Announcements:
1. HW # 2 will be posted online (due Wed)
2. Next lecture will be a review by TA
   to prepare for the midterm

EECS 70A: Network Analysis

Lecture 4

Review & agenda

Last lecture:

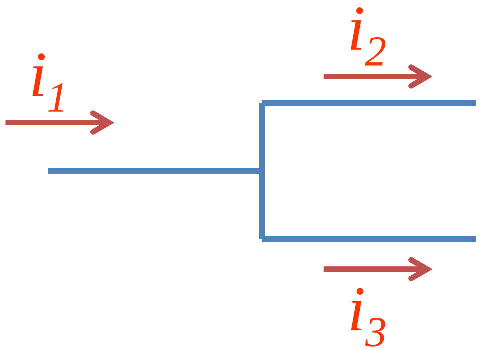
- Resistor circuits
  - Series
  - Parallel
- Kirchoff's current law (KCL)

Today

- Examples of KCL
- Kirchoff's voltage law (KVL)
- Examples with KVL, KCL, Ohm
- Δ-Y transformations

Kirchoff's current law

You have already seen:



$$i_1 = i_2 + i_3$$

Like water in a river…

More generally:

Sum of currents *entering* node = sum of currents *leaving* node.

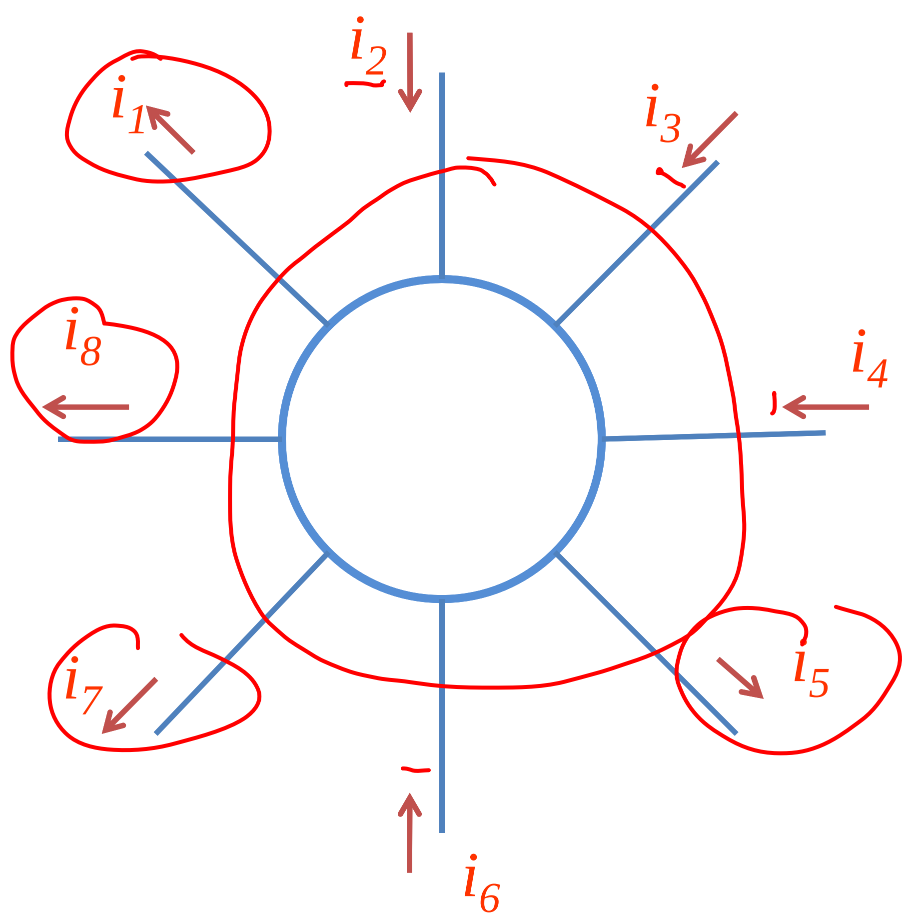
Stated as Kirchoff's current law (KCL):

$$\sum_{n=1}^{N} i_n = 0$$

Current *entering* a node: i_n positive
Current *leaving* a node: i_n negative

KCL examples

Find a relationship among $i_1, i_2, i_3, i_4 \ldots$ (instructor)



