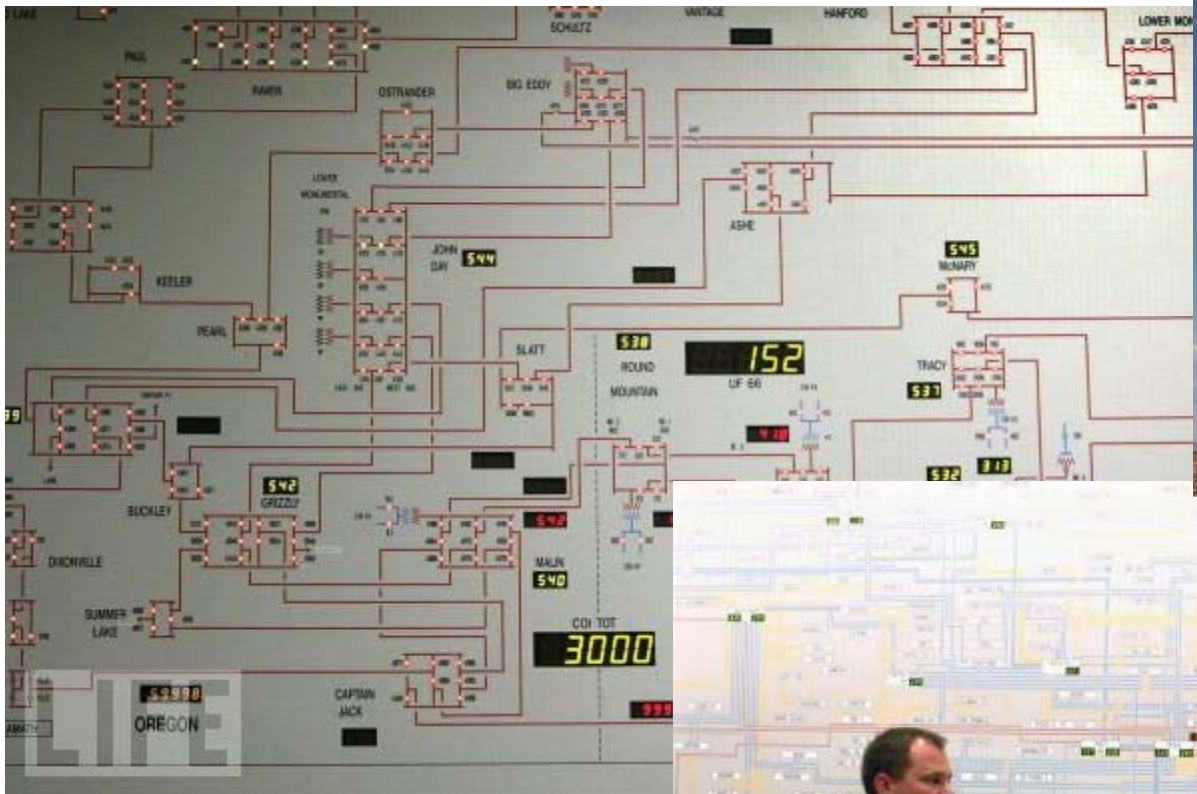


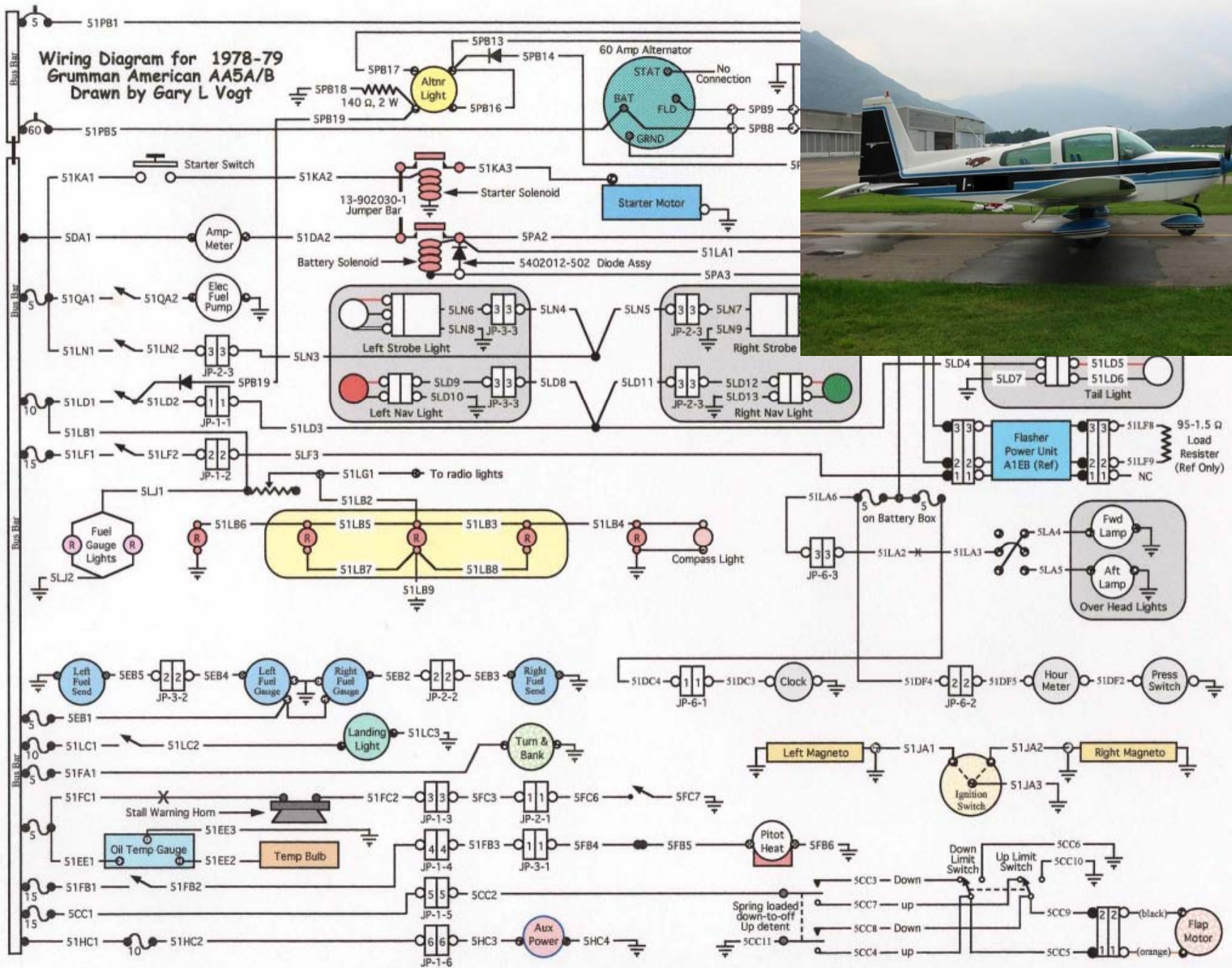
Announcements:

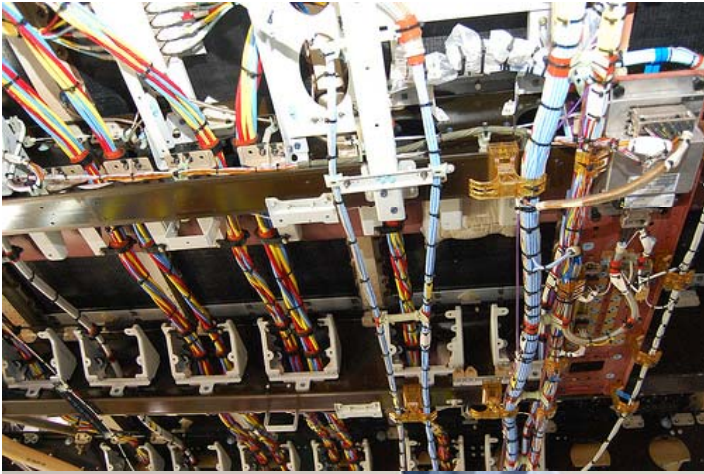
1. Announcements

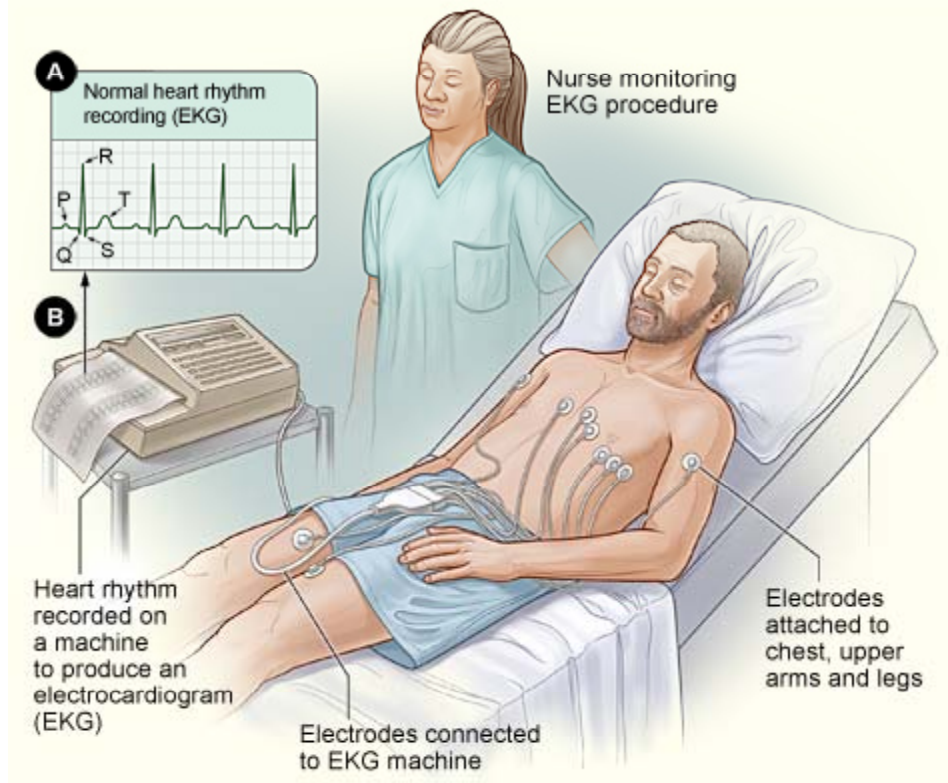
EECS 70A: Network Analysis

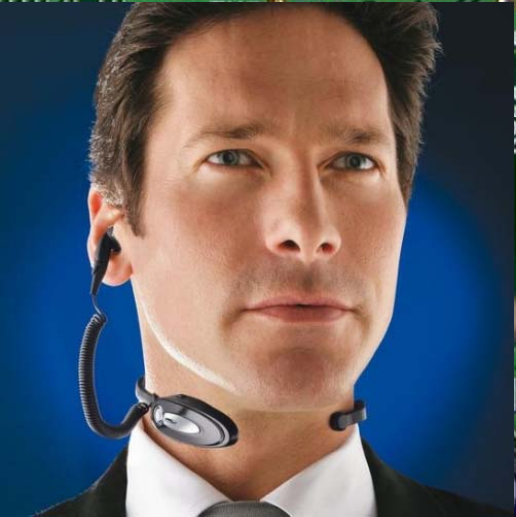
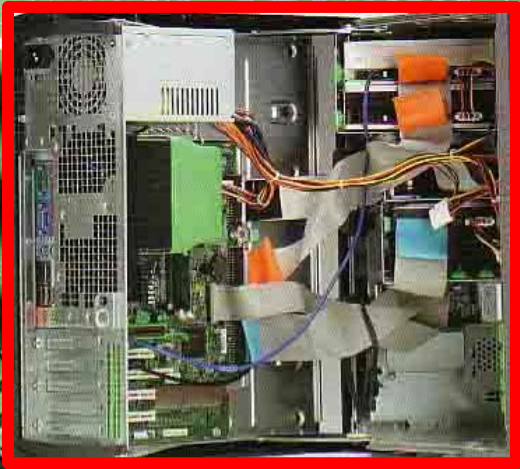
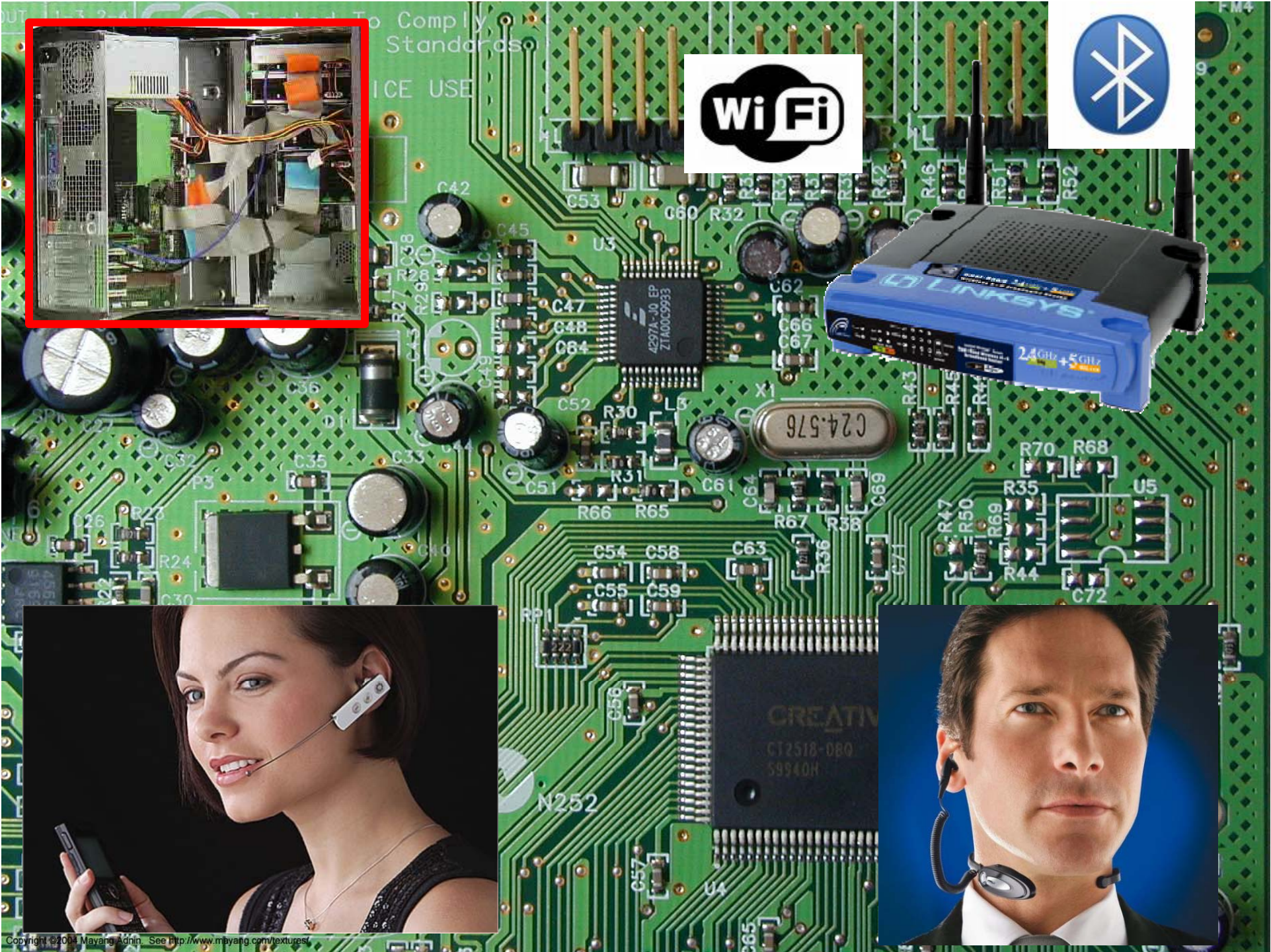
Lecture 5



