Announcements:
1. Announcements

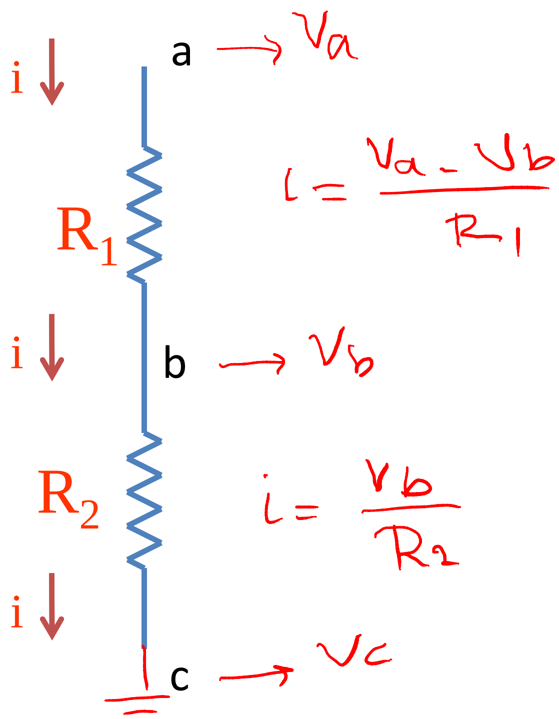
EECS 70A: Network Analysis

Lecture 6

Today's Agenda

- Review of Nodal Analysis
- Mesh Analysis
  - Introduction
    - What is a Mesh?
    - Mesh Current
  - Method
- Mesh Analysis with Current Source

Node Voltage(review)



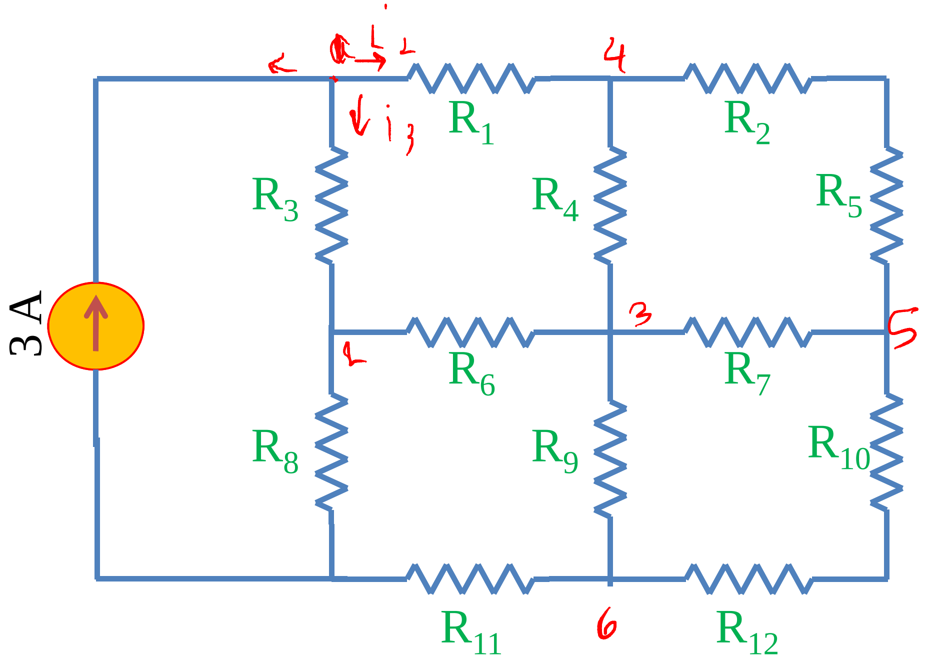
V_{ab} is the voltage drop across resistor 1

Nodal Analysis(Review)

Based on KCL, Use node voltages as circuits variables.

1. Define a reference node.
2. Label remaining nodes. (n-1 nodes)
3. Apply KCL + ohm to all nodes and supernodes
    1. Express all I's in terms of v's
4. Apply KVL to loops with voltage source
5. Solve the n-1 simultaneous equations, to find V's
6. Use Ohm's law to find the currents.