TOTAL CURRENT COMING IN

$i_2 + i_3 + i_4 + i_6$

OUT

$i_8 + i_1 + i_5 + i_7$

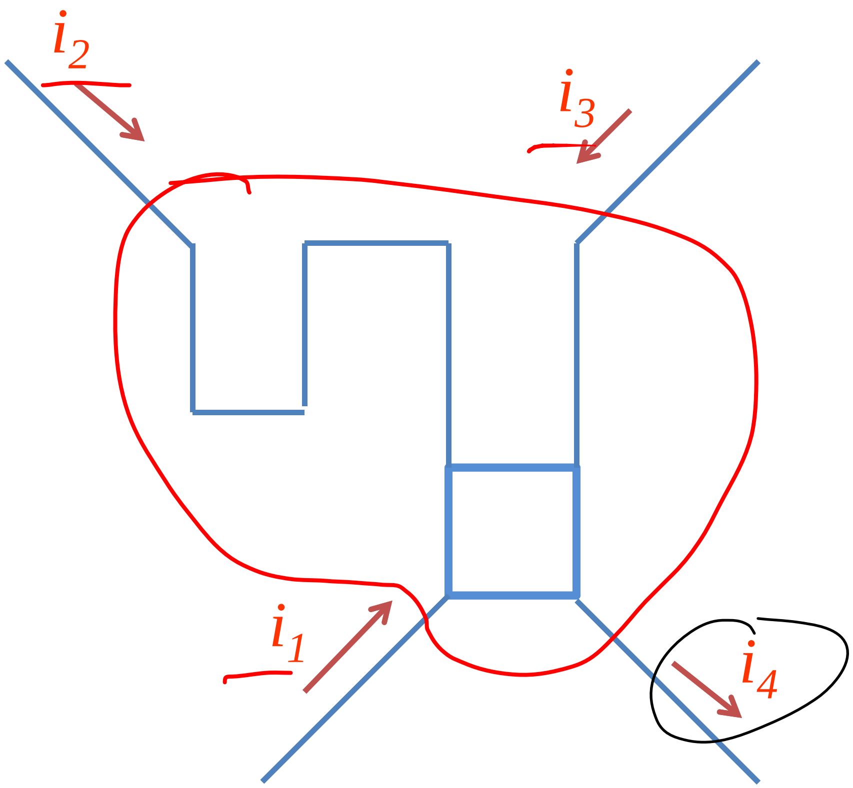
KCL

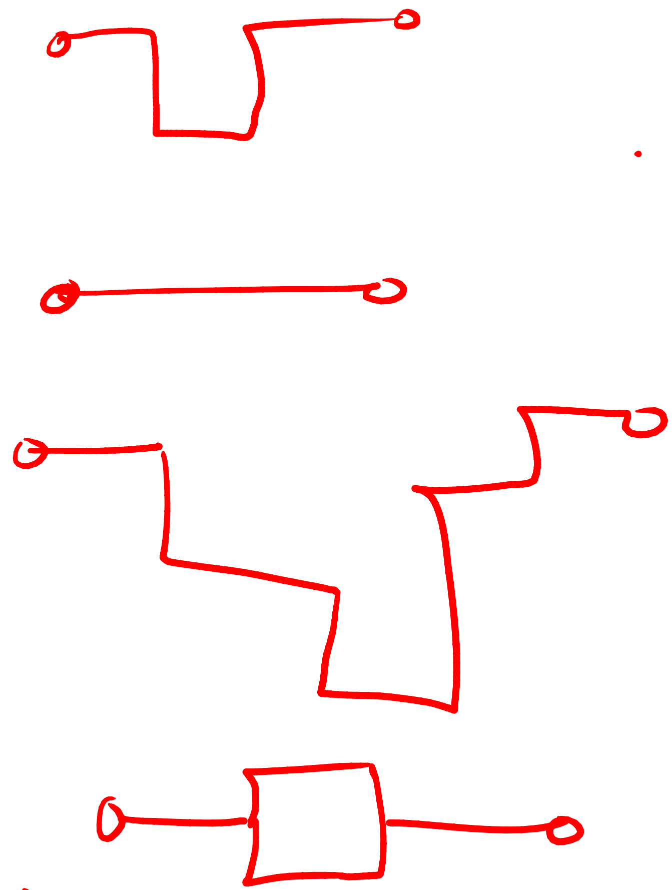
$i_2 + i_3 + i_4 + i_6$

$= i_8 + i_1 + i_5 + i_7$

KCL examples

Find a relationship among $i_1, i_2, i_3, i_4 ...$ (students)

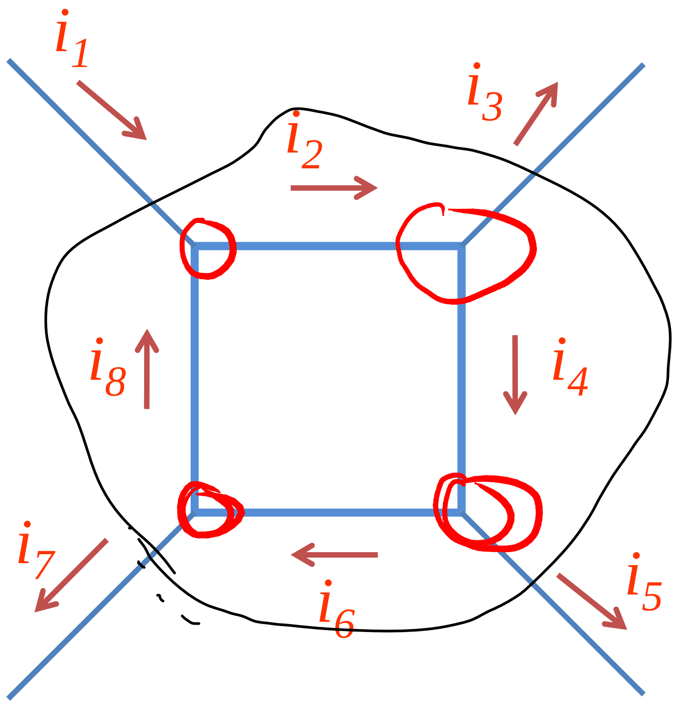




$$i_1 + i_2 + i_3 = i_4$$

KCL examples

Find a relationship among $i_1, i_2, i_3, i_4 \ldots$ (instructor)



IN i_2

OUT $i_3 + i_4$

KCL $i_2 = i_3 + i_4$

$i_1 + i_8 = i_2$

$i_4 = i_5 + i_6$

$i_6 = i_7 + i_8$

IN

i_1

OUT

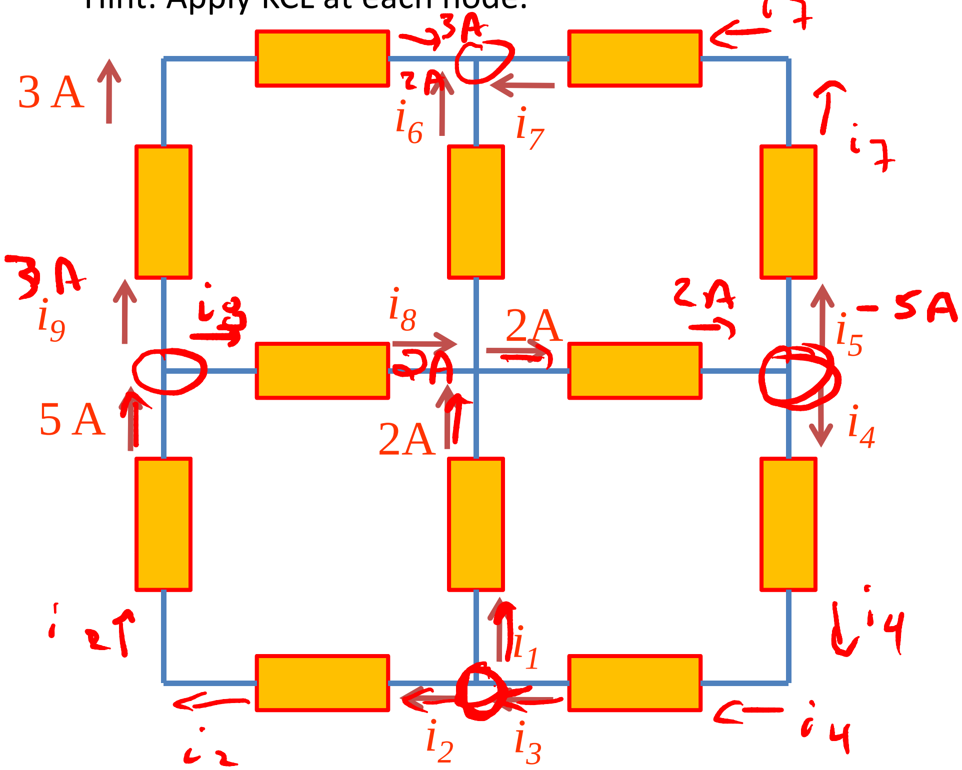
$i_3 + i_5 + i_7$

KCL $\Rightarrow i_1 = i_3 + i_5 + i_7$

KCL examples

a) Find the # of nodes in this circuit. (Instructor) 9

b) Find is i_1 thru i_9 in this circuit. (Instructor)
Hint: Apply KCL at each node.



$-i_3 = i_1 + i_2$

$i_4 + i_5 = 2A$

$i_9 = 3A$

i_8:

$5A = 3A + i_8$

$\Rightarrow i_8 = 2A$

$i_7 = i_5$

$i_6 = 2A$

$\underbrace{2A + 3A + i_7}_{IN} = \underbrace{0}_{OUT}$

$\Rightarrow i_7 = -5A$

IN = OUT

$2A = i_5 + i_4$

$2A = (-5A) + i_4$

$\Rightarrow i_4 = 7A$ $\quad i_2 = 2A$

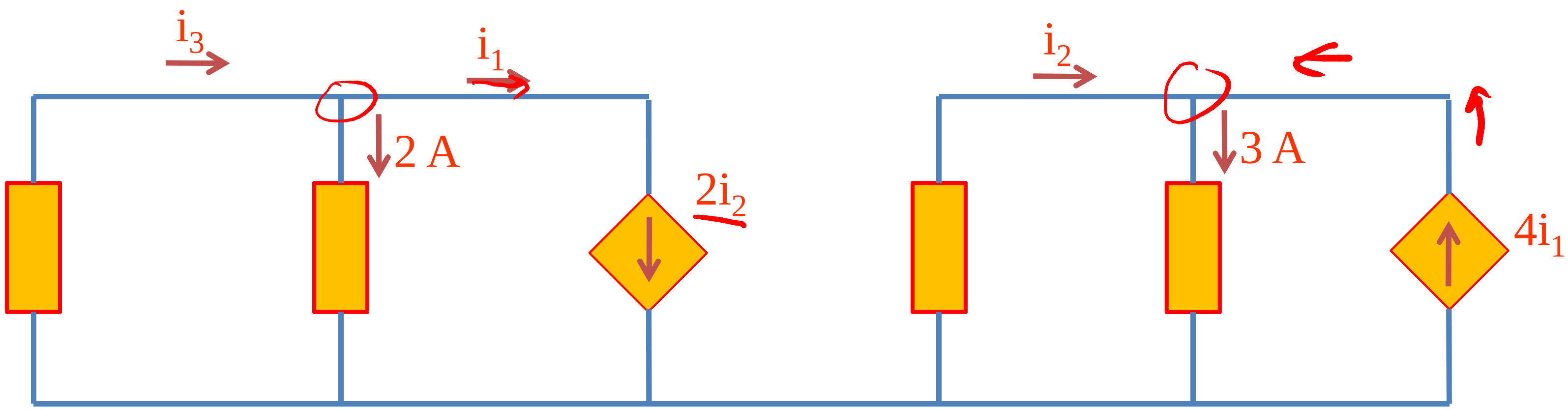
$i_3 = i_4 = 7A$

$i_1 = 5A$

KCL examples

a) Find the # of nodes in this circuit. (Instructor) 3

b) Find is i_1 i_2 & i_3 in this circuit. (Instructor)
Hint: Apply KCL at each node.



KCL

$i_3 = 2A + i_1$ (*)

$i_1 = 2 i_2$ (**)

$i_2 + 4 i_1 = 3A$ (***)

3 eq 3 unknowns

Sub. (**) into (***)

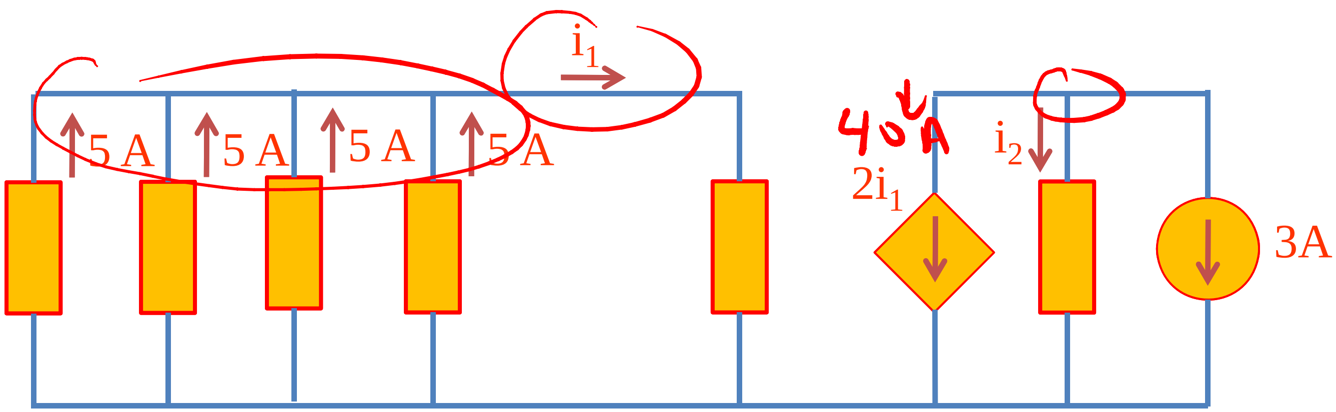
$i_2 + 4(2 i_2) = 3A \Rightarrow i_2 = \frac{1}{3} A$

sub into (**) $i_1 = \frac{2}{3} A$

sub into (*) $i_3 = 2A + \frac{2}{3} A = 2\frac{2}{3} A$

a) Find the # of nodes in this circuit. (students)
b) Find is i_1 & i_2 in this circuit. (students)
Hint: Apply KCL at each node.

KCL examples



$i_1 = 20A$

$i_2 = -43 A$

IN = OUT

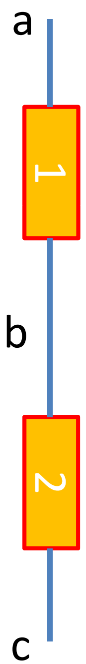
$0 = 40A + i_2 + 3A$

$\Rightarrow i_2 = -43A$

Questions?

Voltage addition in circuits

From lecture #2:



$$V_{ab} \equiv \int_a^b E dx$$

$$V_{bc} \equiv \int_b^c E dx$$

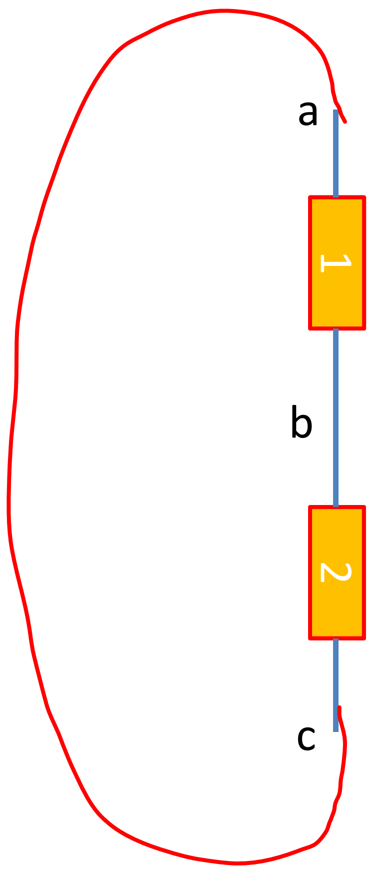
$$\Rightarrow V_{ac} \equiv \int_a^c E dx = \int_a^b E dx + \int_b^c E dx = V_{ab} + V_{bc}$$

$$V_{ac} = V_{ab} + V_{bc}$$

V_{ab} = "voltage drop" across element # 1

V_{bc} = "voltage drop" across element # 2

Closing the loop:



$$V_{ac} = V_{ab} + V_{bc}$$

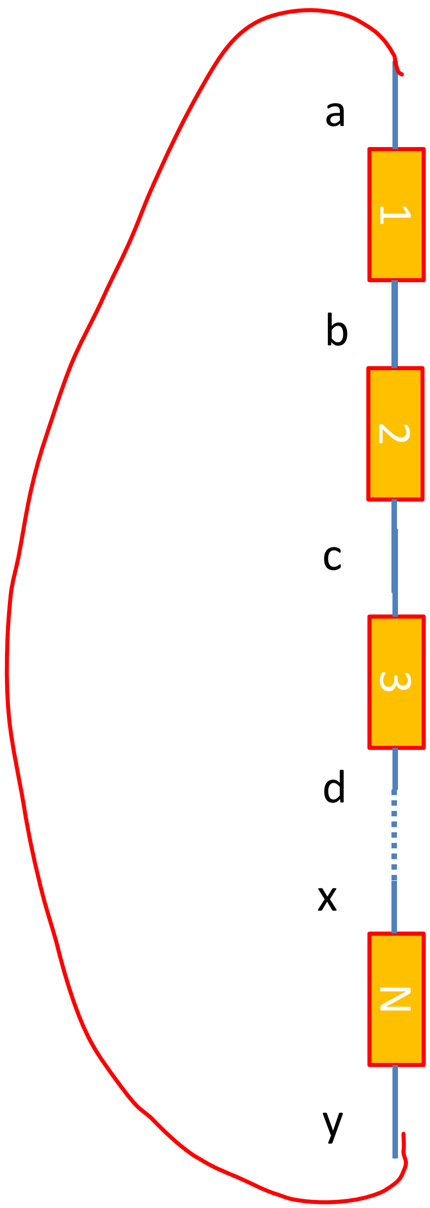
V_{ab} = "voltage drop" across element # 1