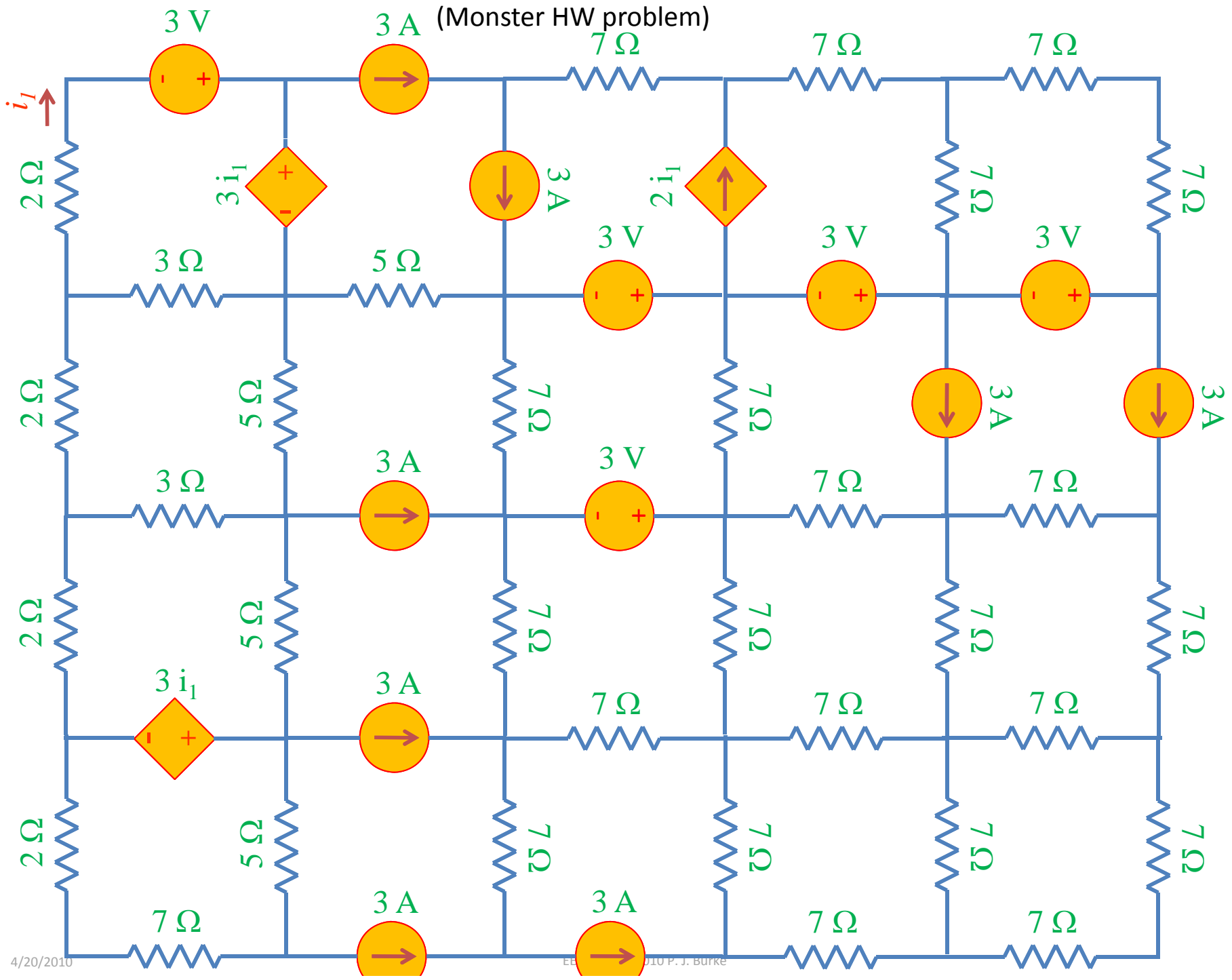


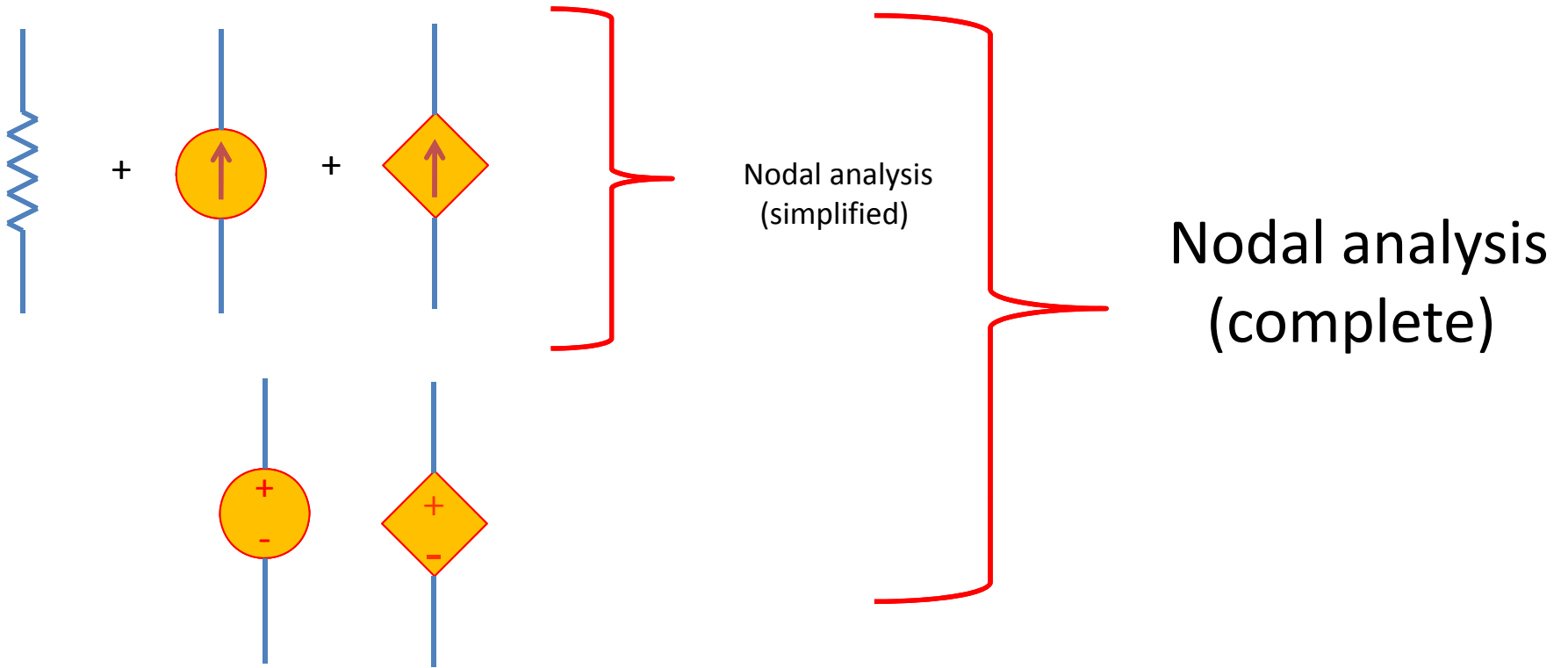




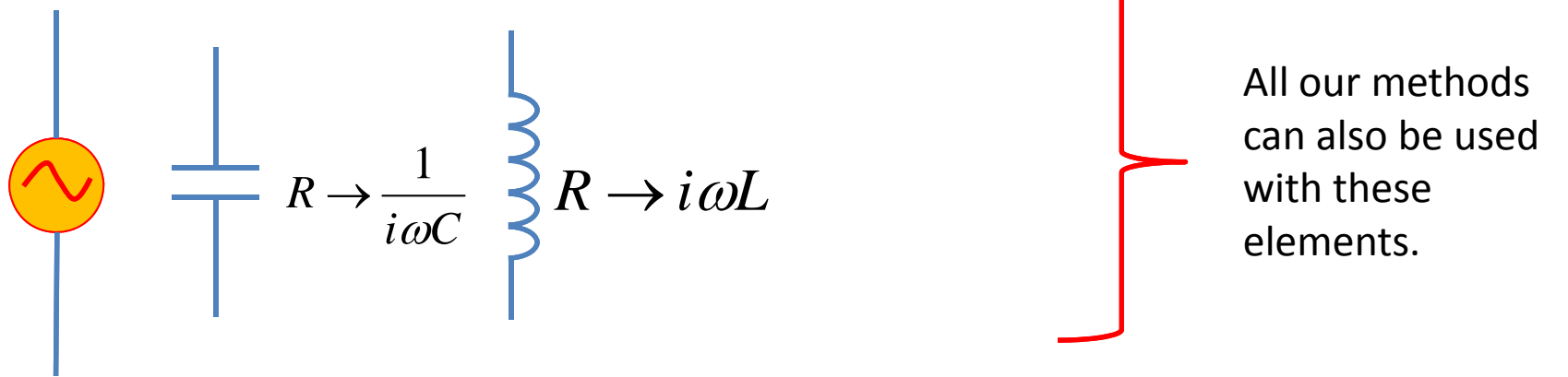


3 A (Monster HW problem)



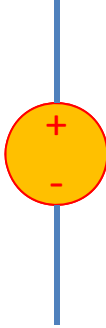


Later in course:





+



+



Mesh analysis
(simplified)

Mesh analysis
(complete)