Apply KCL + Ohm to All Nodes and Supernodes



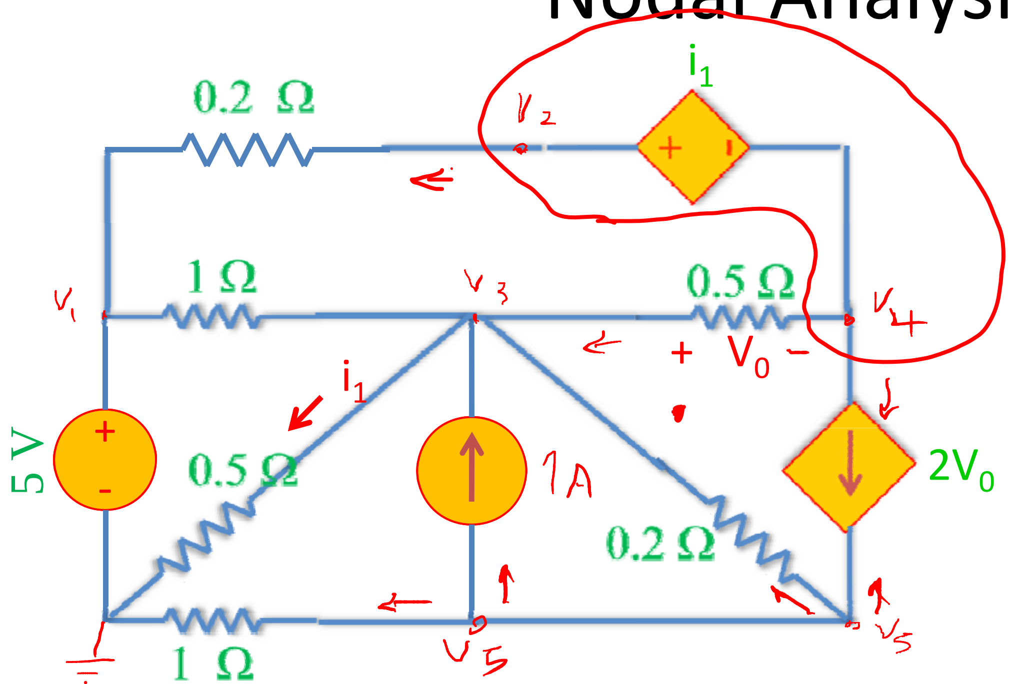
$$i_3 = \frac{V_1 - V_2}{R_3}, \quad \frac{V_1 - V_4}{R_1} = i_2$$

KCL ①:

$$\frac{V_n - V_k}{R_{n,k}}$$

$$\frac{V_3 - V_4}{R_4} + \frac{V_3 - V_2}{R_6} + \frac{V_3 - V_5}{R_7} + \frac{V_3 - V_6}{R_9} = 0$$

Nodal Analysis-Example



$v_1 = 5\,v$

③ $\frac{V_3 - V_1}{1} + 2V_3 + 5(V_3 - V_5) + 2(V_3 - V_4) - 1 = 0$

$-V_1 + 10V_3 - 5V_5 - 2V_4 = 1$

$+5$

$10V_3 - 2V_4 - 5V_5 = 6 \quad (1)$

(2)

⑤ $V_5 + 1 + 5(V_5 - V_3) - 2(V_3 - V_4) = 0$

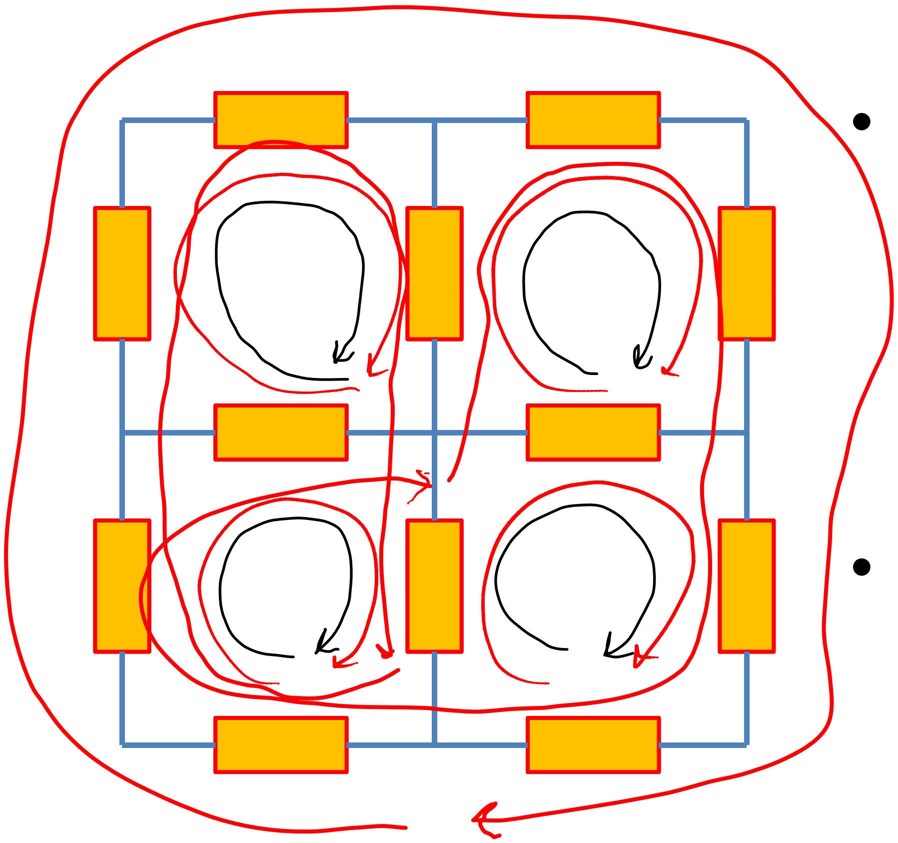
(3)