V_{bc} = "voltage drop" across element # 2

$V_{ac} = 0$

$\Rightarrow V_{ab} + V_{bc} = 0$ KVL

Generalize loop to N-elements:



$$V_{ay} = V_{ab} + V_{bc} + V_{cd} + \ldots + V_{xy}$$

V_{ab} = “voltage drop” across element # 1

V_{bc} = “voltage drop” across element # 2

V_{cd} = “voltage drop” across element # 3

V_{xy} = “voltage drop” across element # N

$V_{ay} = 0$

$V_{ab} + V_{bc} + V_{cd} + \ldots V_{xy} = 0$

KVL

Kirchoff's voltage law

$$\sum_{n=1}^{N} v_n = 0$$

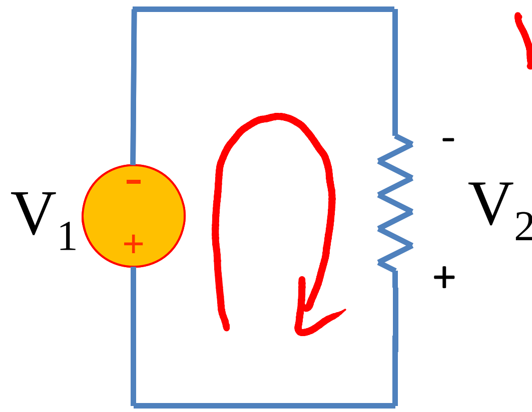
around *any* closed loop.

voltage *drops*

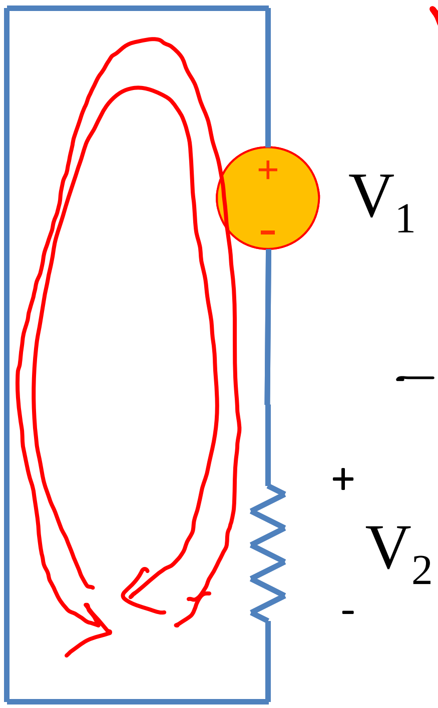
If the voltage is *dropping* as you go around the loop, the voltage drop v_n is *positive*.

KVL application

If the voltage is *dropping* as you go around the loop, the voltage drop v_n is *positive*.

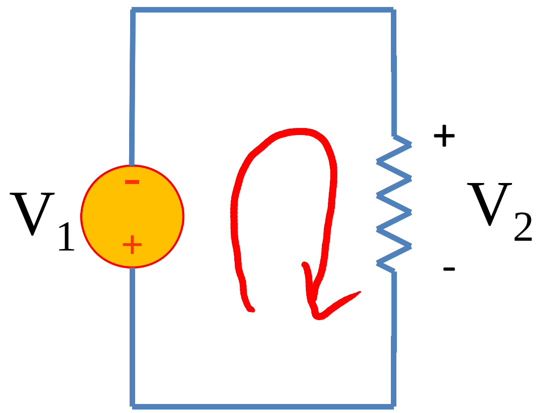


$V_1 - V_2 = 0$

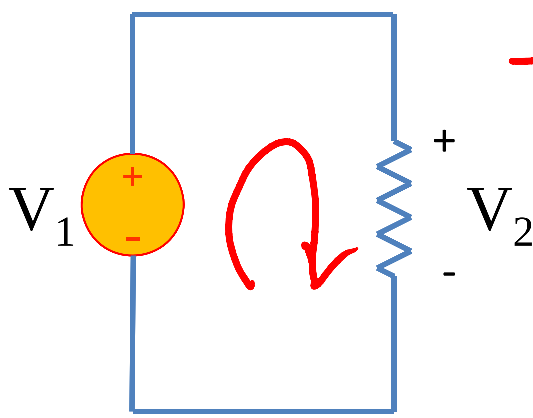


$V_1 + V_2 = 0$

$-V_2 - V_1 = 0$



$V_1 + V_2 = 0$

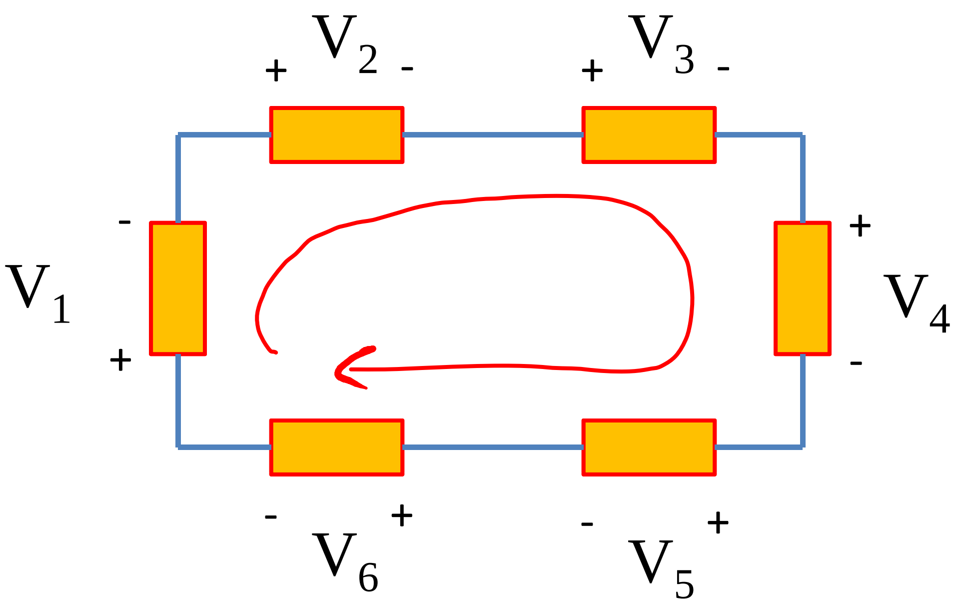


$-V_1 + V_2 = 0$

KVL examples

If the voltage is *dropping* as you go around the loop, the voltage drop v_n is *positive*.

Apply KVL to the circuit below (instructor)

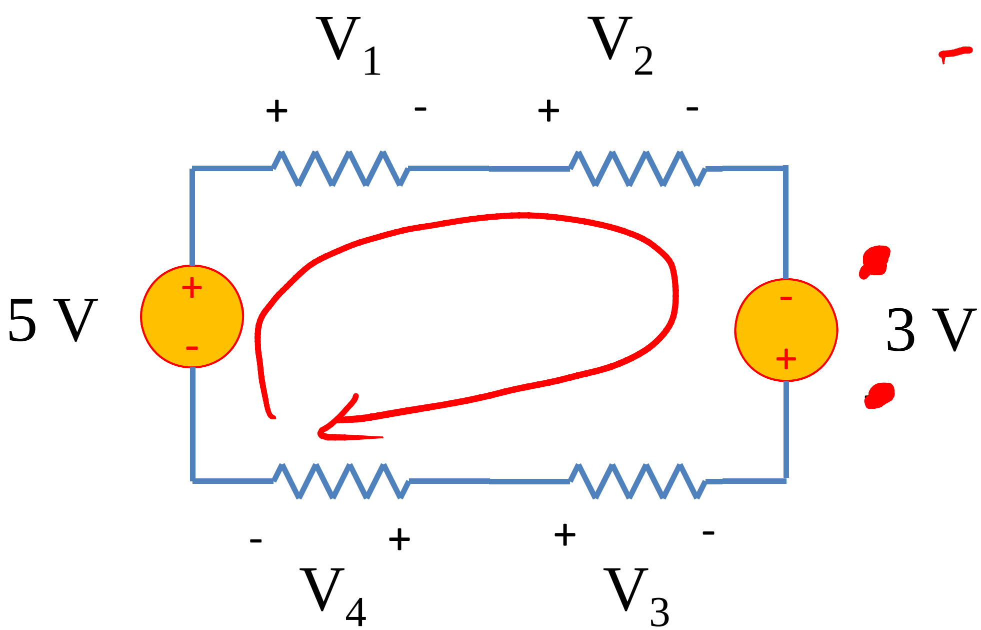


$V_1 + V_2 + V_3 + V_4 + V_5 + V_6 = 0$

KVL examples

If the voltage is *dropping* as you go around the loop, the voltage drop v_n is *positive*.

