

Announcements:

1. Next quiz will be due on Monday next week
2. Next HW will be due on Wednesday next week
3. Graded midterms are being scanned today;
available for pickup Wednesday from TAs

EECS 70A: Network Analysis

Lecture 7

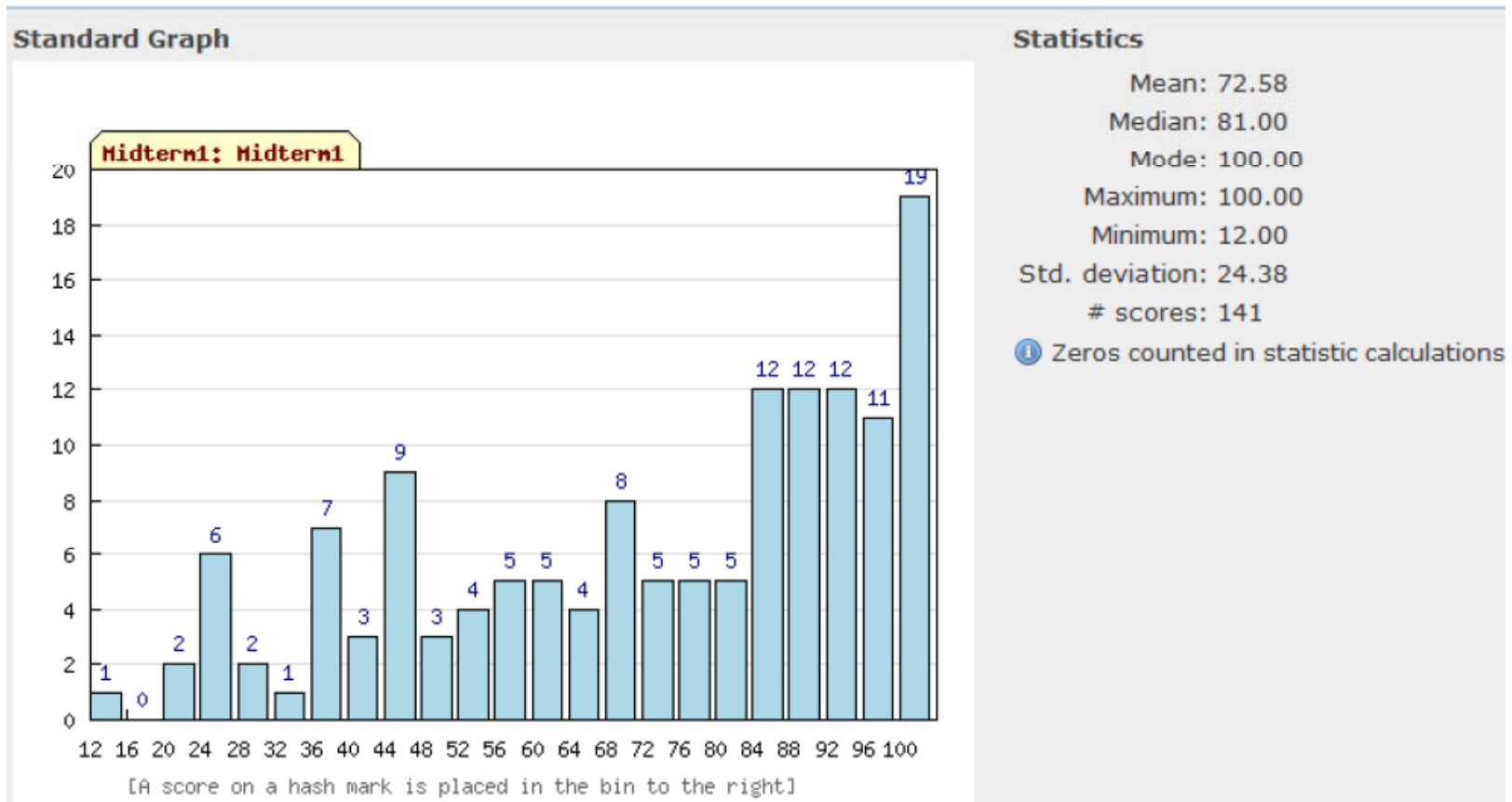
Today's Agenda

- TLTC Midterm Student Feedback Survey
- Midterm 1 results
- Review of Nodal Analysis
- Review of Mesh Analysis
- Example problems using both techniques
- Thevinin/Norton theorem

TLTC Midterm Feedback Survey

- The good:
 - Recorded lectures
 - online notes/tablet pc
- To improve:
 - Need more complex examples in class
- To drop:
 - Demos

