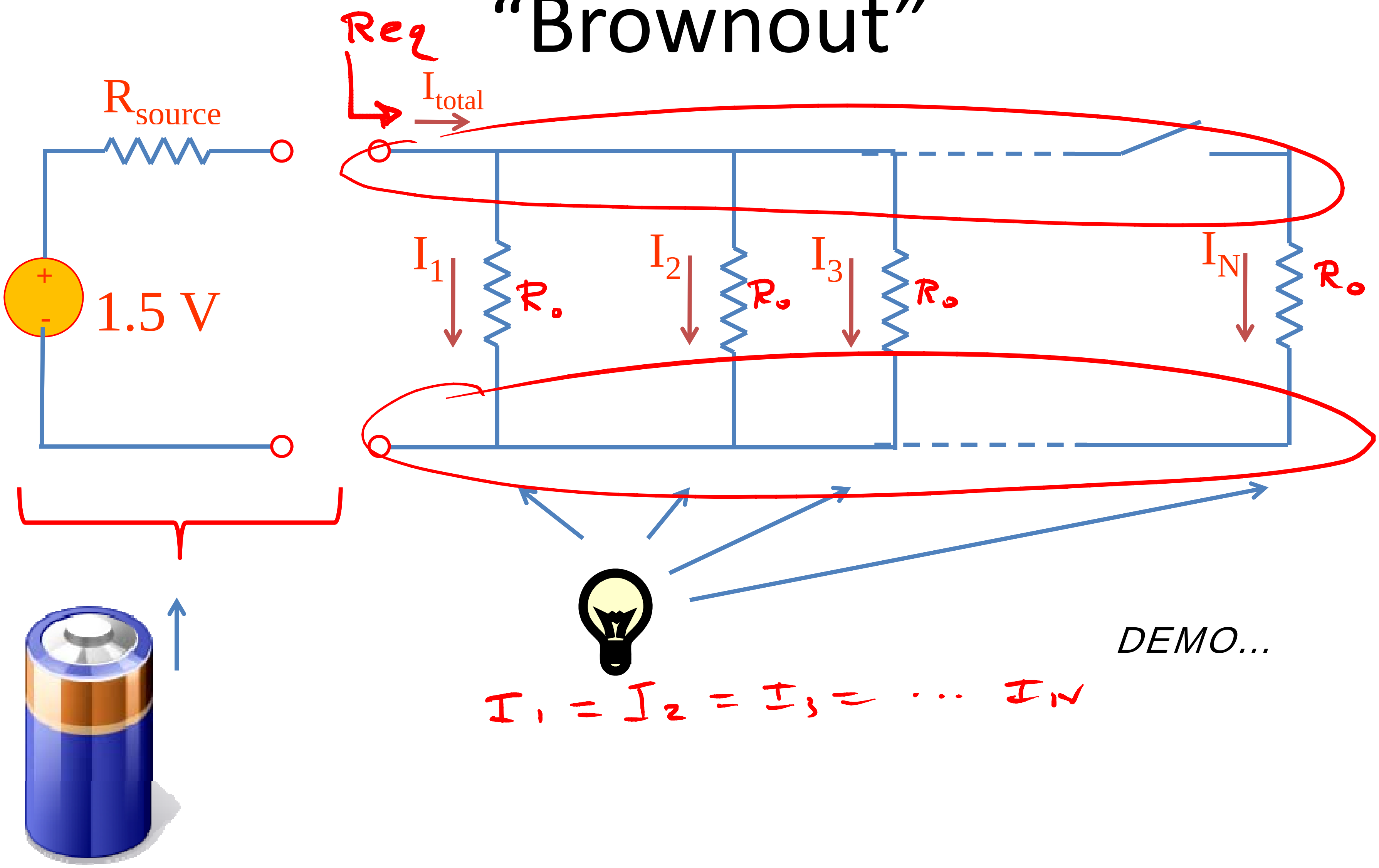
$$R_{source} >> R_{load}$$

$V_{load} \to 0$

$\approx \frac{R_{load}}{R_{source}} V_{source}$

We say R_{load} "*loads down*" the source.

“Brownout”

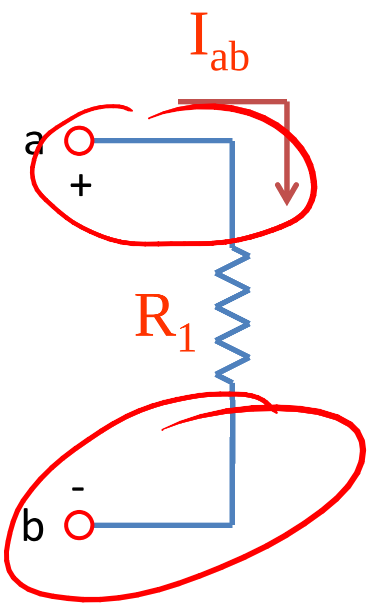


Battery/light bulbs

“Fanout”

Questions?