Apply KVL to the circuit below (instructor)

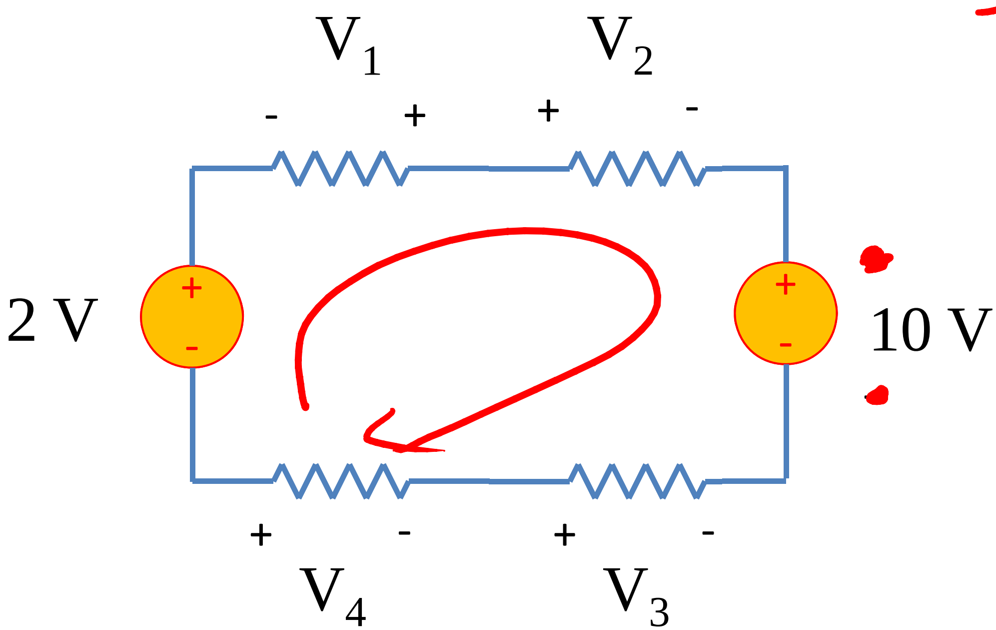


$$-5V + V_1 + V_2 - 3V - V_3 + V_4 = 0$$

KVL examples

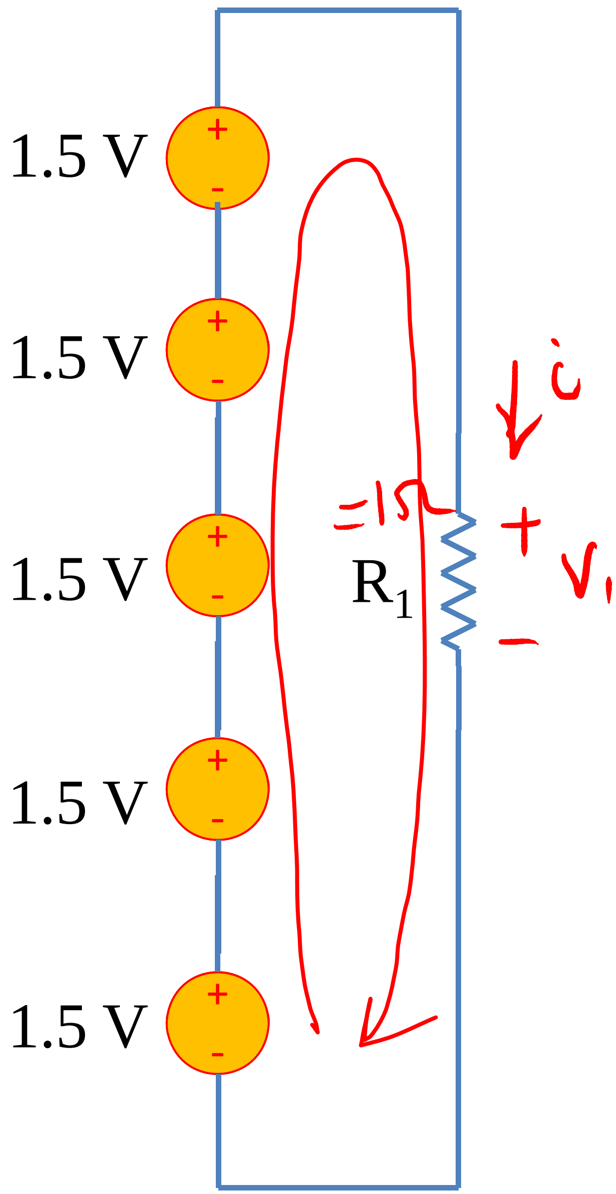
If the voltage is *dropping* as you go around the loop, the voltage drop v_n is *positive*.

Apply KVL to the circuit below (student)



$$-2V - V_1 + V_2 + 10V - V_3 - V_4 = 0$$

Find the voltage across R1 (student)