Midterm results



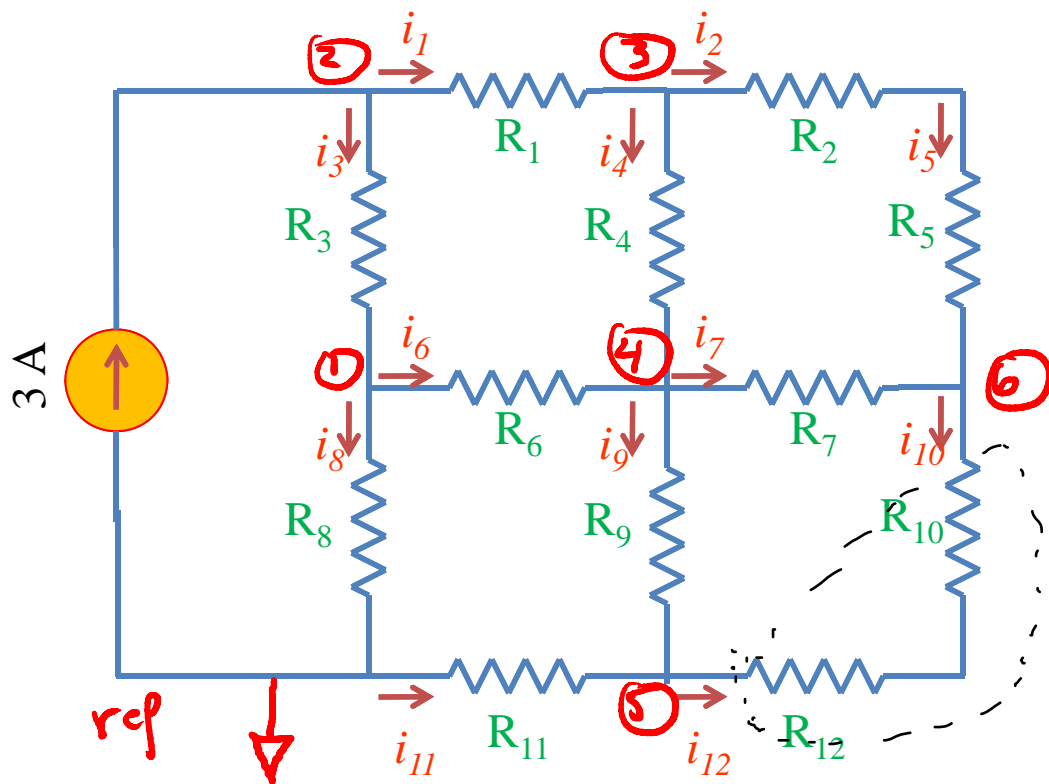
Detailed solutions posted online. Please make sure you understand them!

Nodal analysis summary

1. Define reference node
2. Label remaining nodes (e.g. V_1, V_2, V_3, \dots)
3. Apply KCL + Ohm's law
4. Solve for nodal voltages (e.g. using Kramer's rule)
5. Solve for currents

Nodal analysis example

1. Define a reference node.
2. Label remaining nodes.
3. Apply KCL + ohm.



$G_1 = \frac{1}{R_1}$
 $G_2 = \frac{1}{R_2}$ etc

6 variables

$V_1, V_2, V_3, V_4, V_5, V_6$

Need 6 eqns.

$i_{11} + i_9 = i_{12}$

$$\frac{0 - V_5}{R_{11}} + \frac{V_4 - V_5}{R_9} = \frac{V_5 - V_6}{R_{12} + R_{10}}$$

① $i_3 = i_6 + i_7$

$i_8 + i_6 - i_3 = 0$

$$\frac{V_1 - 0}{R_8 + 1/R_6} + \frac{V_1 - V_4}{R_6} - \frac{V_2 - V_1}{R_3} = 0$$

$$V_1 \left(\frac{1}{R_8} + \frac{1}{R_3} \right) + V_4 \left(-\frac{1}{R_6} \right) + V_2 \left(-\frac{1}{R_3} \right) = 0$$

$$\textcircled{1} V_1 (G_8 + G_3) + V_4 (-G_6) + V_2 (-G_3) = 0$$

Typical notation:

i_1 is current through R_1 . (Same as before)
 V_1 is voltage of node 1 relative to reference node. (Different from before)

We will do this entire problem in class...