② & ④ $5(V_2 - V_1) + 2(V_3 - V_4) + 2(V_3 - V_4) = 0$

KVL: $V_2 - V_4 = i_1 \Rightarrow V_2 - V_4 = 2V_3 \quad (4)$

$V_2 - i_1 = V_4$

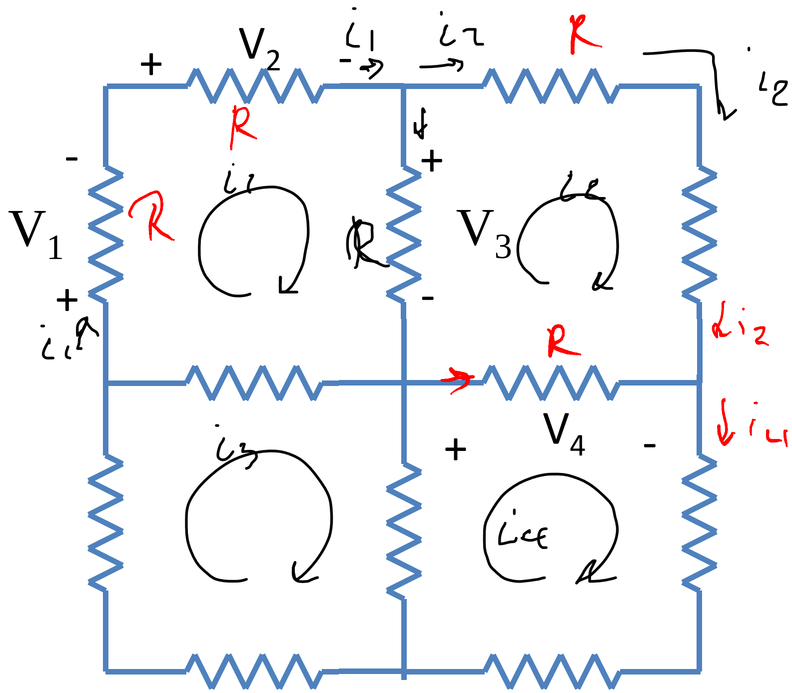
Mesh Analysis-Introduction
What is a Mesh?



- A loop is a closed path with no node passed more than once.
- A mesh is a loop that does not contain any other loops within it.

Mesh Analysis-Introduction

Mesh Current vs. Element Current



- The current through a mesh is known as mesh current.
- Direction of the mesh current is arbitrary-conventionally assumed to be clockwise.
- The current through an element can be the same as mesh current or the subtraction of two mesh currents.

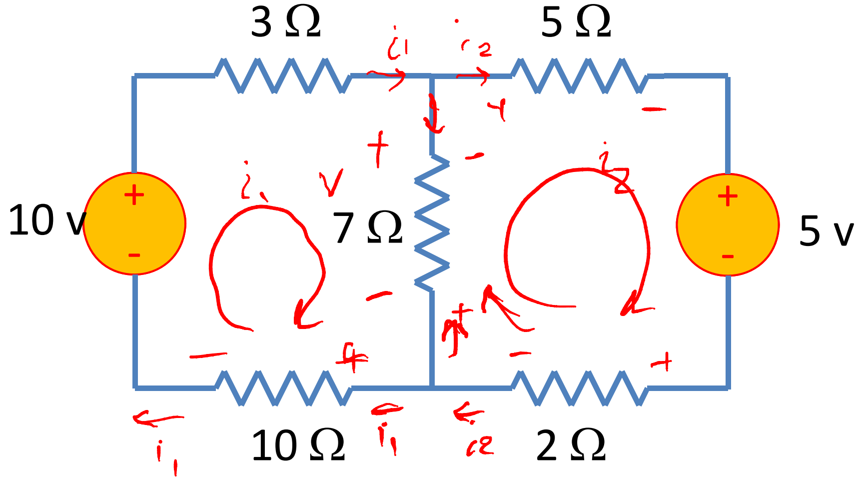
$$V_3 = R_x (i_1 - i_2)$$

$$V_1 = i_1 \times R$$

$$V_4 = R_x (i_4 - i_2)$$

Mesh Analysis-Method

- Assign mesh currents $i_1, i_2, \ldots i_n$
- Apply KVL+ Ohm's law to each mesh
- Solve the equations for $i_1, i_2, \ldots i_n$