Diode



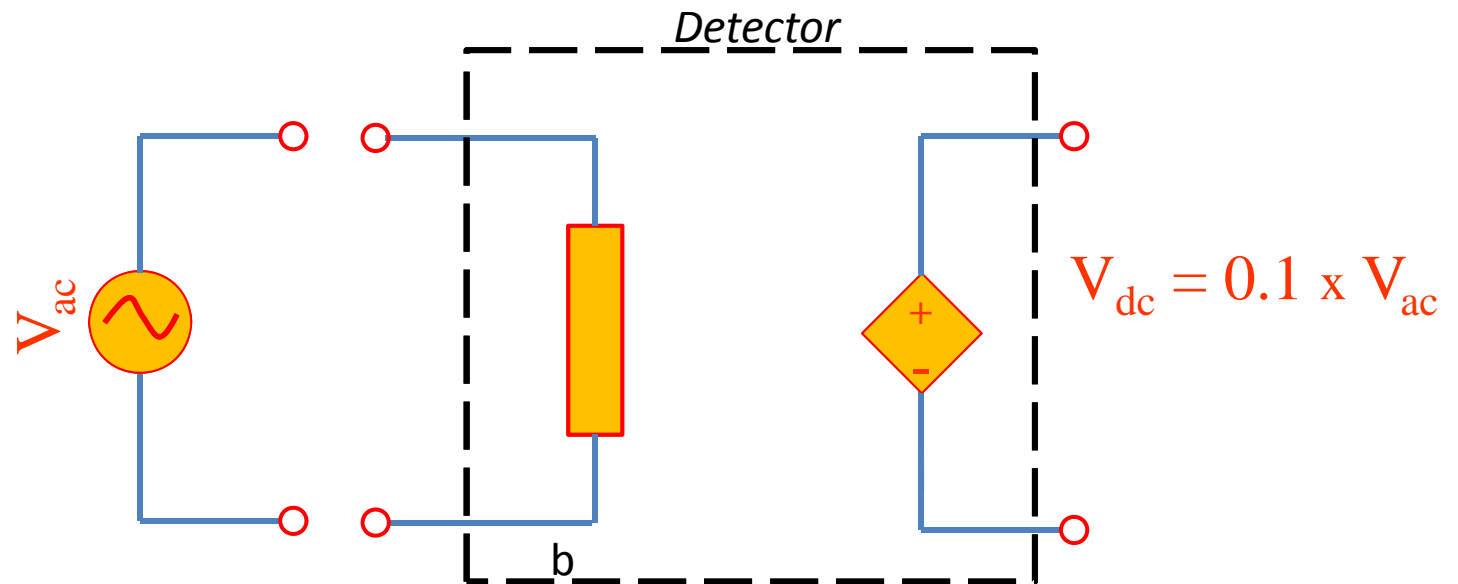
$$I = I_0 \left(e^{qV/kT} - 1 \right)$$

Nodal/mesh analysis won't work on this but...

Solar cell is a diode.
Behaves like:

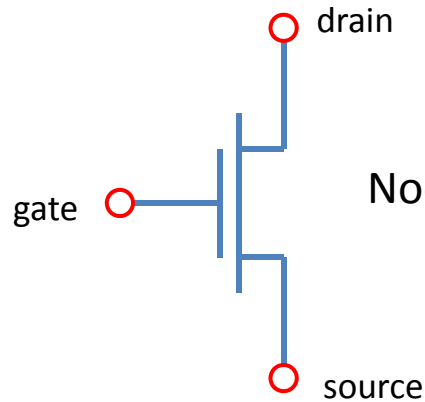


Radio wave detector is a diode.
Behaves like:



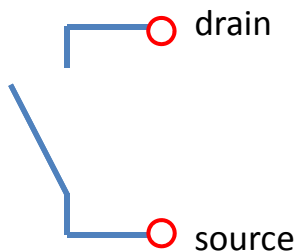
Nodal/mesh analysis works on these equivalent circuits!

Transistors



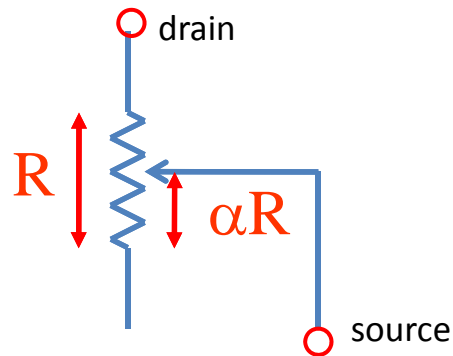
Nodal/mesh analysis won't work on this but...

In some cases behaves like *switch*:



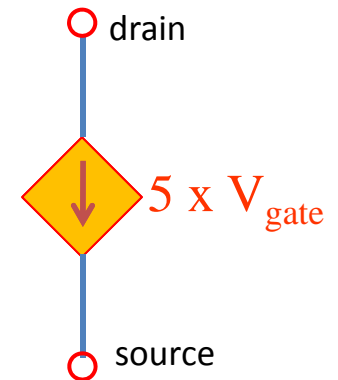
V_{gate}	Switch
+ 5 V	Open
0 V	Closed

In some cases behaves like *potentiometer*:



$$R_{sd} = (\text{constant}) \times V_{gate}$$

In some cases behaves like *VCCS*:



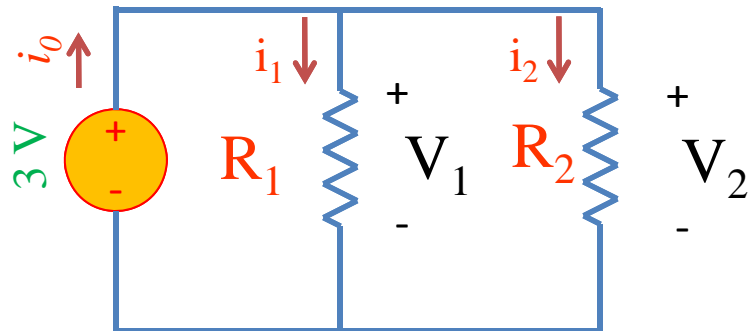
Nodal/mesh analysis works on these equivalent circuits!

Nodal vs. mesh analysis

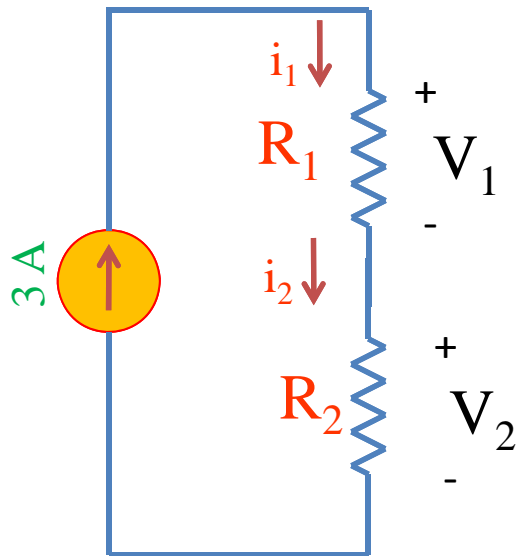
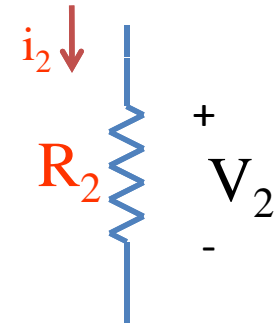
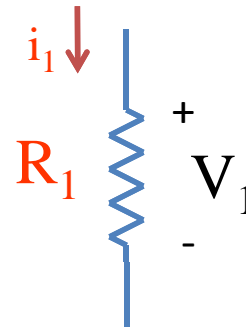
Consider 2 examples, each with 2 resistors.

For both problems, find i_1 , i_2 , V_1 , V_2 .

Is it easier to solve for the currents or the voltages first?



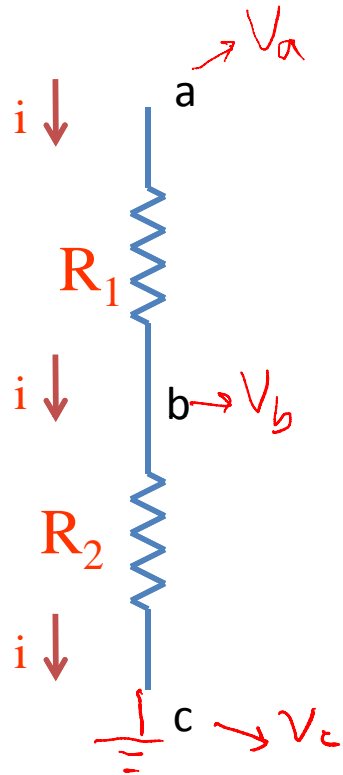
$$V_1 = V_2 = 3V$$



$$i_1 = i_2 = 3A$$

Notation: two elements in series

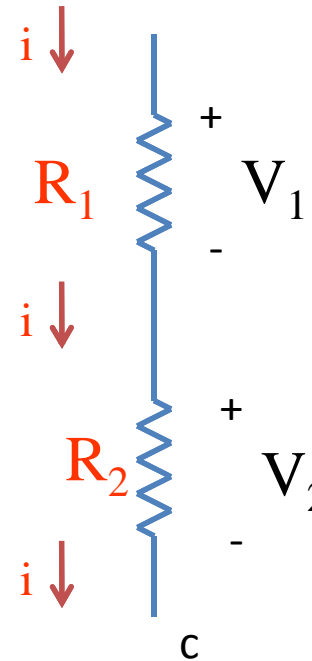
Textbook chapter 2 notation:



$$i = \frac{V_{ab}}{R_1} = \frac{V_a - V_b}{R_1}$$

$$i = \frac{V_{bc}}{R_2} = \frac{V_b - V_c}{R_2}$$

V_{ab} is the voltage drop across resistor 1



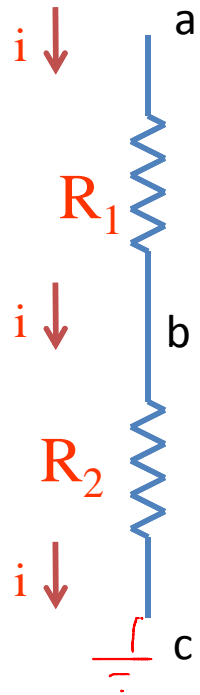
$$i = \frac{V_1}{R_1}$$

$$i = \frac{V_2}{R_2}$$

V_1 is the voltage drop across resistor 1

Nodal analysis

1. Define a reference node.
2. Label remaining nodes



$$i = \frac{V_{ab}}{R_1}$$

$$i = \frac{V_{bc}}{R_2}$$

$$V_c = 0v, V_a = 5v, V_b = 3v$$

$$V_{ab} = V_a - V_b = 5v - 3v = 2v$$

$$V_{bc} = 3v - 0v = 3v$$

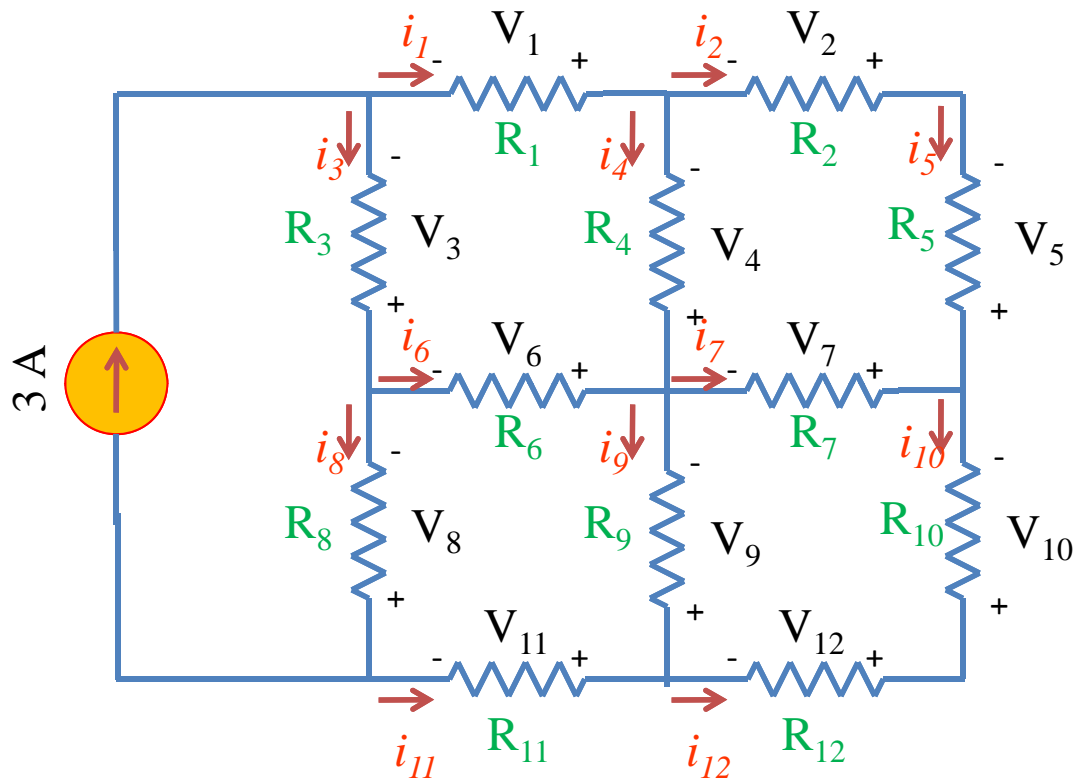
$$V_b = 0, V_{ab} = 2v, V_{bc} = 3v$$

$$V_{ab} = V_a - V_b = 2v \Rightarrow V_a = 2v$$

$$V_{bc} = V_b - V_c = 3v \Rightarrow V_c = -3v$$

V_{ab} is the voltage drop
across resistor 1

Example (Ch. 2 notation)



Label voltages as drops across resistors.

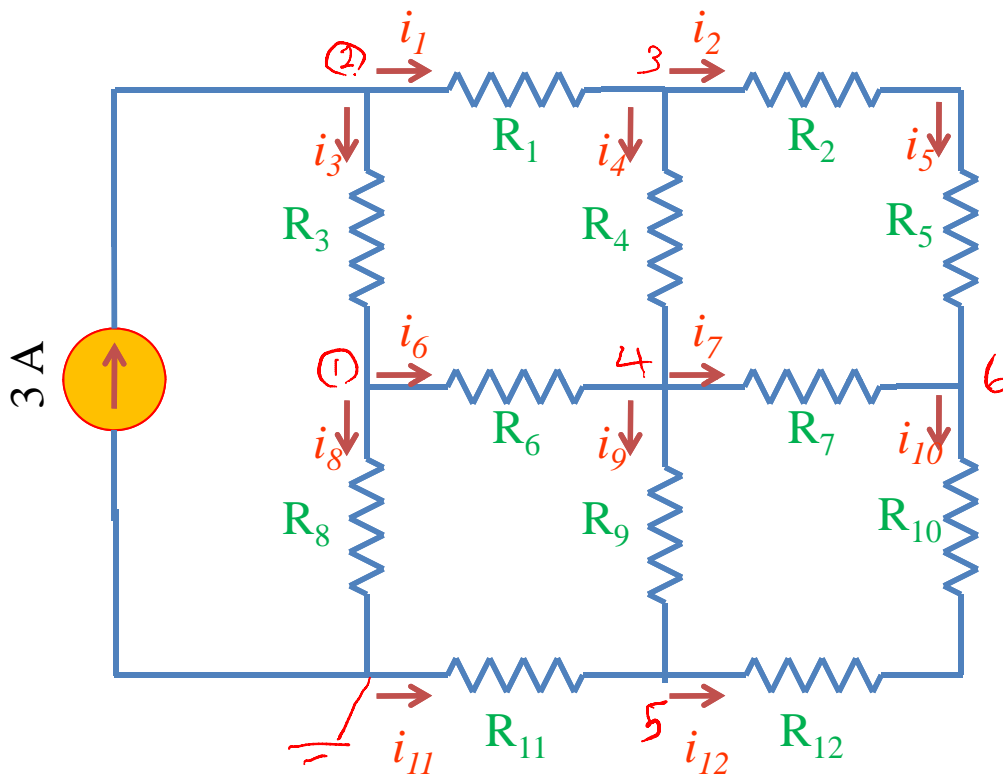
Typical notation:

V_1 is voltage drop across R_1 .

i_1 is current through R_1 .

Same circuit: Nodal analysis

1. Define a reference node.
2. Label remaining nodes.
3. Apply KCL + ohm.



$$\textcircled{1} \quad i_8 + i_6 - i_3 = 0$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\frac{V_1 - 0}{R_8} + \frac{V_1 - V_4}{R_6} - \frac{V_2 - V_1}{R_3} = 0$$

n nodes \rightarrow $n-1$ KCL

Typical notation:

i_1 is current through R_1 . (Same as before)

V_1 is voltage of node 1 relative to reference node. (Different from before)

Kramer's rule

Solve for x, y, z in terms of known constants $a_{1-3}, b_{1-3}, c_{1-3}, d_{1-3}$:

$$a_1x + b_1y + c_1z = d_1$$

x, y, z are node voltages

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}, \quad y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\Delta}$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\Delta}$$

Determinants:

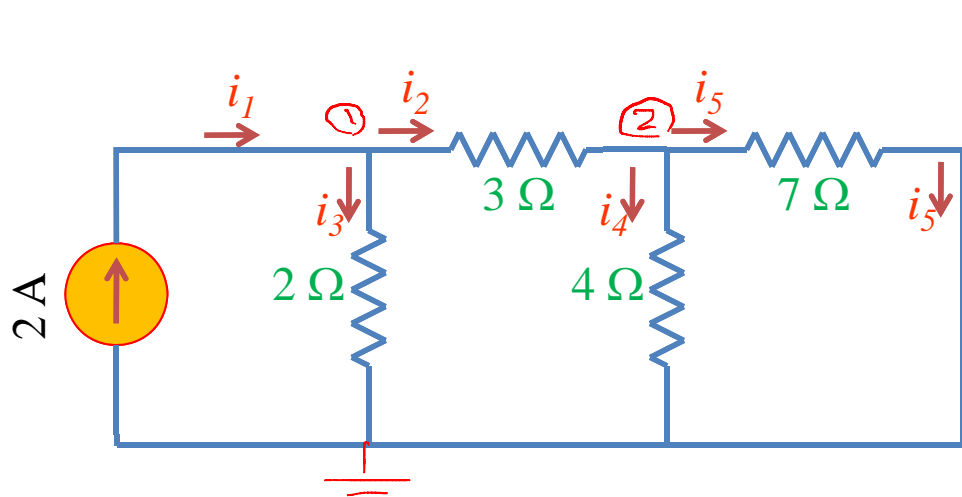
Solve for x,y,z in terms of known constants $a_{1-3}, b_{1-3}, c_{1-3}, d_{1-3}$:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \cancel{ab} - \cancel{cd} \quad ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \dots \quad a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{matrix} + \\ + \\ + \end{matrix}$$

Nodal analysis example



$$\textcircled{1} \quad -i_1 + i_3 + i_2 = 0$$

$$\star \quad 2A + \frac{V_1}{2\Omega} + \frac{V_1 - V_2}{3\Omega} = 0$$

$$\textcircled{2} \quad -i_2 + i_4 + i_5 = 0$$

$$\star \star \quad \frac{V_2 - V_1}{3\Omega} + \frac{V_2}{4\Omega} + \frac{V_2}{7\Omega} = 0$$

$$\star \rightarrow V_1 \left(\frac{1}{2} + \frac{1}{3} \right) - \frac{V_2}{3} = -2 \xrightarrow{\times 6} \begin{cases} 5V_1 - 2V_2 = -12 \\ -28V_1 + 61V_2 = 0 \end{cases}$$

$$\star \star \rightarrow \frac{-V_1}{3} + V_2 \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{7} \right) = 0 \xrightarrow{\times 84} \begin{cases} 5V_1 - 2V_2 = -12 \\ -28V_1 + 61V_2 = 0 \end{cases}$$

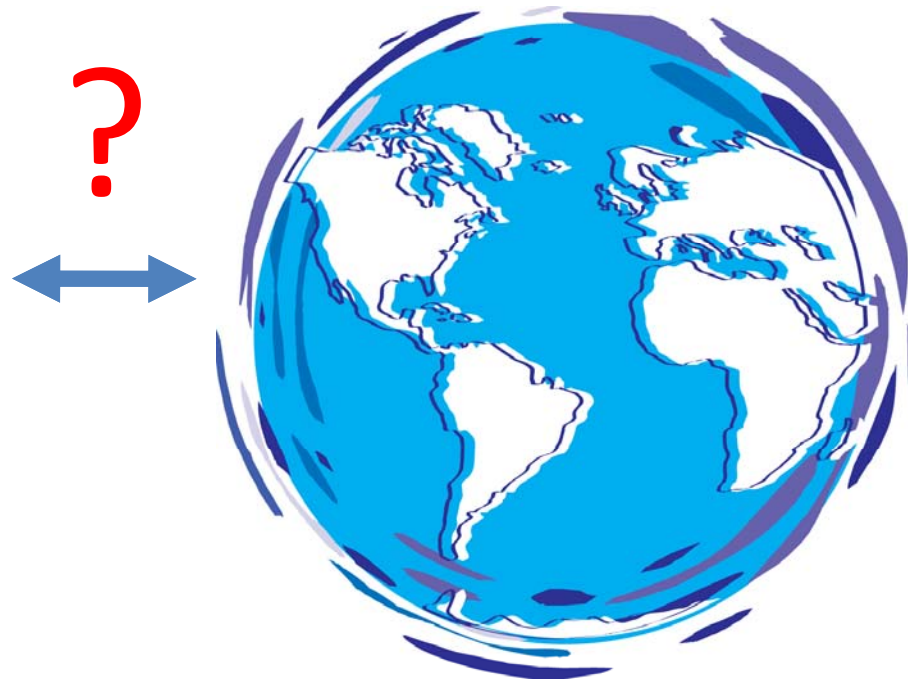
$$V_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} -12 & -2 \\ 0 & 61 \end{vmatrix}}{\begin{vmatrix} 5 & -2 \\ -28 & 61 \end{vmatrix}} = \frac{-12 \times 61 - 0}{5 \times 61 - (-2) \times (-28)} = \frac{732}{249} = 2.93 \text{ V}$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 5 & -2 \\ -28 & 0 \end{vmatrix}}{249} = \frac{0 - (-12) \times (-28)}{249} = 1.35 \text{ V}$$

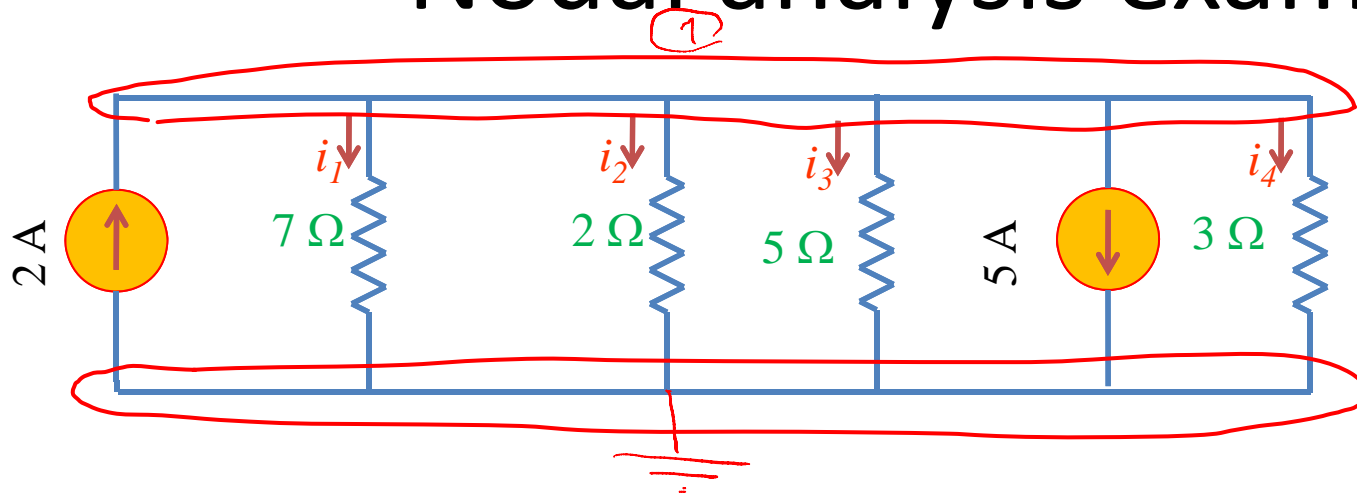
$$i_3 = \frac{2.93 \text{ V}}{2\Omega}$$

$$i_2 = \frac{V_1 - V_2}{3\Omega} = 0.53 \text{ A}$$

Ground?



Nodal analysis example



$$\text{KCL : } i_1 + i_2 + i_3 + i_4 + 5\text{A} - 2\text{A} = 0$$

$$\frac{V_1}{7} + \frac{V_1}{2} + \frac{V_1}{5} + \frac{V_1}{3} + 5 - 2 = 0$$

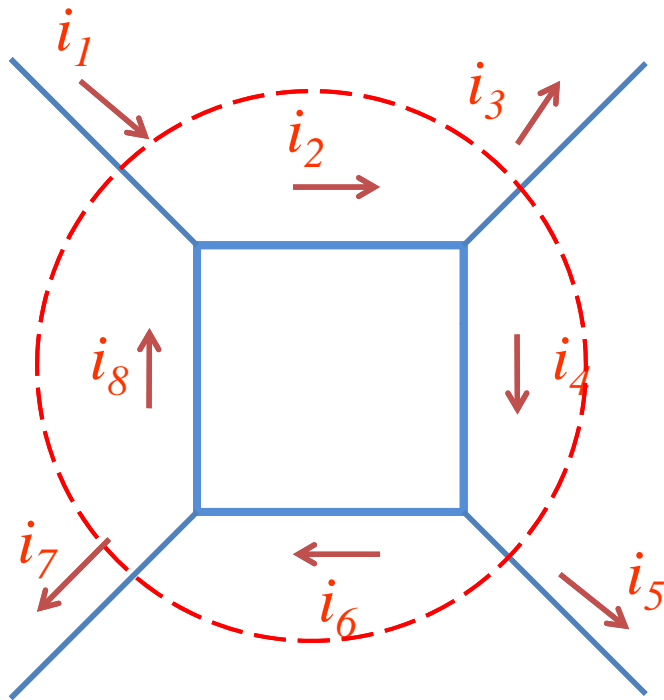
$$V_1 \left(\frac{1}{7} + \frac{1}{2} + \frac{1}{5} + \frac{1}{3} \right) = -3$$

$$V_1 = -2.5\text{V}$$

Questions?