Using these techniques, you can attempt the "monster problem" as extra credit on HW3...

$$\begin{aligned}
 & (G_8 + G_3 + G_6) V_1 + (-G_3) V_2 + (0) V_3 + (-G_6) V_4 + (0) V_5 + (0) V_6 = (0) \\
 & (-G_3) V_1 + (\cancel{G_1 + G_2} + G_1 + G_3) V_2 + (-G_1) V_3 + (0) V_4 + (0) V_5 + (0) V_6 = (3) \\
 & (0) V_1 + (-G_1) V_2 + (G_1 + G_4) V_3 + (-G_4) V_4 + (0) V_5 + (-G_5) V_6 = (0) \\
 & (G_6) V_1 + (0) V_2 + (G_4) V_3 + (-G_4) V_4 + (G_9) V_5 + (\cancel{G_7} + G_7) V_6 = (0) \\
 & (0) V_1 + (0) V_2 + (0) V_3 + (-G_9) V_4 + (G_9 + G_{12} + G_4) V_5 + (-G_{10} - G_{12}) V_6 = (0) \\
 & (0) V_1 + (0) V_2 + \left(\frac{G_2 \cdot G_5}{G_2 + G_5} \right) V_3 + (G_7) V_4 + \left(\frac{G_{12} G_{10}}{G_{12} + G_{10}} \right) V_5 + \left(\frac{-G_{10} G_{12}}{G_2 + G_5} - \frac{G_{12} G_{10}}{G_{12} + G_{10}} - G_7 \right) V_6 = (0)
 \end{aligned}$$

WRONG * WRONG

$$V_1 = \frac{|N_1|}{|D|}$$

$$V_2 = \frac{|N_2|}{|D|}$$

$$V_1 = \frac{|N_1|}{|D|}$$

$$G_{21} \equiv \frac{G_2 + G_3}{G_2 G_3}$$

$$G_{12}^{-1} \equiv \frac{G_{12} + G_{10}}{G_{10} G_{12}}$$

$$|N_1| = \begin{pmatrix} 0 & -G_2 & 0 & -G_6 & 0 & 0 \\ 3 & G_1 + G_3 & -G_1 & 0 & 0 & 0 \\ 0 & -G_1 & G_1 + G_4 & -G_4 & 0 & -G_5 \\ 0 & 0 & G_4 & -(**) & G_4 & G_7 \\ 0 & 0 & 0 & -G_4 & (***) & G_{10} G_{12} \\ 0 & 0 & G_{23} & G_7 & G_{12} G_{10} & (**) \end{pmatrix}$$

(∴ same)

$$D = \begin{pmatrix} G_1 + G_3 + G_6 & \cdot & \cdot & \cdot & \cdot & \cdot \\ -G_3 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ G_6 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$V_1 = \frac{|N_1|}{|D|}$$

$$G_{21} \equiv \frac{G_2 + G_3}{G_2 G_3}$$

$$G_{12}^{-1} \equiv \frac{G_{12} + G_{10}}{G_{10} G_{12}}$$

$$|N_1| = \begin{pmatrix} 0 & -G_2 & 0 & -G_6 & 0 & 0 \\ 3 & G_1 + G_3 & -G_1 & 0 & 0 & 0 \\ 0 & -G_1 & G_1 + G_9 & -G_9 & 0 & -G_5 \\ 0 & 0 & G_4 & -(**) & G_9 & G_7 \\ 0 & 0 & 0 & -G_9 & (***) & G_{10} G_{12} \\ 0 & 0 & G_{23} & G_7 & G_{12} G_{10} & (**) \end{pmatrix}$$

(∴ same)

$$D = \begin{pmatrix} G_1 + G_3 + G_6 & \cdot & \cdot & \cdot & \cdot & \cdot \\ -G_3 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ G_6 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$\begin{array}{c|ccccc}
 0 & -G_2 & 0 & -G_6 & 0 & 0 \\
 3 & G_1 G_3 & -G_1 & 0 & 0 & 0 \\
 0 & -G_1 & G_1 G_4 & -G_9 & 0 & -G_5 \\
 0 & 0 & G_4 & -(**) & G_9 & G_7 \\
 0 & 0 & 0 & -G_9 & (***) & G_{10} G_{12} \\
 0 & 0 & G_{23} & G_7 & G_{1210} & (**)
 \end{array} =$$

$$0 \mid -3 \mid m_3 \mid +0 \mid \mid \emptyset \mid 0 \mid +0 \mid \mid$$

$$= -3 \left| \begin{array}{ccccc}
 -G_2 & 0 & -G_6 & 0 & 0 \\
 -G_1 & G_1 G_4 & & & \\
 0 & G_4 & & & \\
 0 & 0 & & & \\
 0 & G_{23} & & &
 \end{array} \right|$$