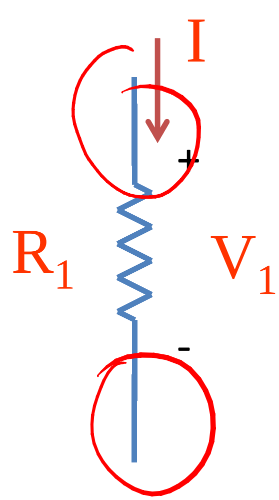
Notation: one element



$$V_{ab} = I_{ab}\, R_1$$

V_{ab} is the voltage *drop* from a to b.

Textbook chapter 2 notation:

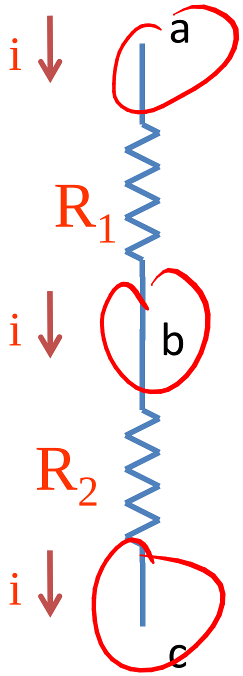


$$V_1 = I\, R_1$$

In chapter 2, text does not label each node.
V_1 is voltage drop across resistor 1.

Notation: two elements in series

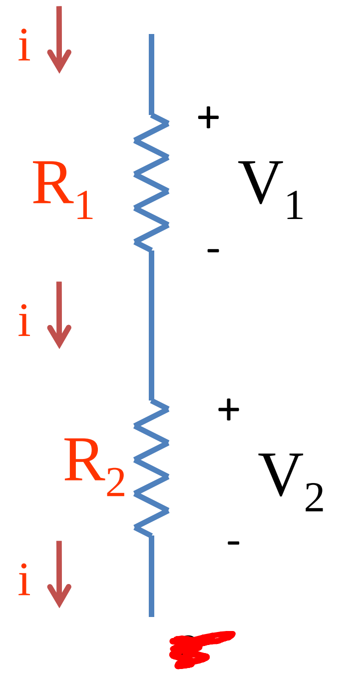
Textbook chapter 2 notation:



$$i = \frac{V_{ab}}{R_1}$$

$$i = \frac{V_{bc}}{R_2}$$

V_{ab} is the voltage drop across resistor 1



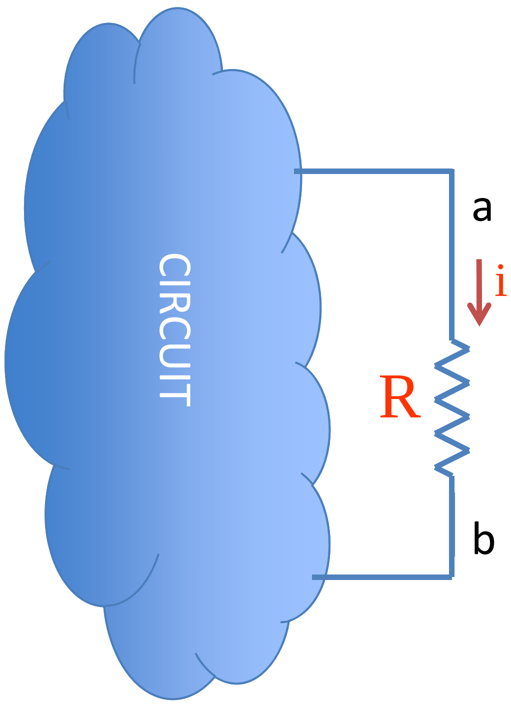
$$i = \frac{V_1}{R_1}$$

$$i = \frac{V_2}{R_2}$$

V_1 is the voltage drop across resistor 1

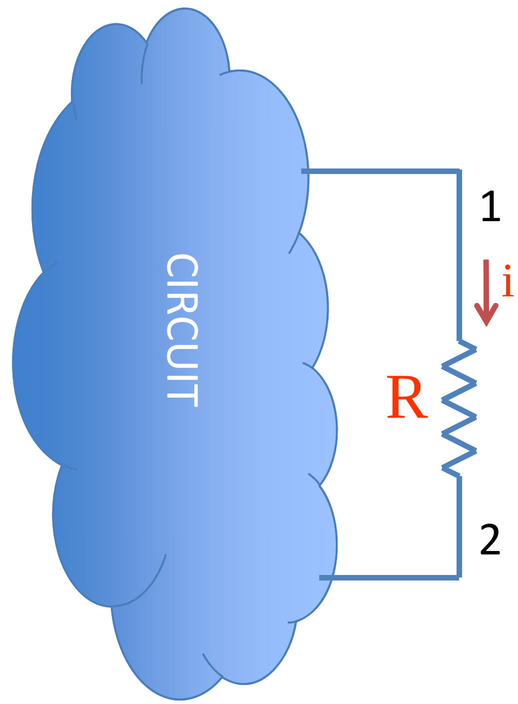
Letters and numbers

Both can be used to label nodes, resistors, voltages, currents, etc.



$$i = \frac{V_{ab}}{R}$$

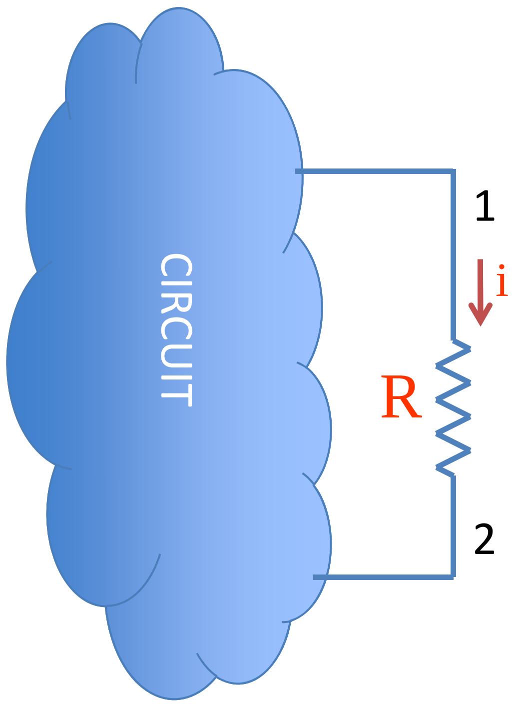
V_{ab} is the voltage drop across resistor 1



$$i = \frac{V_{12}}{R}$$

V_{12} is the voltage drop across resistor 1

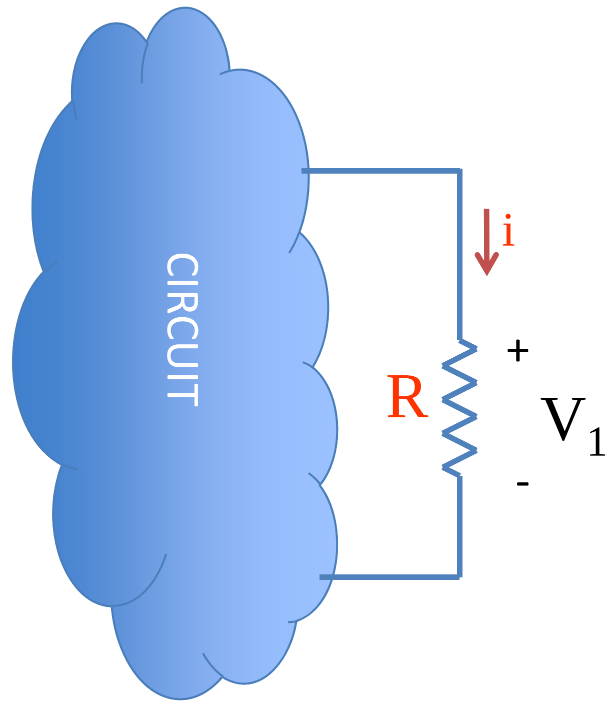
Chapter 2 notation



$$i = \frac{V_{12}}{R}$$

V_{12} is the voltage drop across resistor 1

Textbook chapter 2 notation:



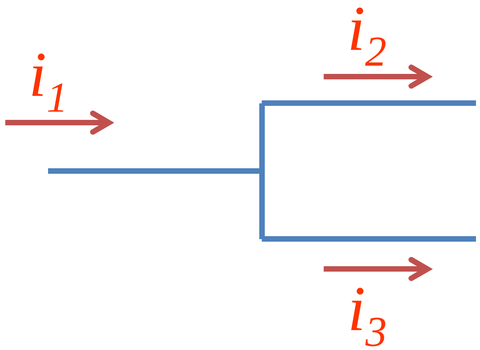
$$i = \frac{V_1}{R_1}$$

V_1 is the voltage drop across resistor 1

Questions?