KCL examples

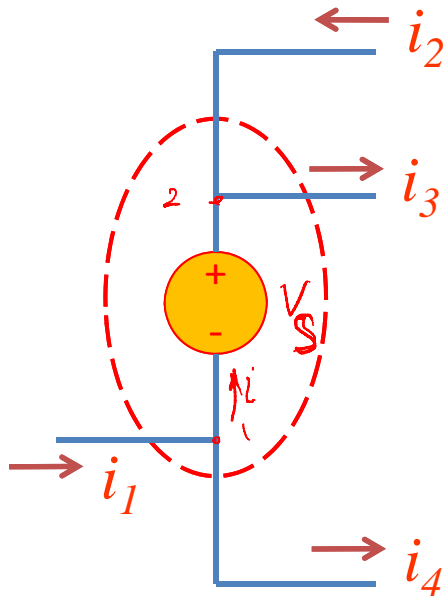
From Lecture 3, Week 2: Find a relationship among $i_1, i_2, i_3, i_4, \dots$



$$i_1 = i_7 + i_5 + i_5$$

“Supernode”

A node with a voltage source in it...



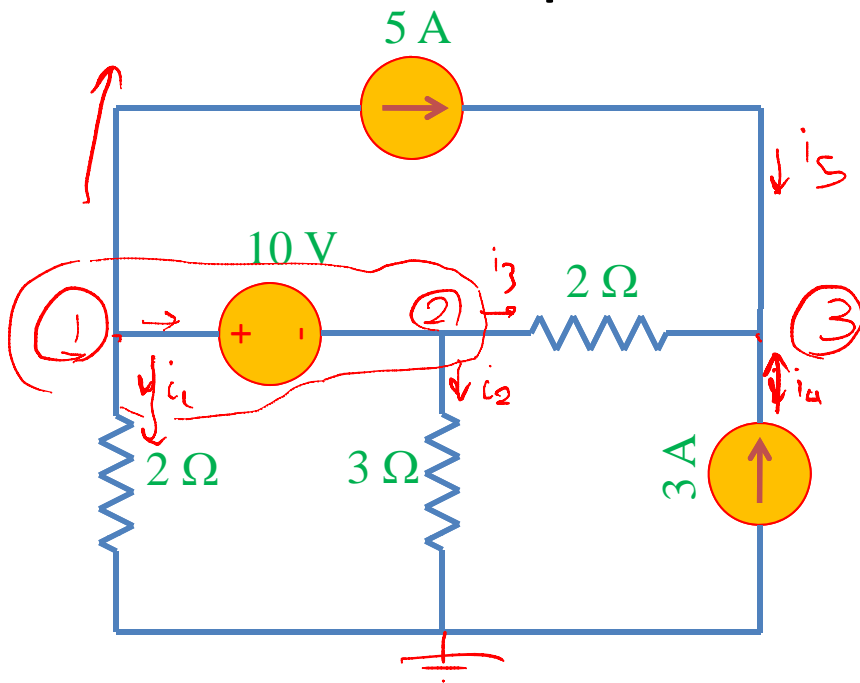
KCL: $i_1 + i_2 = i_3 + i_4$

$\rightarrow V_2 - V_1 = V_S$

Must define a supernode if a voltage source appears when doing nodal analysis...
(unless one end of voltage source connected to reference node)

1. Define a reference node.
2. Label remaining nodes.
3. Apply KCL + ohm to all nodes **and supernodes**
4. **Apply KVL to loop with voltage source**

Example nodal w/voltage source



$$\textcircled{3} \quad -i_3 - i_5 - i_4 = 0$$

$$\downarrow$$

$$-\frac{V_3 - V_2}{2} - 5 - 3 = 0 \quad *$$

$$5 + i_3 + i_2 + i_1 = 0$$

$$5 + \frac{V_2 - V_3}{2} + \frac{V_2}{3} + \frac{V_1}{2} = 0 \quad **$$

$$V_1 - V_2 = 10 \text{ v} \quad ***$$

$$V_1 = 10 + V_2 \quad \xrightarrow{\text{r. **}}$$

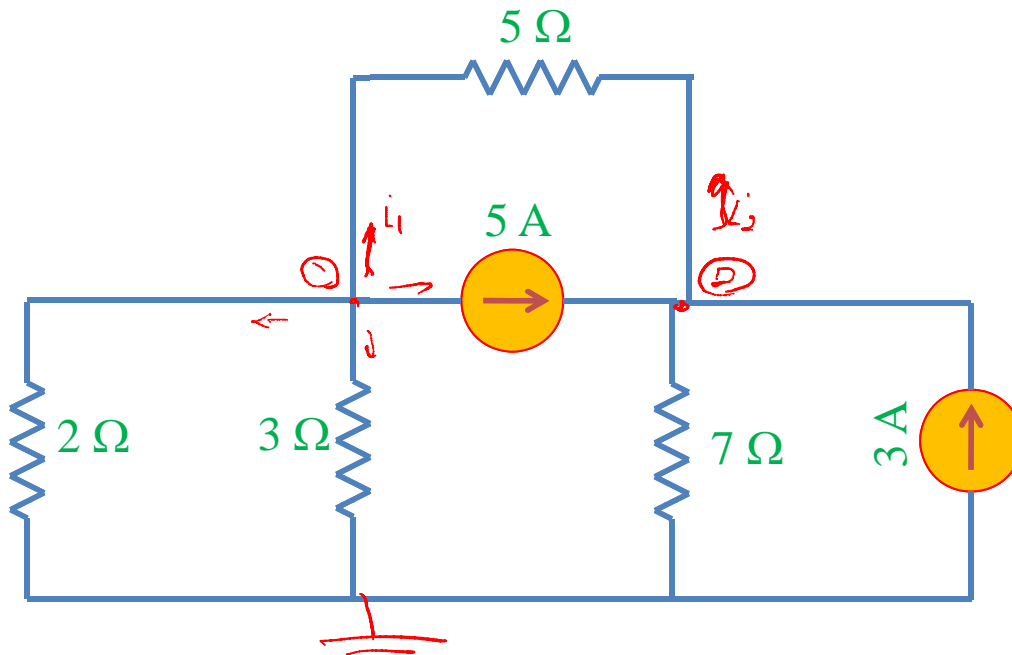
$$\left\{ \begin{aligned} 5 + \frac{10 + V_2}{2} + V_2 \left(\frac{1}{2} + \frac{1}{3} \right) - \frac{V_3}{2} &= 0 \\ -\frac{V_3}{2} + \frac{V_2}{2} &= 8 \end{aligned} \right.$$

$$V_2 = -2.4 \text{ v}, \quad V_3 = 13.6 \text{ v}$$

$$V_1 = 7.6 \text{ v}$$

Questions?

Example nodal w/voltage source



$$\textcircled{1} \quad \frac{v_1}{2} + \frac{v_1}{3} + \frac{v_1 - v_2}{5} + 5 = 0$$

$$v_1 \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} \right) - \frac{v_2}{5} = -5$$

$$31v_1 - 6v_2 = -150 \quad *$$

$$\textcircled{2} \quad \frac{v_2 - v_1}{5} + \frac{v_2}{7} - 5 - 3 = 0$$

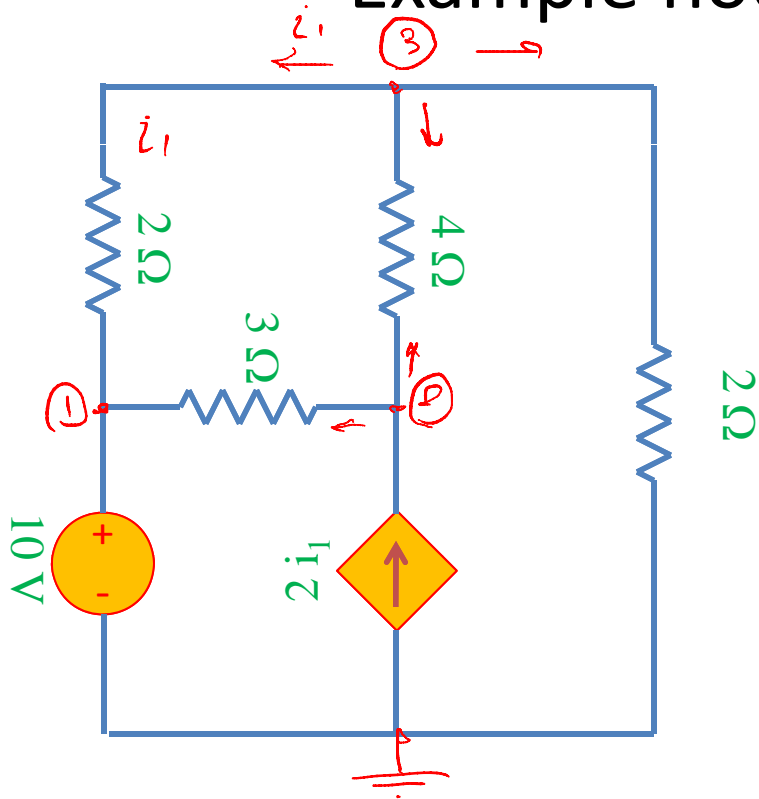
$$-\frac{v_1}{5} + v_2 \left(\frac{1}{5} + \frac{1}{7} \right) = 8$$