Mesh analysis summary

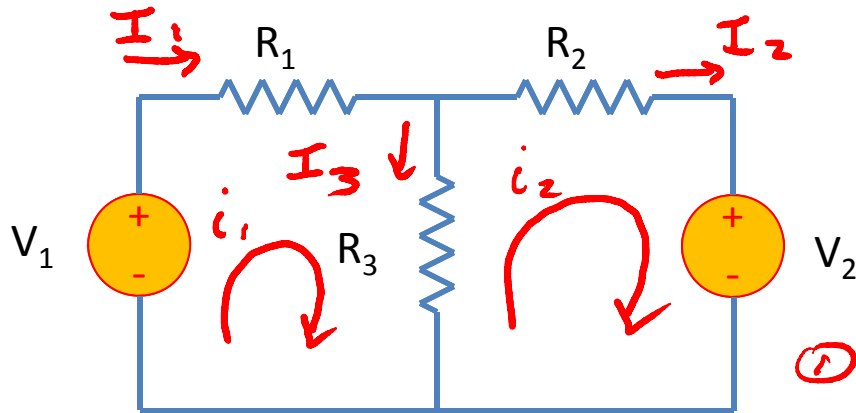
1. Assign mesh currents i_1, i_2, \dots, i_n
2. Apply KVL to each mesh
3. Solve for mesh currents (e.g. using Kramer's rule)
4. Then solve for voltages

$$i_1 = I_1$$

$$i_2 = I_2$$

Assigning mesh currents

$$I_3 = i_1 - i_2$$



$$\textcircled{1} -V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0$$

$$\textcircled{2} -R_3 (i_1 - i_2) + R_2 i_2 + V_2 = 0$$

2 eqns. 2 unk. currents

$$\textcircled{1} (R_1 + R_3) i_1 + (-R_3) i_2 = (V_1)$$

$$\textcircled{2} (-R_3) i_1 + (R_2 + R_3) i_2 = (-V_2)$$

Solve for i_1, i_2 :

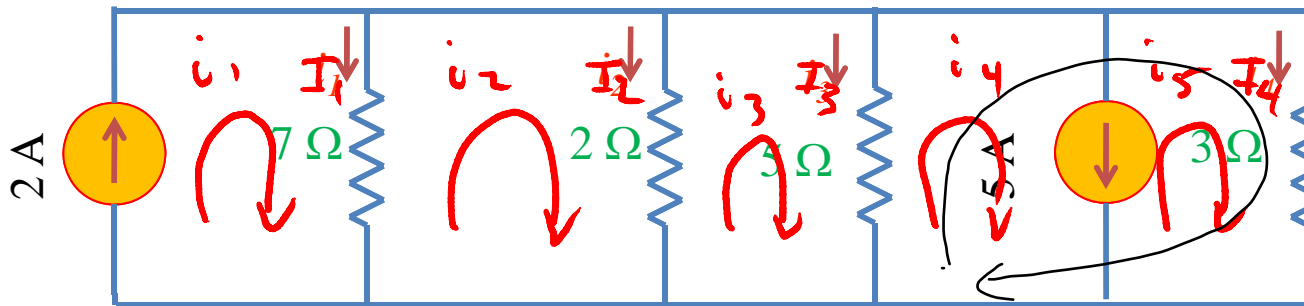
$$i_1 = \frac{\begin{vmatrix} V_1 & -R_3 \\ -V_2 & R_2 + R_3 \end{vmatrix}}{\begin{vmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{vmatrix}} =$$

$$i_2 = \frac{\begin{vmatrix} R_1 + R_3 & V_1 \\ -R_3 & -V_2 \end{vmatrix}}{\begin{vmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{vmatrix}} =$$

$$\frac{V_1 (R_2 + R_3) - (-V_2) (-R_3)}{(R_1 + R_3)(R_2 + R_3) - (-R_3)(-R_3)}$$

$$\frac{(R_1 + R_3)(-V_2) - (V_1)(-R_3)}{(R_1 + R_3)(R_2 + R_3) - (R_3)(-R_3)}$$

Nodal vs. mesh analysis?