KVL

Mesh1:

$$-10 + 3i_1 + 7(i_1 - i_2) + 10i_1 = 0$$

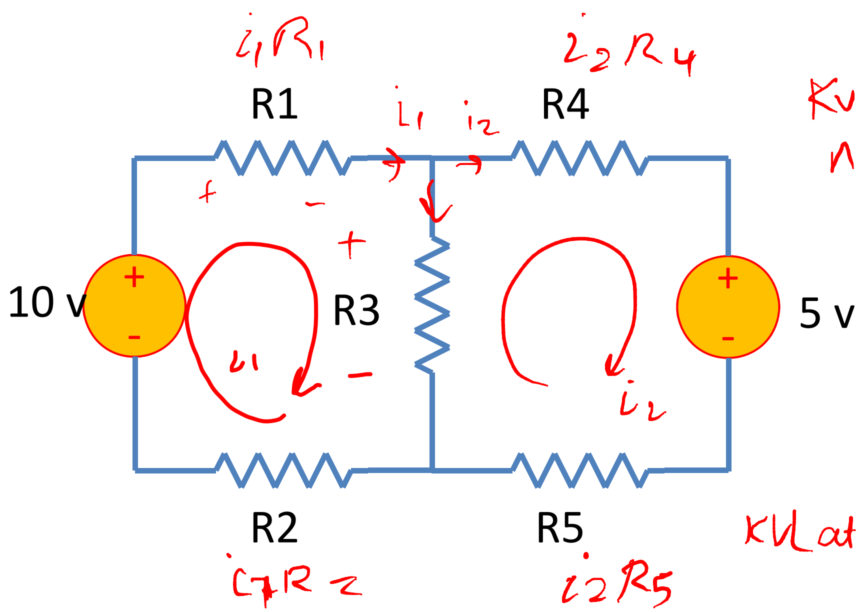
$$20i_1 - 7i_2 = 10 \quad (1)$$

Mesh2:

$$7(i_2 - i_1) + 5i_2 + 5 + 2i_2 = 0$$

$$-7i_1 + 14i_2 = -5 \quad (2)$$

Apply KVL+ Ohm's Law to Each Mesh



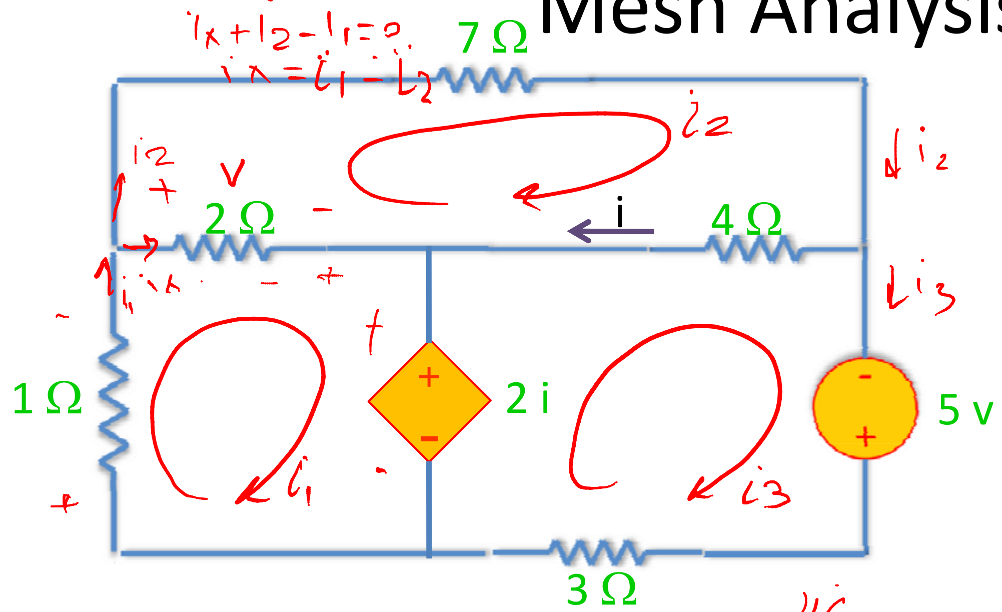
KVL n: $V = R\, i_n$ (R only in mesh n)