$-1.5V - 1.5V - 1.5V - 1.5V - 1.5V$

$+V_1 = 0$

$\Rightarrow V_1 = 7.5V$

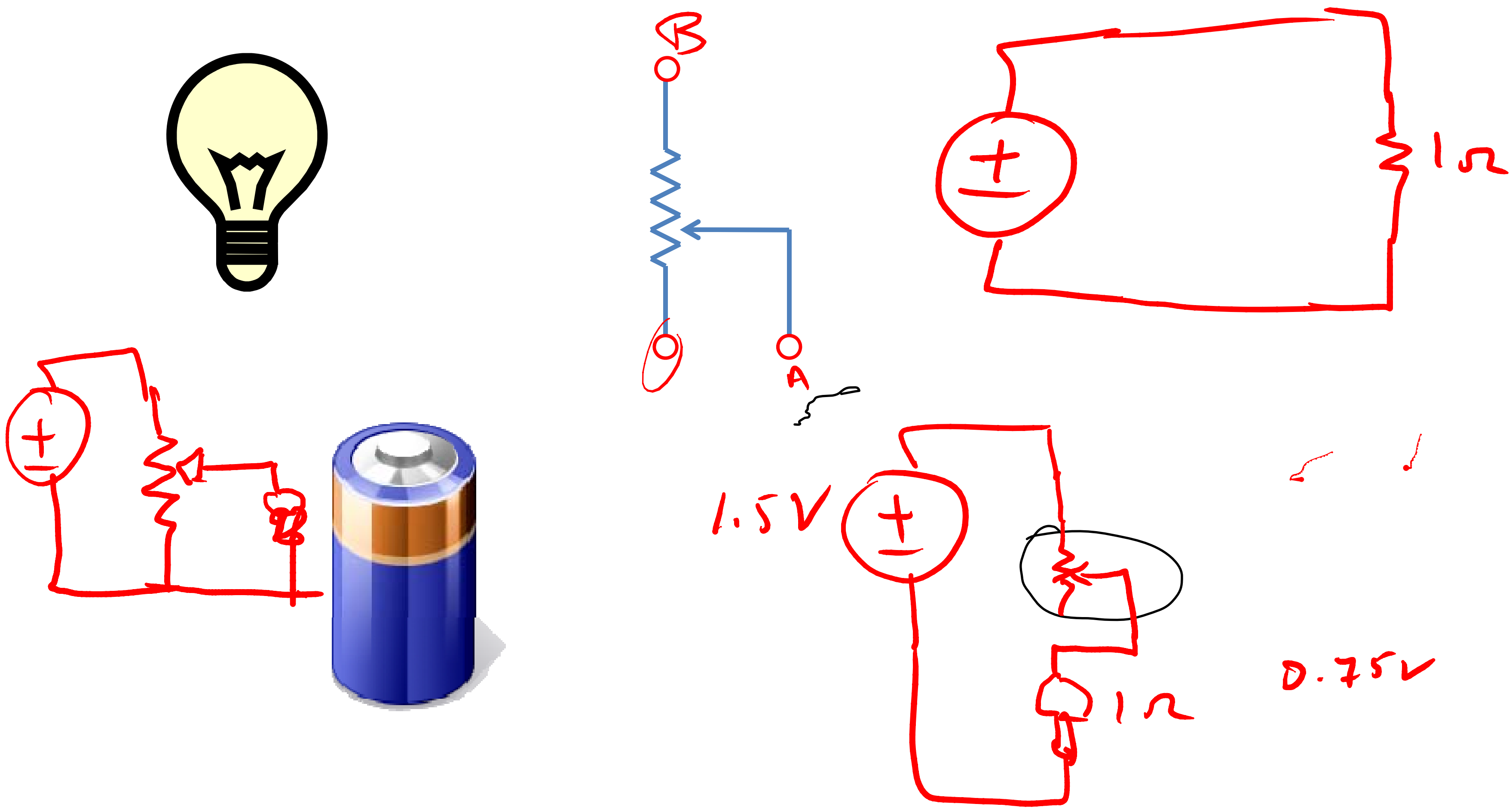
$i_1 = 7.5A$

Demo…

Dimming circuit

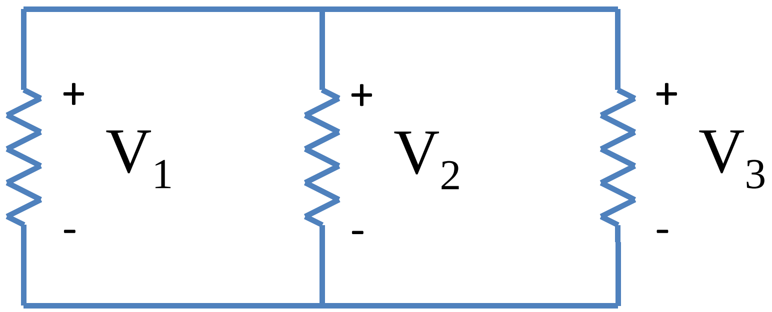
Given the four elements below:

1. Design a circuit that continuously dims the light.
   (It needs to go from completely dim to completely bright.)
2. Calculate the power supplied by the battery when the bulb is brightest and when it is off.

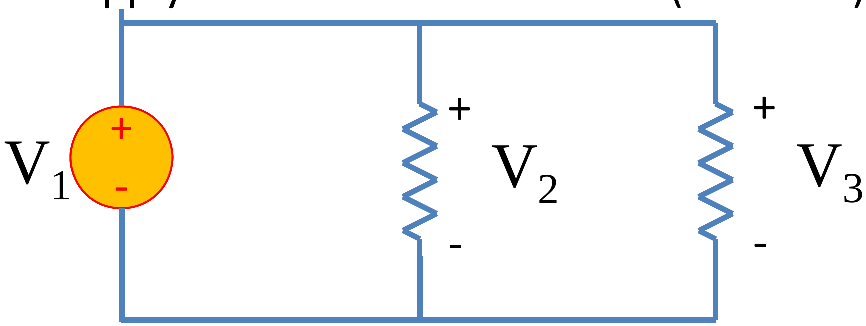


KVL examples

Apply KVL to the circuit below (instructor)

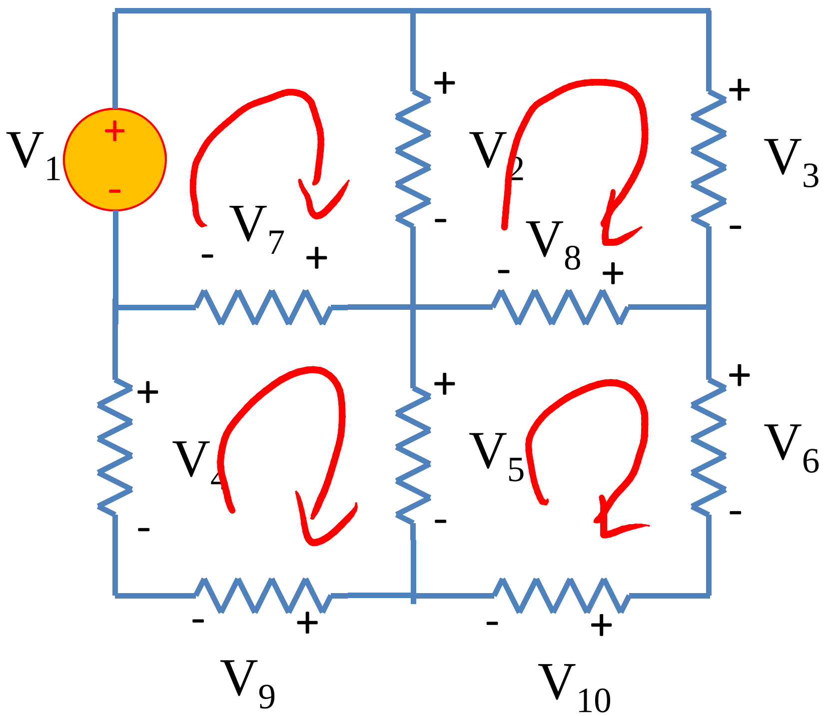


Apply KVL to the circuit below (students)



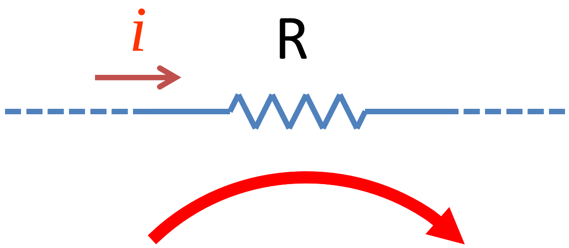
KVL examples

Apply KVL to the circuit below (instructor)

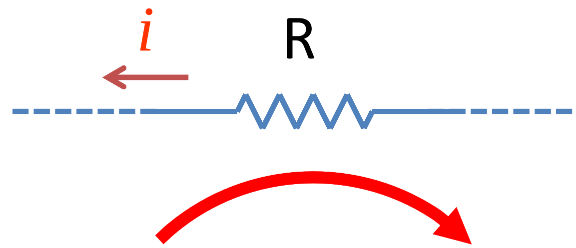


Questions?