5 unknowns

Supermesh

$$\textcircled{5} i_1 = 2A$$

$$\textcircled{1} -7(i_1 - i_2) + 2(i_2 - i_3) = 0$$

$$\textcircled{2} -2(i_2 - i_3) + 5(i_3 - i_4) = 0$$

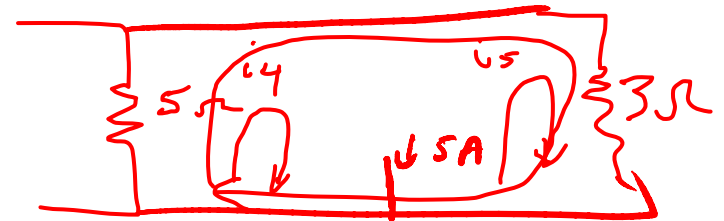
KVL \textcircled{a} supermesh

$$\textcircled{3} -5\Omega(i_3 - i_4) + 3\Omega(i_5) = 0$$

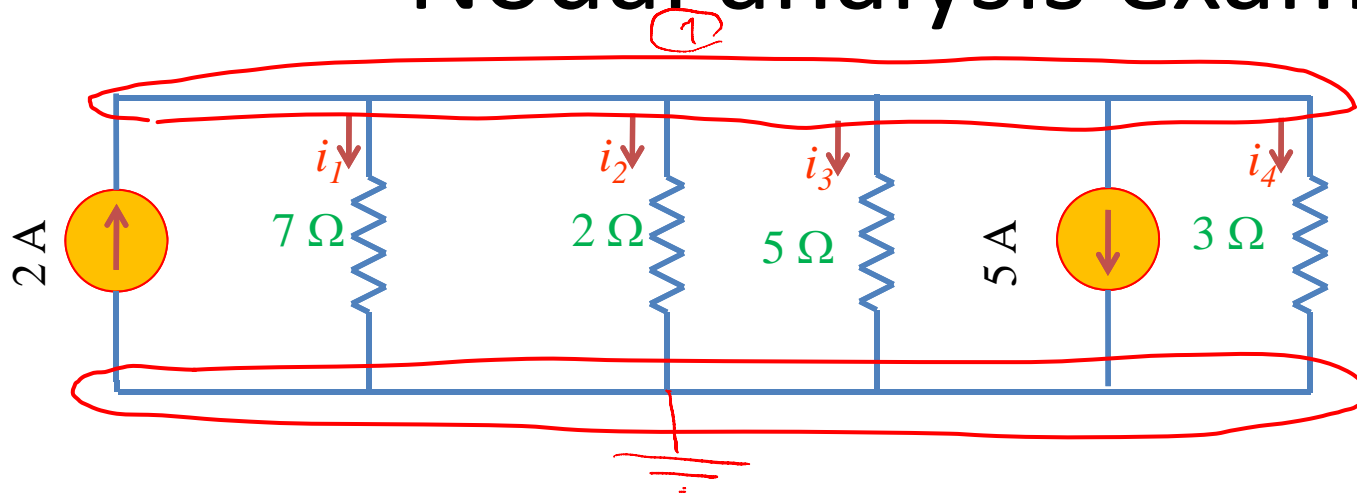
KCL \textcircled{a} supermesh

$$i_4 + 5A = i_5$$

5 eqns. 5 unknowns i_1, i_2, i_3, i_4, i_5



Nodal analysis example



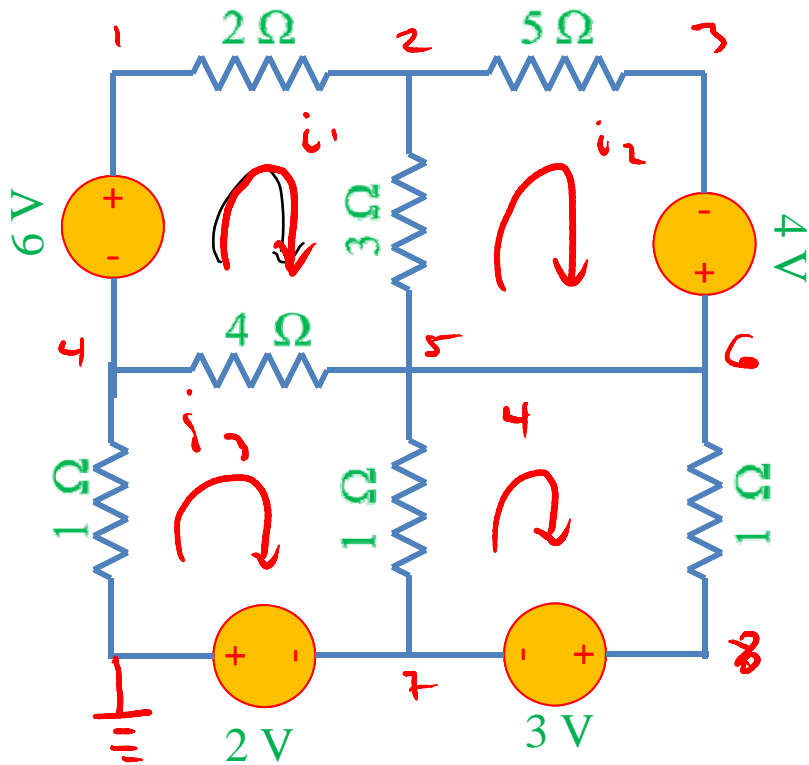
$$\text{KCL} : i_1 + i_2 + i_3 + i_4 + 5\text{A} - 2\text{A} = 0$$