$V = R_3(i_1 - i_2)$

n: $V = R_{n,k}(i_n - i_k)$ (R between mesh n & k)

KVL at K: $V = R_{n,k}(i_k - i_n)$

Mesh Analysis - Example



M1 :

$$1 \times i_1 + 2(i_1 - i_2) + 2i = 0$$

$$i = i_2 - i_3$$

$$i_1(1+2) + i_2(-2+2) - 2i_3 = 0$$

$$3i_1 - 2i_3 = 0 \qquad (1)$$

$4i$

M2 :

$$2(i_2 - i_1) + 7i_2 + 4(i_2 - i_3) = 0$$

$$-2i_1 + 13i_2 - 4i_3 = 0 \qquad (2)$$

M3 :

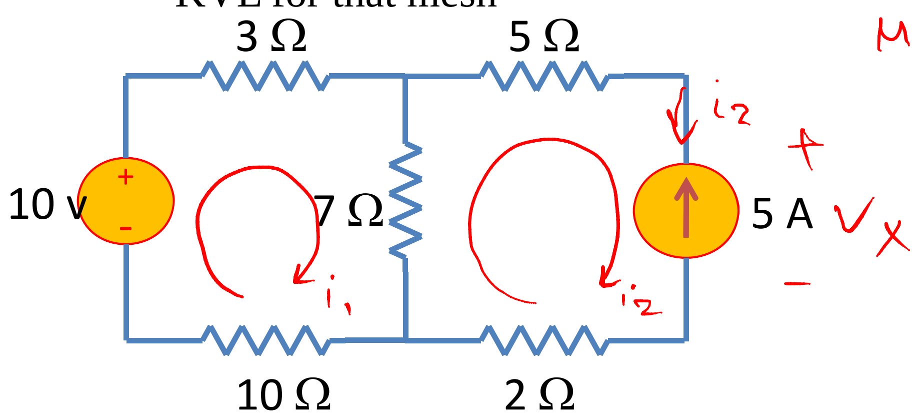
$$-2(i_2 - i_3) + 4(i_3 - i_2) - 5 + 3i_3 = 0$$

$$-6i_2 + 9i_3 = 5 \qquad (3)$$

Mesh Analysis with Current Sources

- CASE 1: current source only in one mesh.

already have the current for that mesh => no need to write KVL for that mesh



M2: $7(i_2 - i_1) + 5i_2 + V_x + 2(i_2) = 0$

$i_2 = -5A$

M1: $-10 + 3i_1 + 7(i_1 - i_2) + 10i_1 = 0$

$20i_1 - 7i_2 = 10$

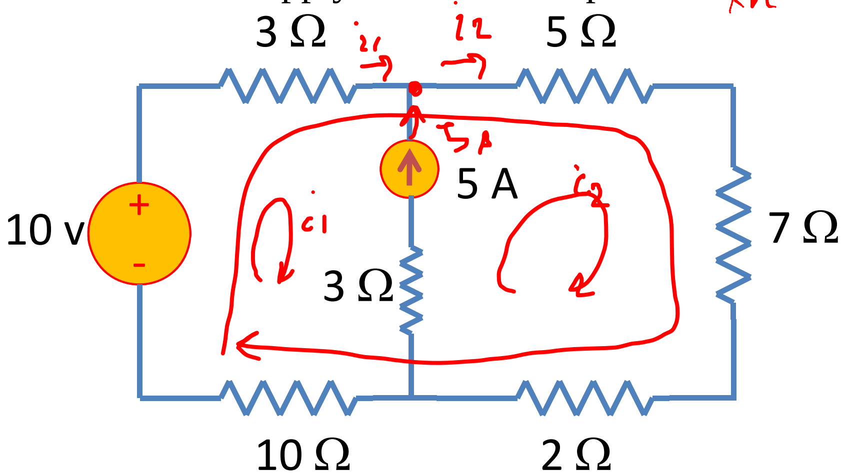
$i_2 \to -5$

$20i_1 + 35 = 10$

$i_1 = \frac{-25}{20} = -1.25\ A$

Mesh Analysis with Current Sources

- CASE 2: current source exits between two meshes. => create a supermesh
  - Apply KVL to the supermesh
  - Apply KCL to the supermesh