Sign of voltage drop



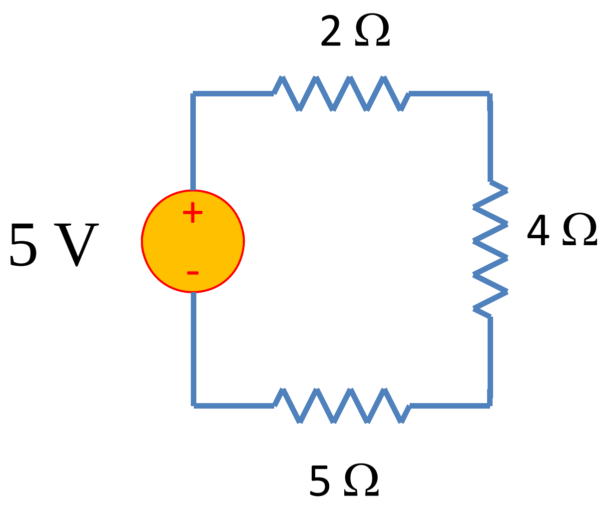
Voltage drop
= + i R



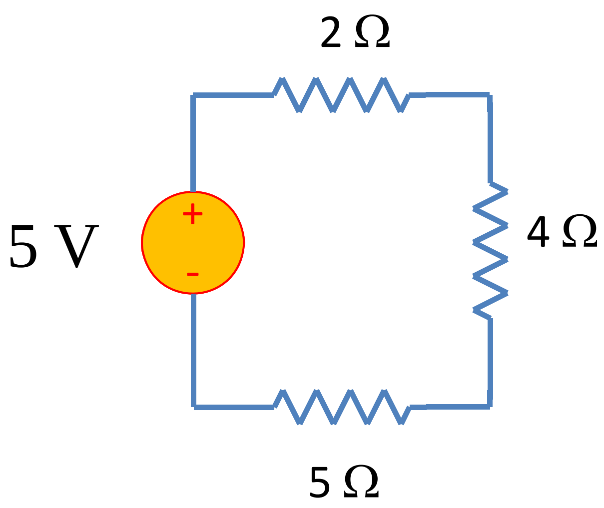
Voltage drop
= - i R

Combining KVL + Ohm

Find the current flowing in this circuit (instructor):

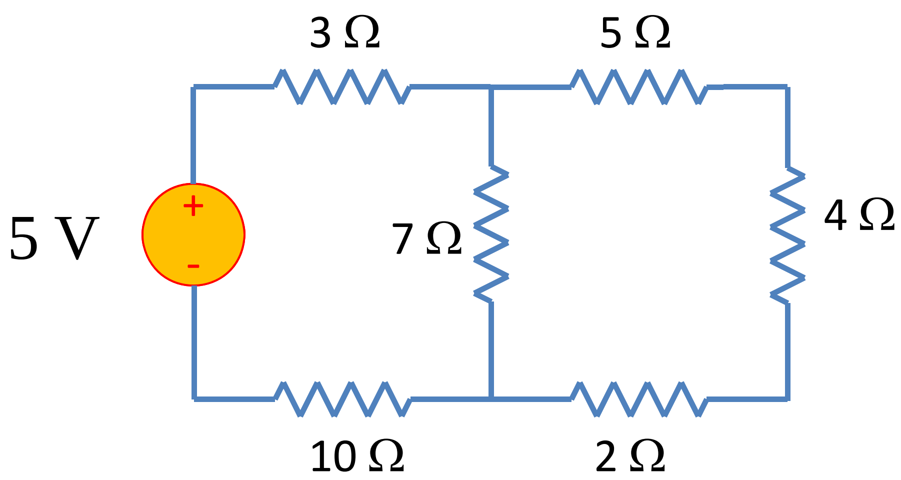


Find the current flowing in this circuit (instructor):



Combining KVL + KCL + Ohm

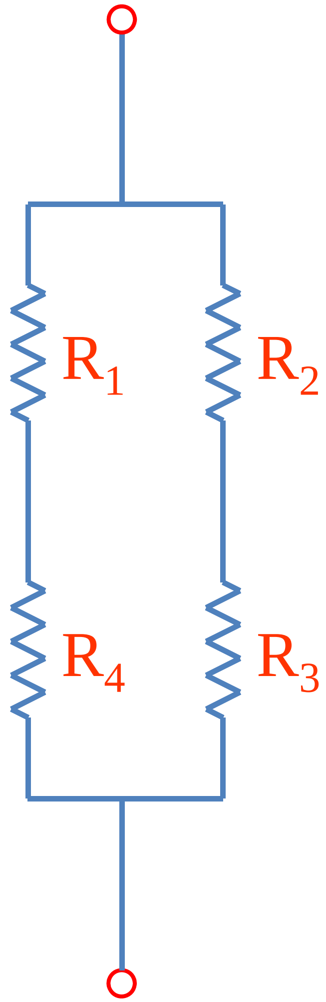
Find the currents flowing in the circuit below (instructor):



Questions?

Example problems

Solve for R_{eq}. (students).



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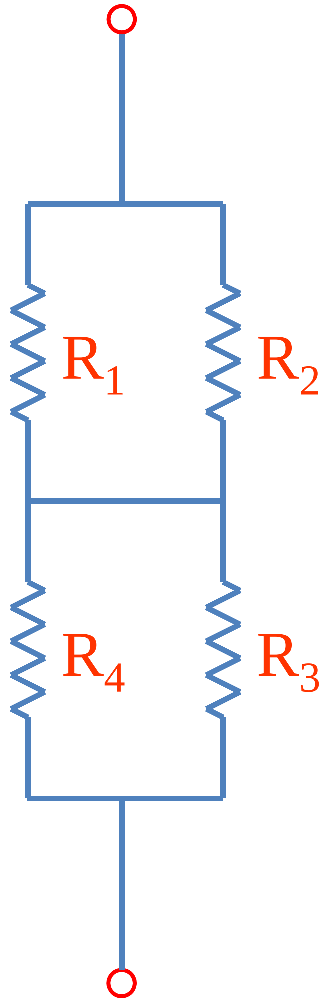

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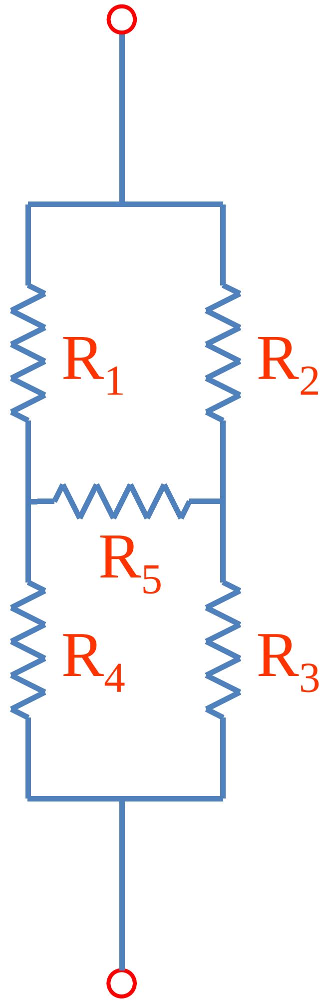
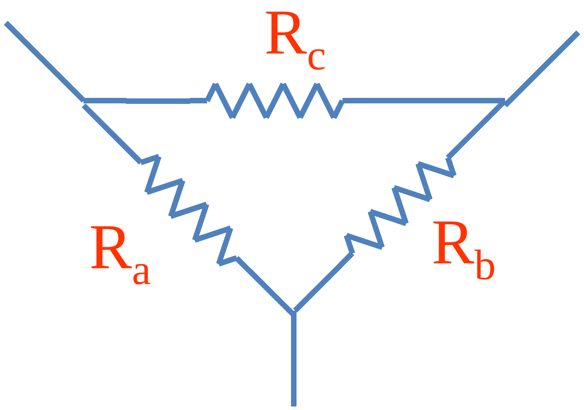