$$\frac{V_1}{7} + \frac{V_1}{2} + \frac{V_1}{5} + \frac{V_1}{3} + 5 - 2 = 0$$

$$V_1 \left(\frac{1}{7} + \frac{1}{2} + \frac{1}{5} + \frac{1}{3} \right) = -3$$

$$V_1 = -2.5\text{V}$$

Nodal vs. Mesh Analysis



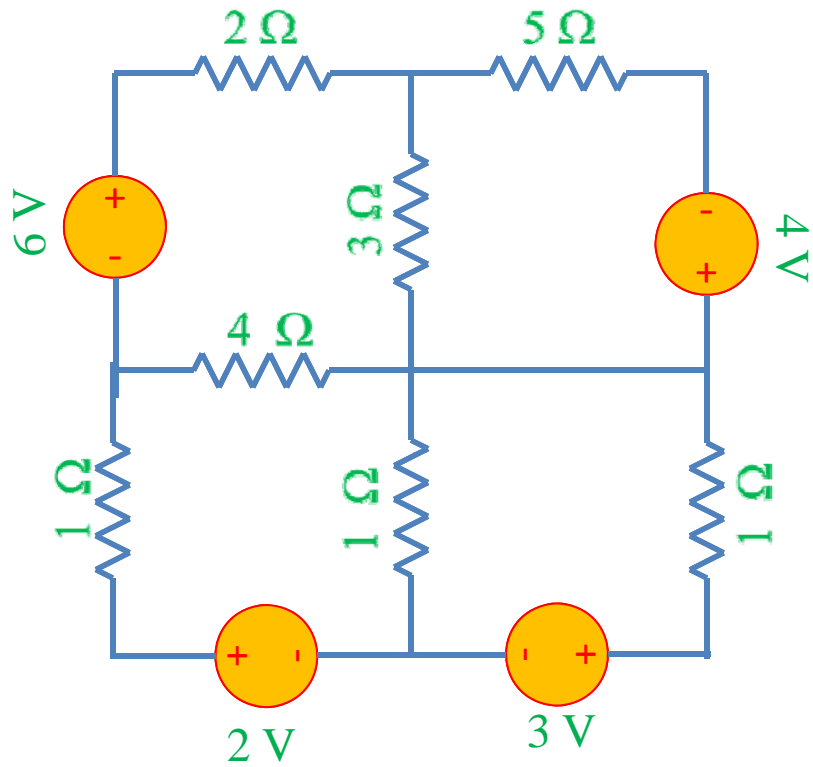
MESH

4 unknowns
4 eqns.