KVL: $-10 + 3i_1 + 5i_2 + 7i_2 + 2i_2 + 10i_1 = 0$

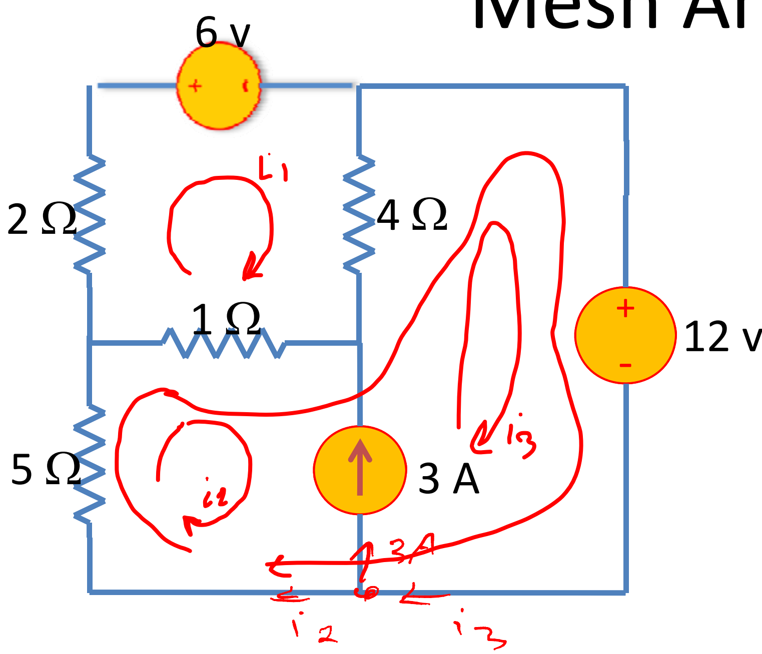
$13i_1 + 14i_2 = 10$ (1)

KCL: $i_1 - 5A = i_2$

$i_1 - i_2 = 5$ (2)

$$\begin{cases} 13i_1 + 14i_2 = 10 \\ i_1 - i_2 = 5 \end{cases} \rightsquigarrow i_1 \,\&\, i_2$$

Mesh Analysis- Example



Mesh 1 :

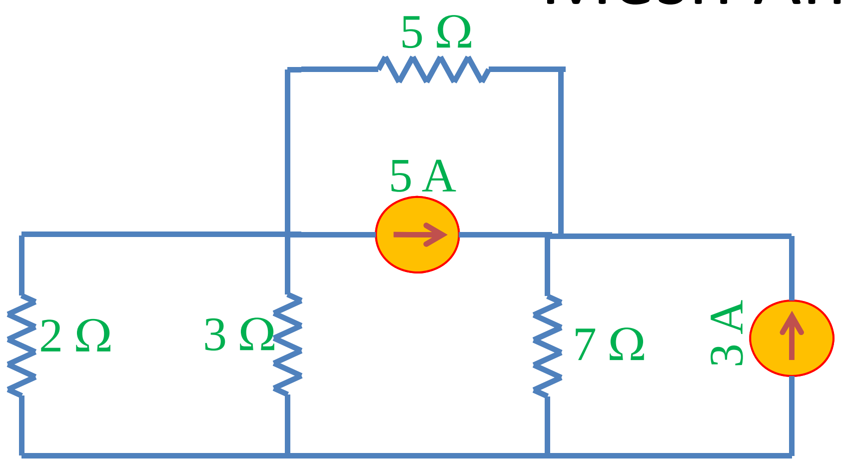
$2i_1 + 6 + 4(i_1 - i_3) + 1(i_1 - i_2) = 0$ (1)

(1) & (2) :

$5i_2 + 1\times(i_2 - i_1) + 4(i_3 - i_1) + 12 = 0$ (2)

KCL $\Rightarrow$ $i_3 = 3 + i_2$ (3)