Δ-Y transformations



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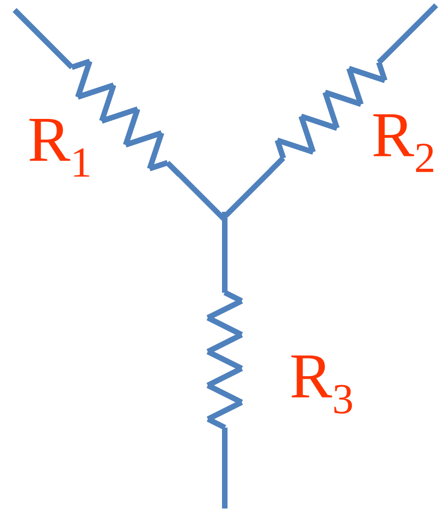


If:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

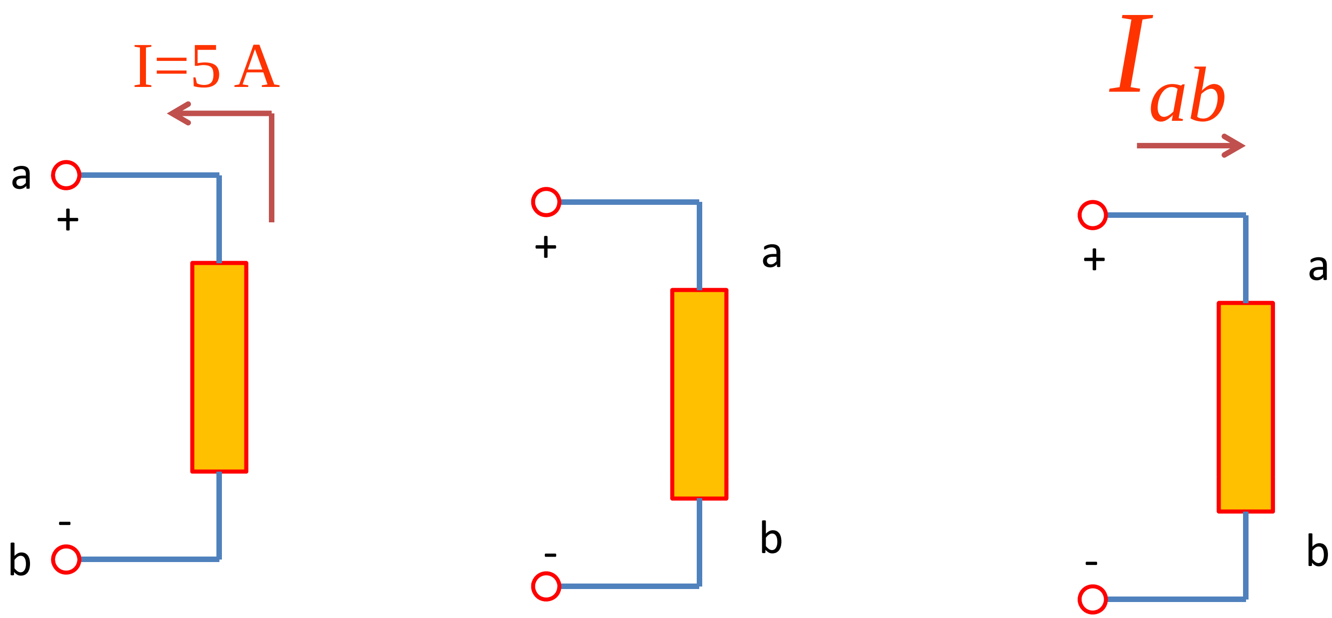
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

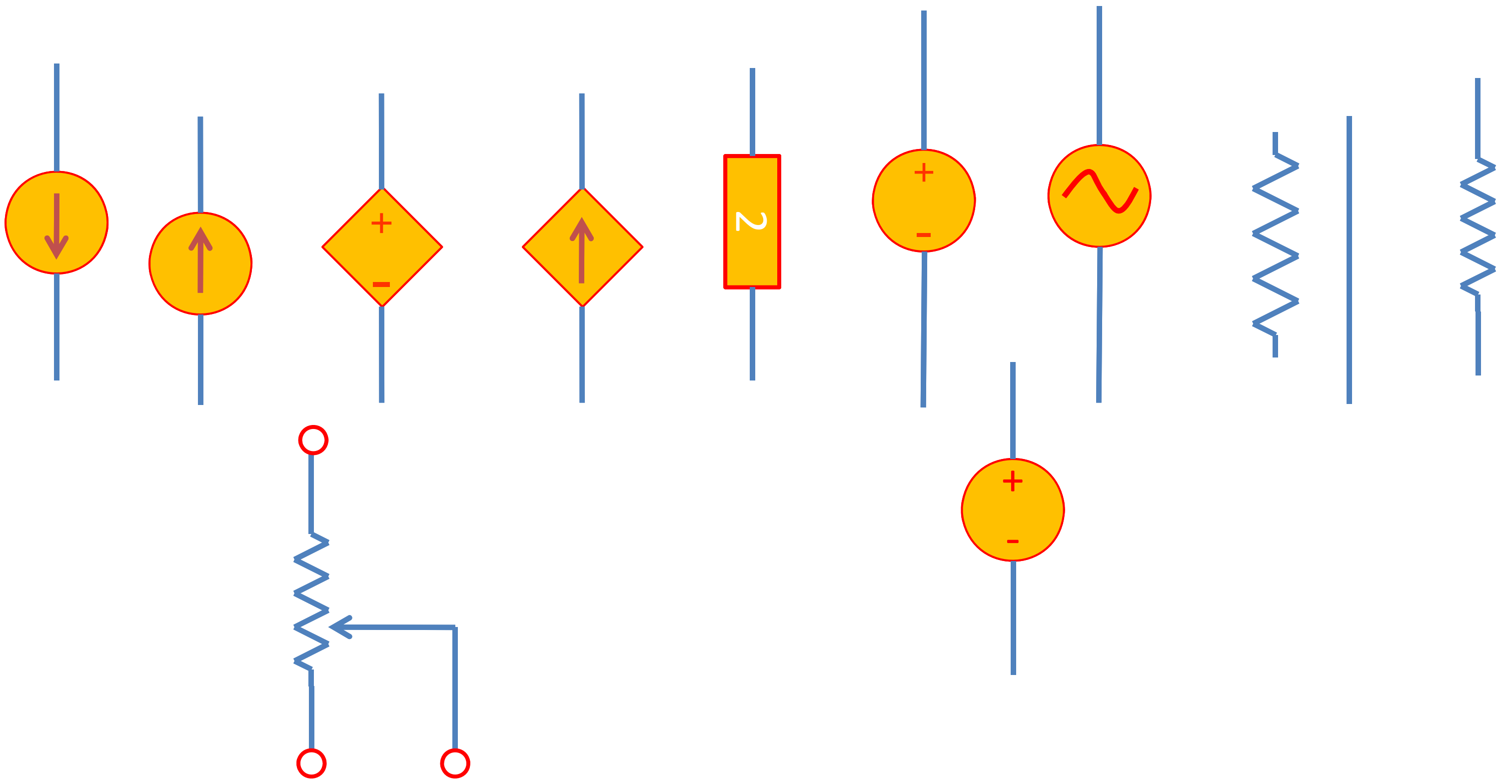
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Symbol library



Symbol library



Symbol & circuit library