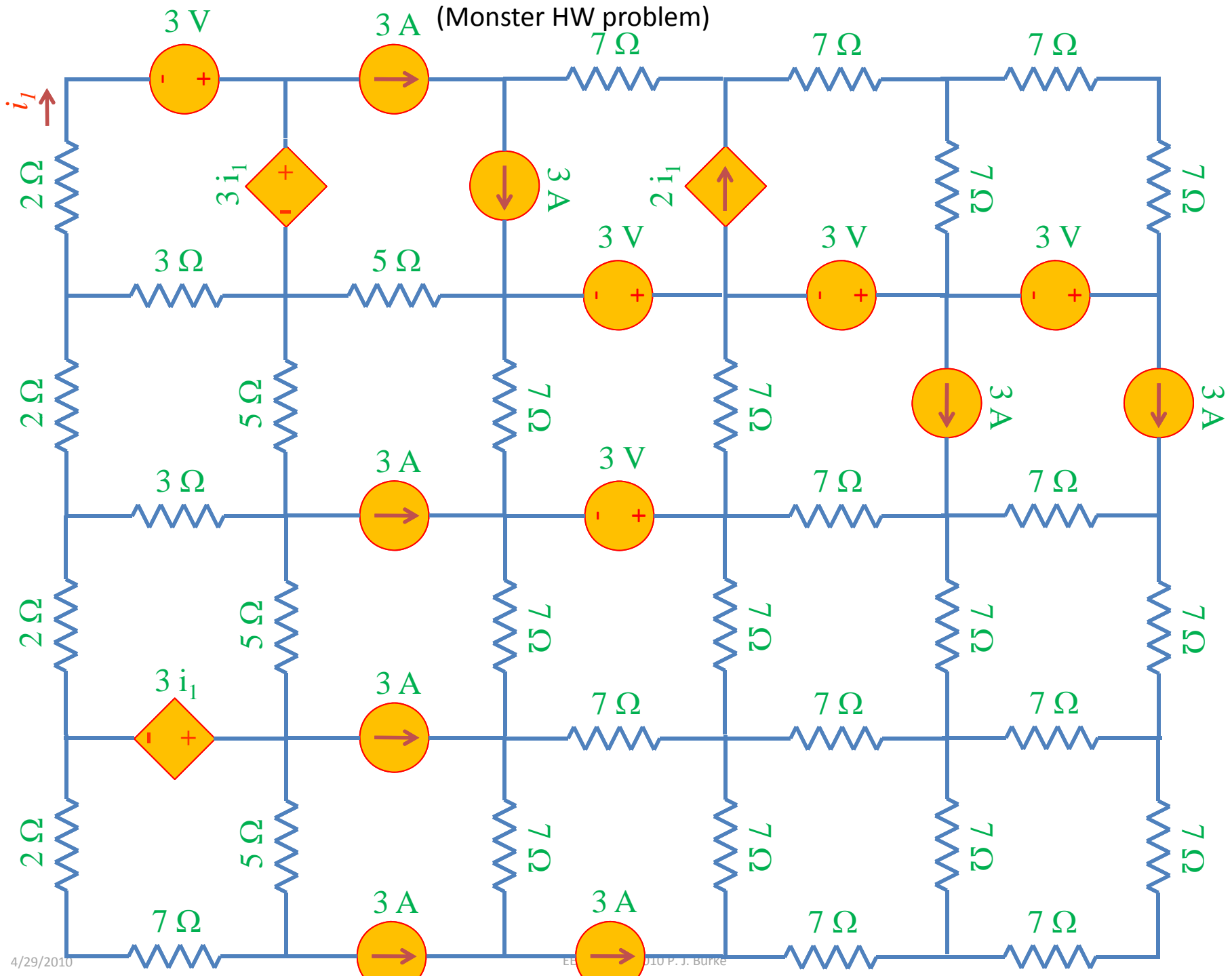
NODAL

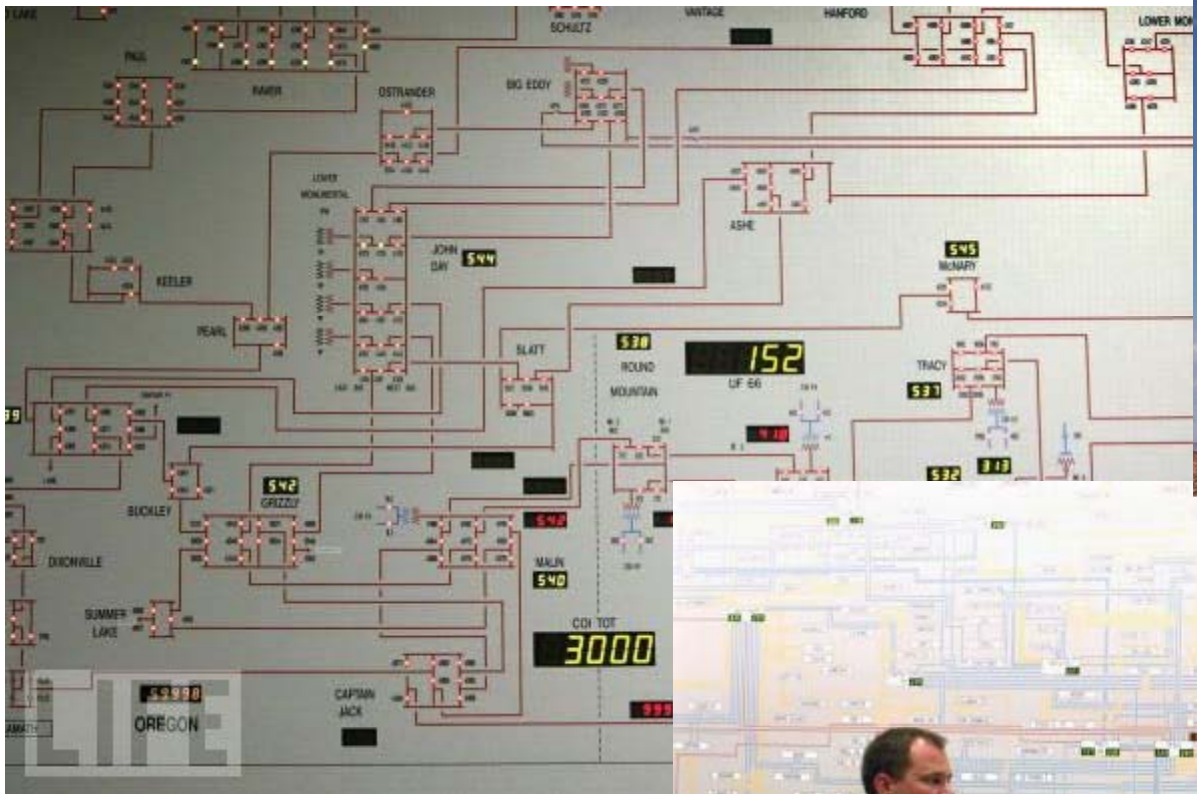
$V_1 - V_8$ unknowns
8 eqns.