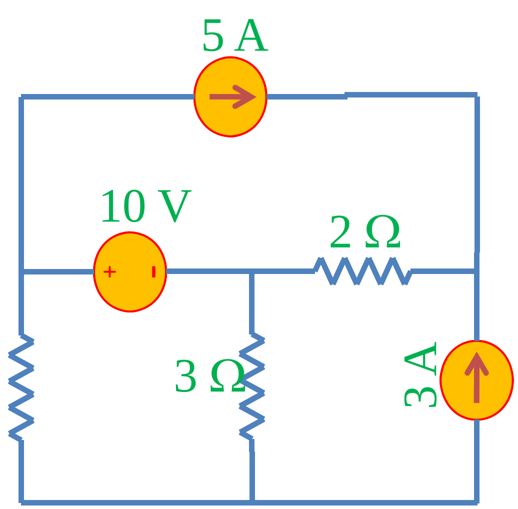
Mesh Analysis- Example

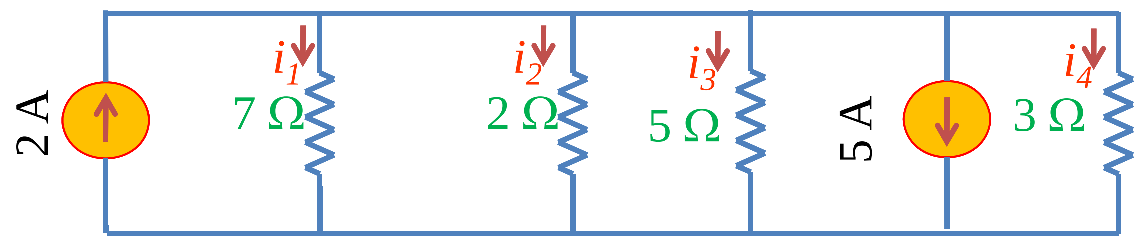


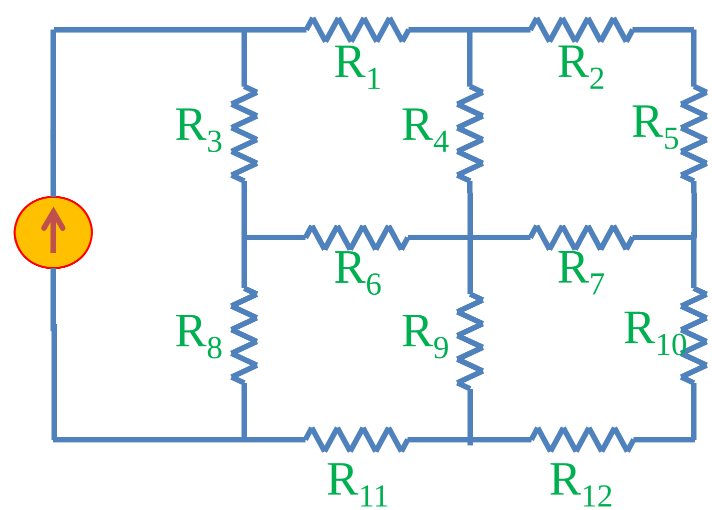
Nodal Versus Mesh Analysis

- The method that results in fewer number of equations is more suitable.
  - Mesh analysis for networks with many series connected elements
  - Nodal Analysis for networks with many parallel connected elements

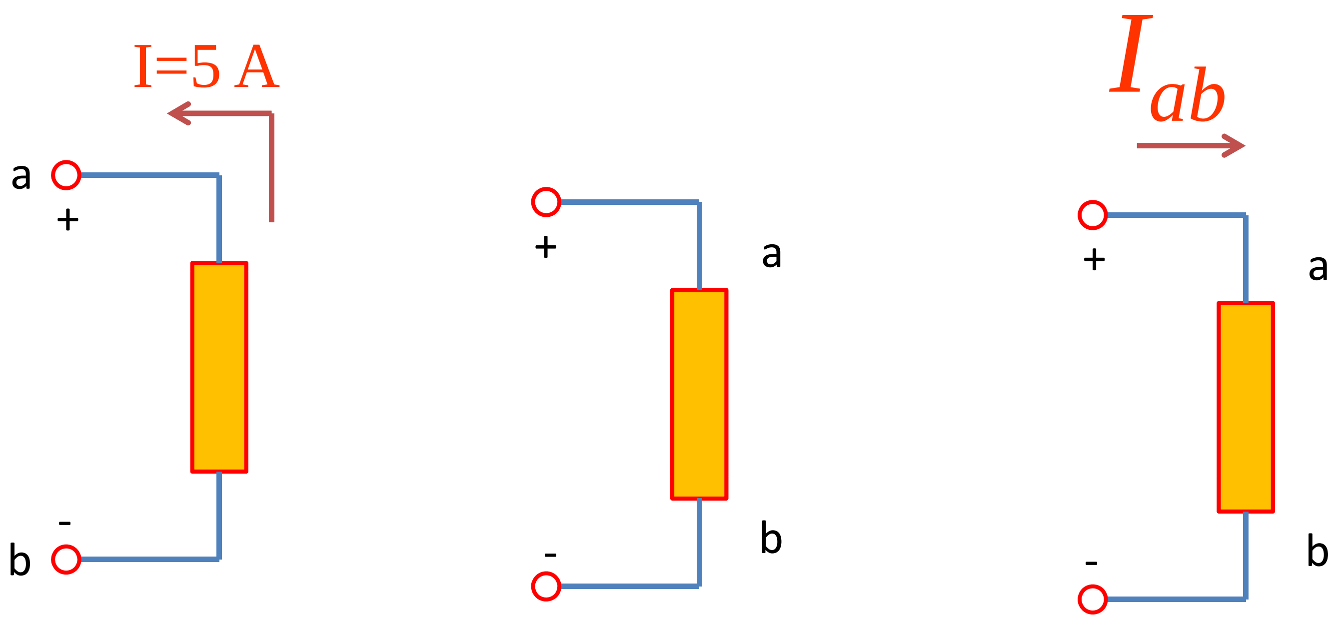
But also depends on the type of the sources.