Kirchoff’s current law

You have already seen:



$$i_1 = i_2 + i_3$$

Like water in a river…

More generally:

Sum of currents *entering* node = sum of currents *leaving* node.

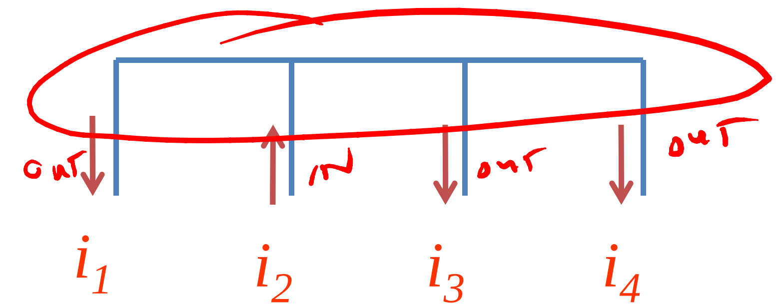
Stated as Kirchoff’s current law (KCL):

$$\sum_{n=1}^{N} i_n = 0$$

Current *entering* a node: i_n positive
Current *leaving* a node: i_n negative

KCL examples

Find a relationship among i_1, i_2, i_3, i_4 (instructor)

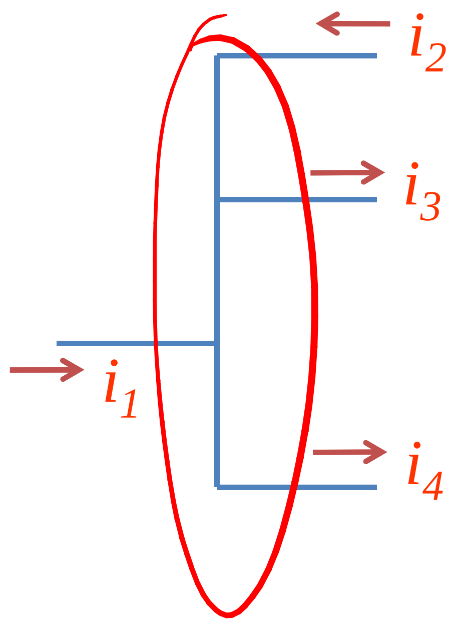


IN: i_2

OUT: $i_1 + i_3 + i_4$

KCL $\Rightarrow$ $i_2 = i_1 + i_3 + i_4$

Find a relationship among i_1, i_2, i_3, i_4 (students)