$$-7v_1 + 12v_2 = 280 \quad **$$

$$\left. \begin{array}{l} 31v_1 - 6v_2 = -150 \quad * \\ -7v_1 + 12v_2 = 280 \quad ** \end{array} \right\}$$

$$55v_1 = -20 \rightarrow v_1 = \frac{-20}{55}$$

Example nodal w/voltage source



$$V_2 =$$

$$V_1 = 10 \text{ v}$$

$$\textcircled{3} \rightarrow \frac{V_3 - V_1}{2} + \frac{V_3 - V_2}{4} + \frac{V_3}{2} = 0$$

$$\leadsto -V_2 + 5V_3 = 20 \text{ v} \quad *$$

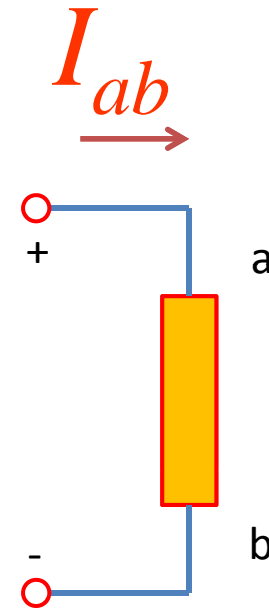
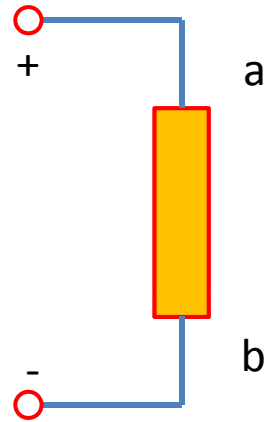
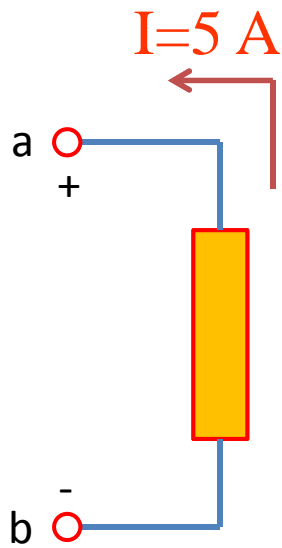
$$\textcircled{2} \frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{4} - 2i_1 = 0$$

$$\downarrow$$

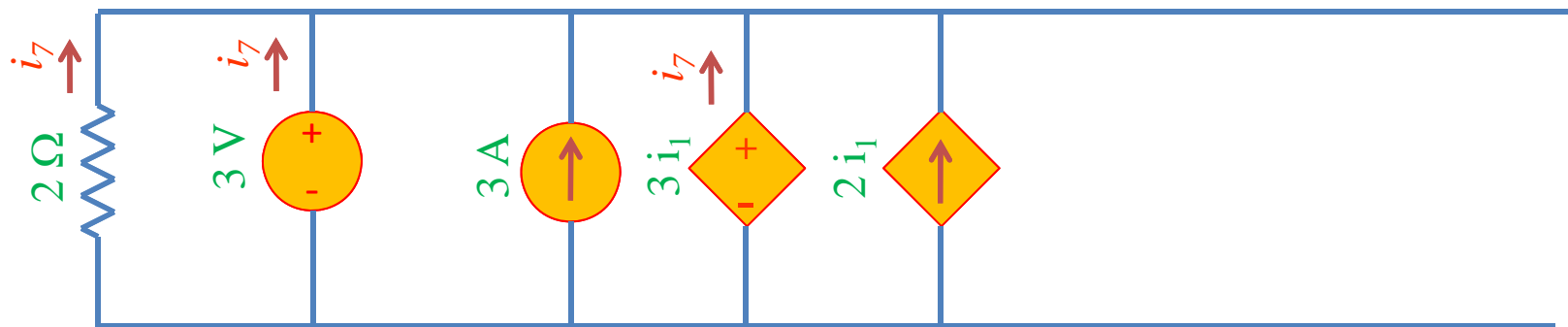
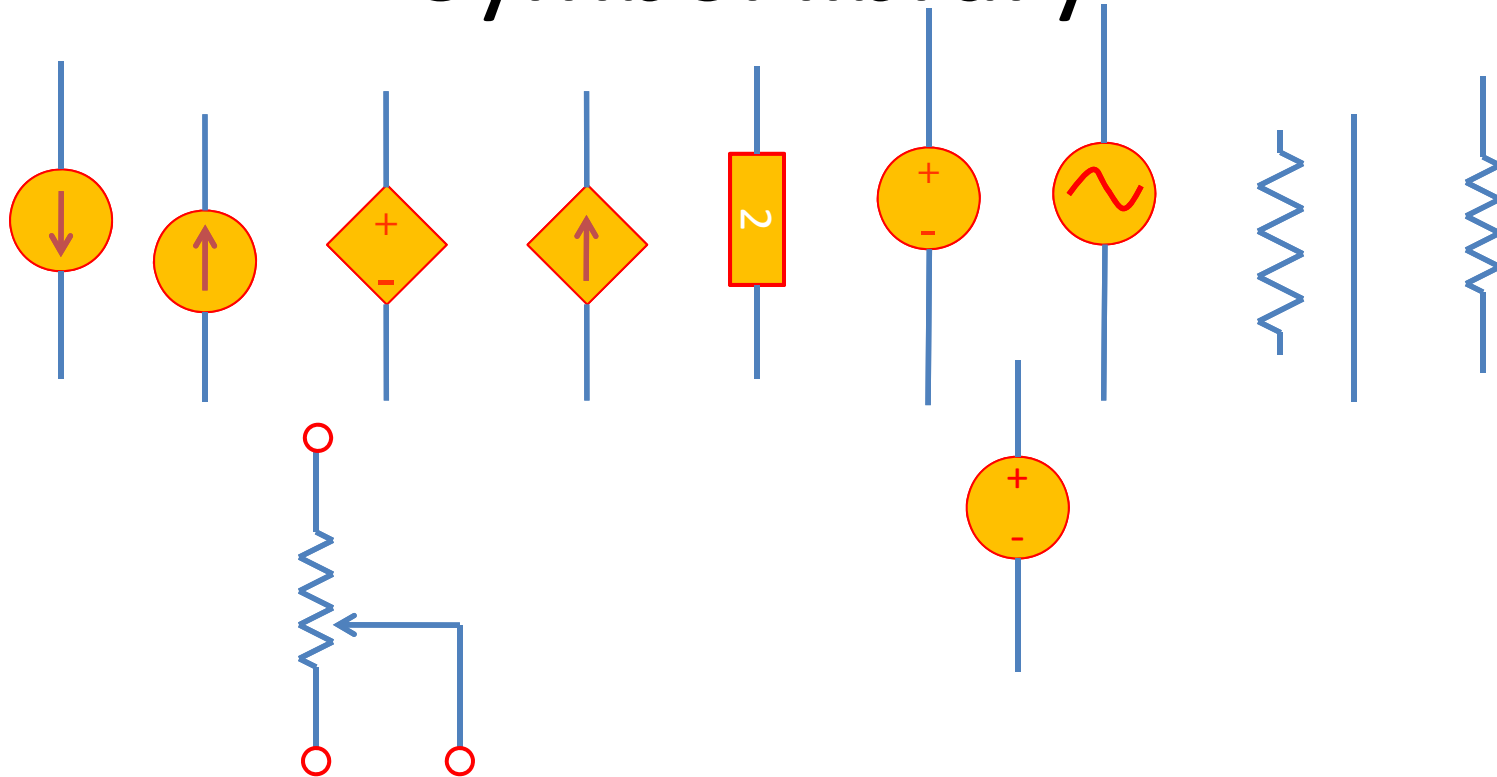
$$\frac{V_3 - 10}{2}$$

$$\leadsto 7V_2 - 15V_3 = 52 \quad **$$

Symbol library



Symbol library



Symbol & circuit library