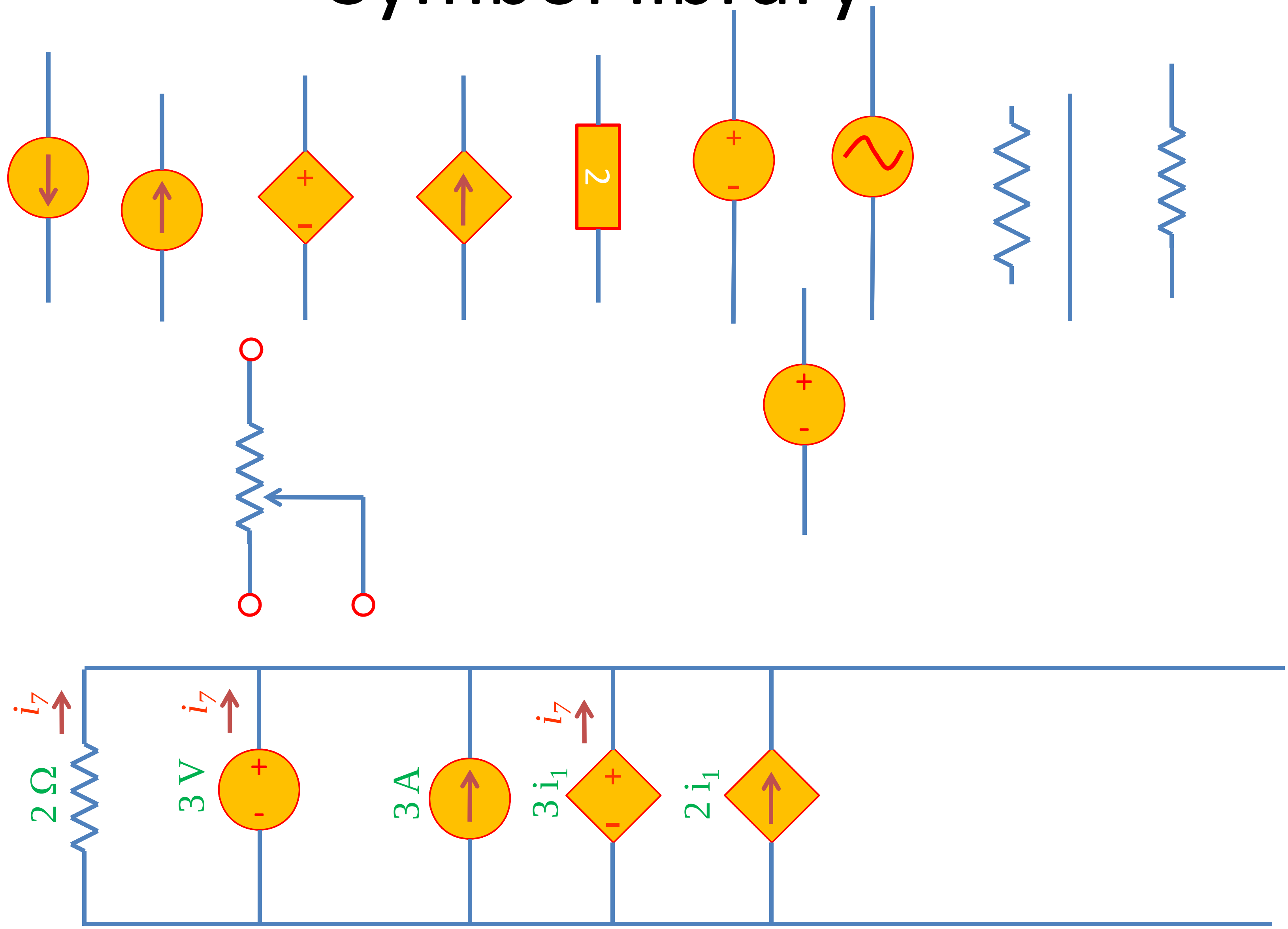




Symbol library



Symbol library



Symbol & circuit library