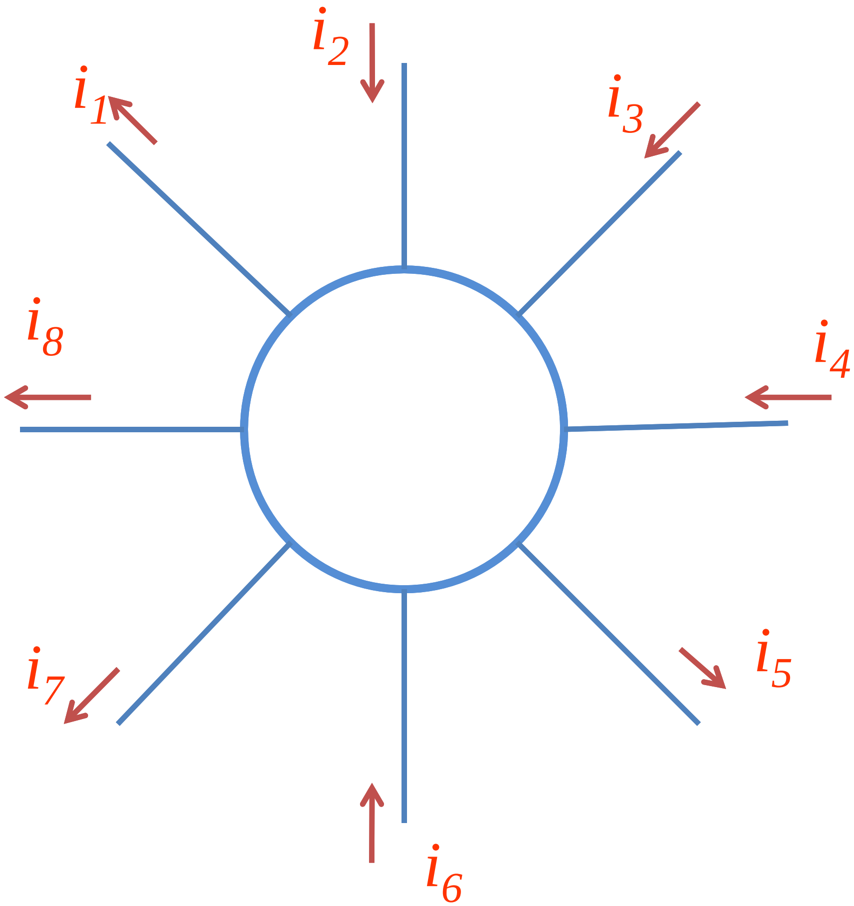
IN $i_1 + i_2$

OUT $i_3 + i_4$

KCL $\Rightarrow$ $i_1 + i_2 = i_3 + i_4$

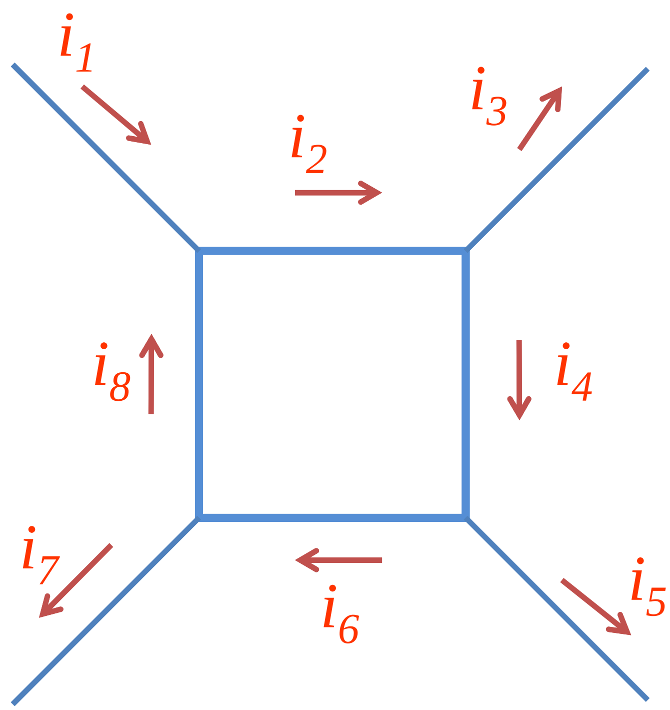
KCL examples

Find a relationship among i_1,i_2, i_3,$i_{4...}$ (instructor)



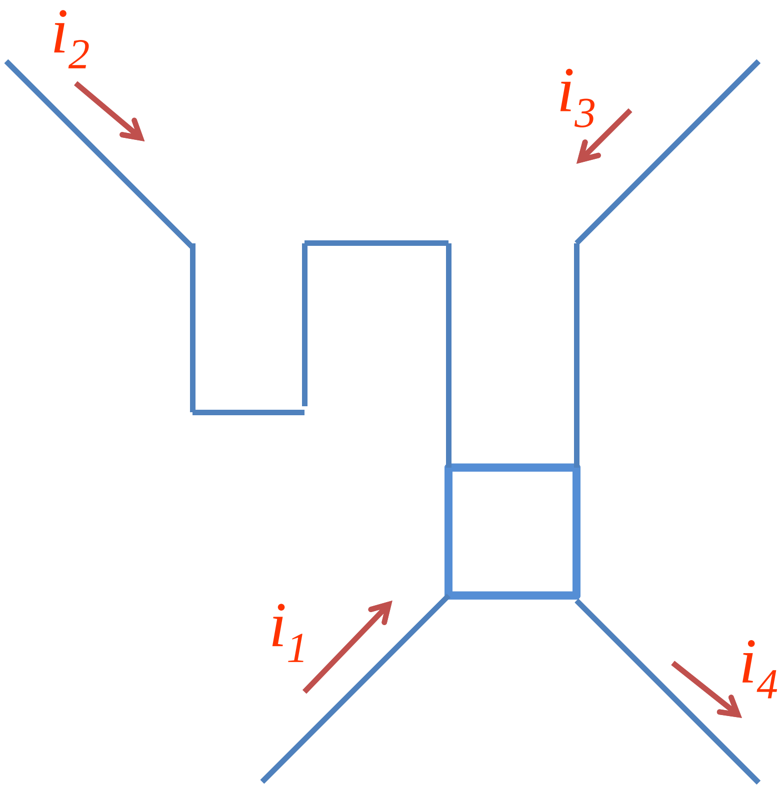
KCL examples

Find a relationship among $i_1, i_2, i_3, i_4 \ldots$ (instructor)



KCL examples

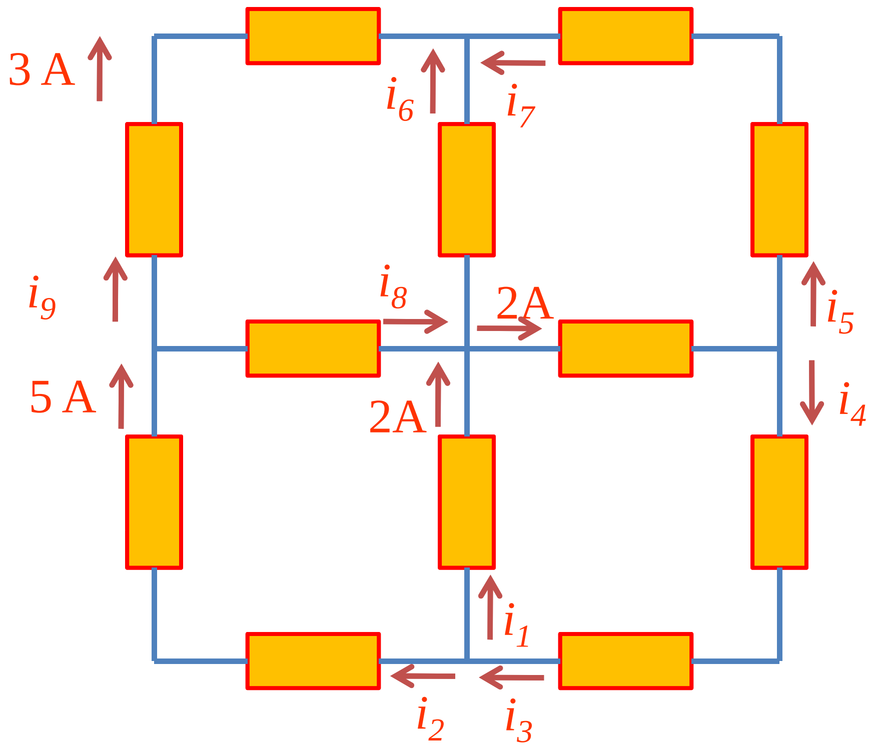
Find a relationship among i_1,i_2, i_3,$i_{4...}$ (students)



KCL examples

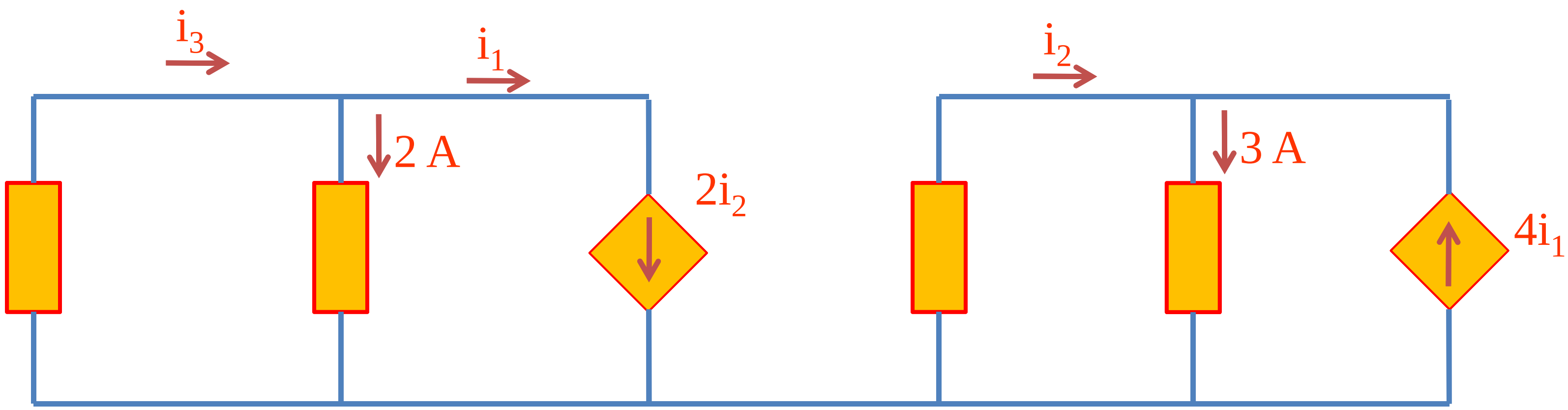
a) Find the # of nodes in this circuit. (Instructor)

b) Find is i_1 thru i_9 in this circuit. (Instructor)
Hint: Apply KCL at each node.



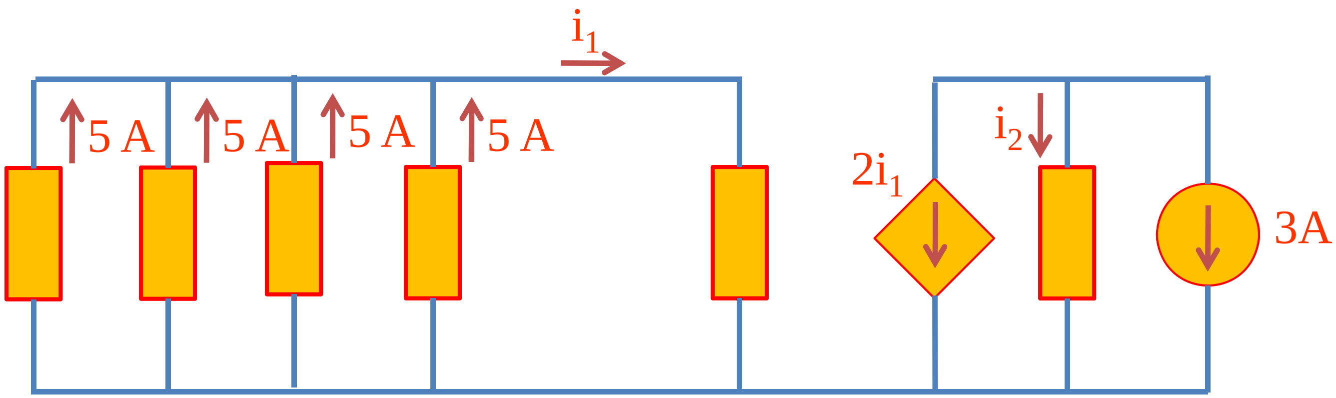
KCL examples

a) Find the # of nodes in this circuit. (Instructor)
b) Find is i_1 i_2 & i_3 in this circuit. (Instructor)
Hint: Apply KCL at each node.



KCL examples

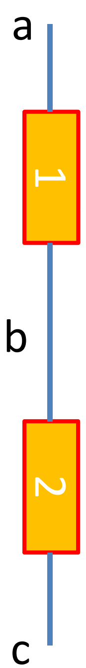
a) Find the # of nodes in this circuit. (students)
b) Find is i_1 & i_2 in this circuit. (students)
Hint: Apply KCL at each node.



Questions?

Voltage addition in circuits

From lecture #2:



$$V_{ab} \equiv \int_a^b E dx$$

$$V_{bc} \equiv \int_b^c E dx$$

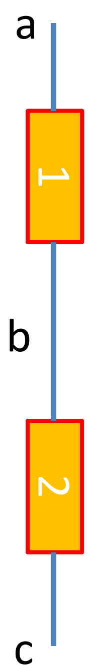
$$\Rightarrow V_{ac} \equiv \int_a^c E dx = \int_a^b E dx + \int_b^c E dx = V_{ab} + V_{bc}$$

$$V_{ac} = V_{ab} + V_{bc}$$

V_{ab} = “voltage drop” across element # 1

V_{bc} = “voltage drop” across element # 2

Closing the loop:

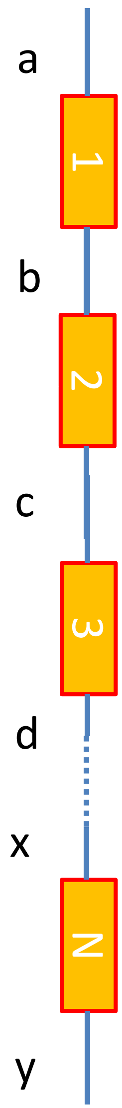


$$V_{ac} = V_{ab} + V_{bc}$$

V_{ab} = “voltage drop” across element # 1

V_{bc} = “voltage drop” across element # 2

Generalize loop to N-elements:



$$V_{ay} = V_{ab} + V_{bc} + V_{cd} + \ldots + V_{xy}$$

V_{ab} = “voltage drop” across element # 1

V_{bc} = “voltage drop” across element # 2

V_{cd} = “voltage drop” across element # 3

V_{xy} = “voltage drop” across element # N

Kirchoff's voltage law

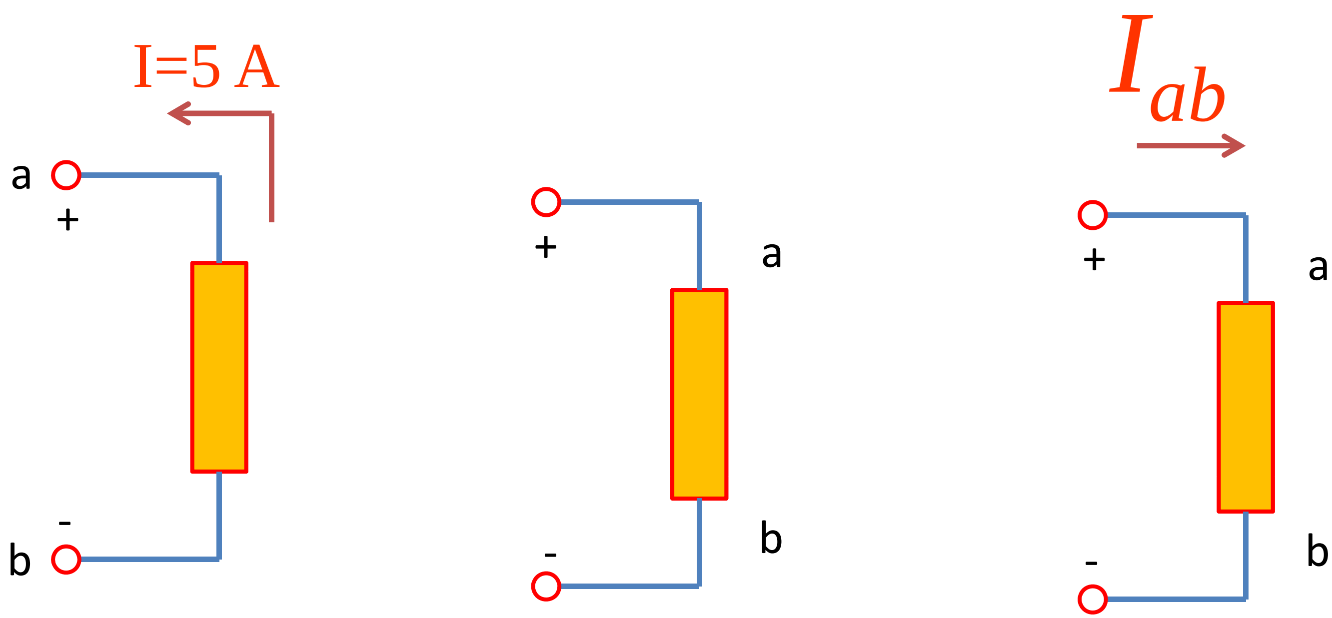
$$\sum_{n=1}^{N} v_n = 0$$

around *any* closed loop.

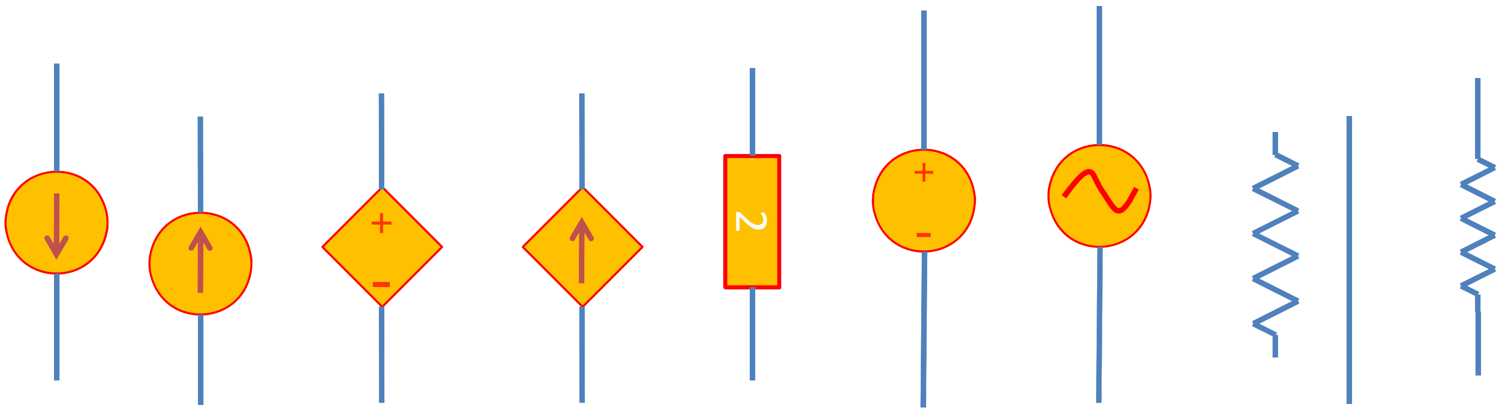
voltage *drops*

Questions?

Symbol library



Symbol library



Symbol & circuit library