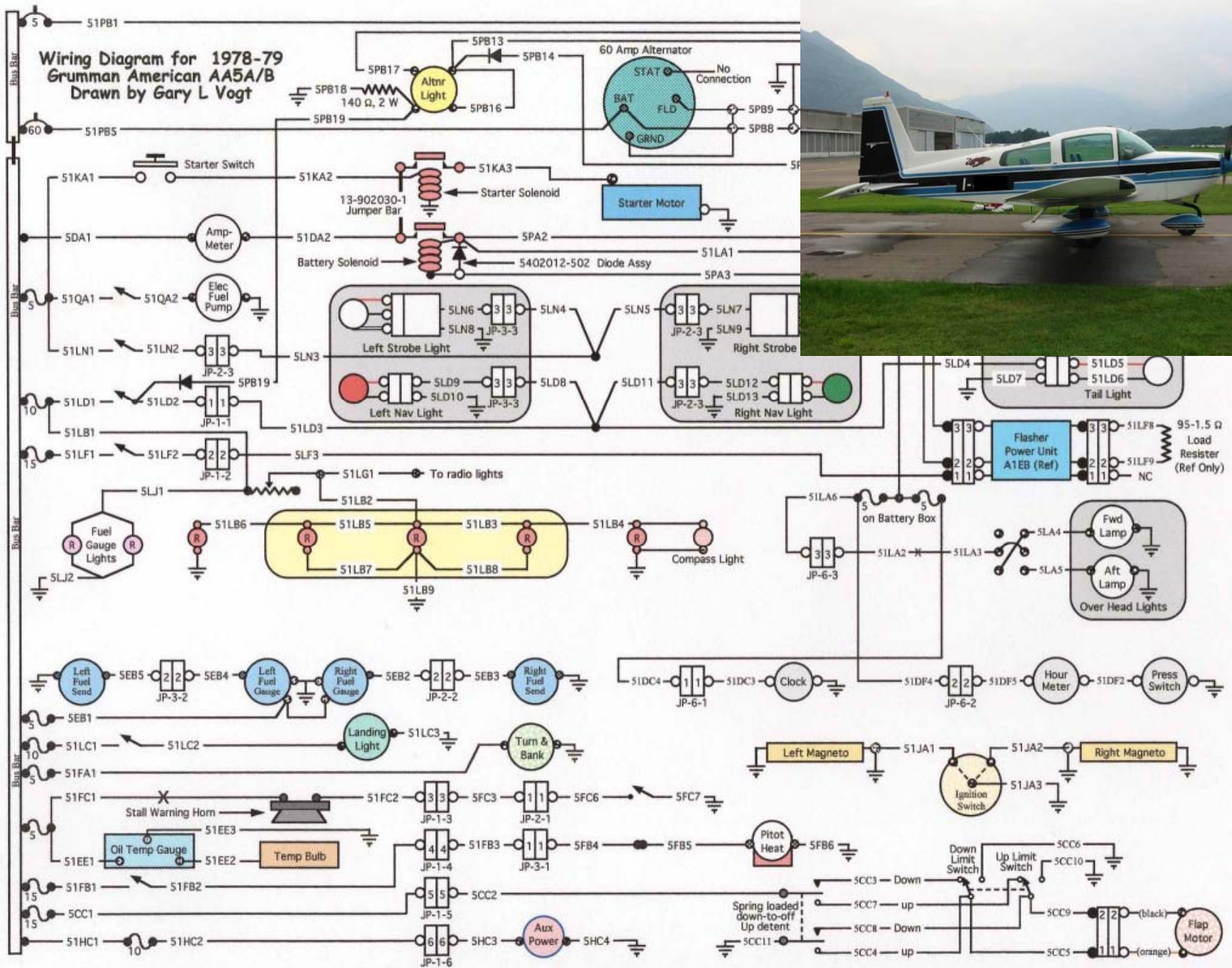
Nodal vs. Mesh Analysis

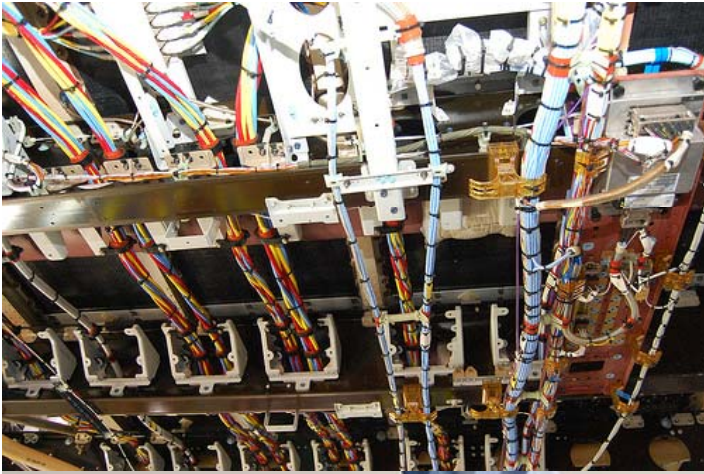


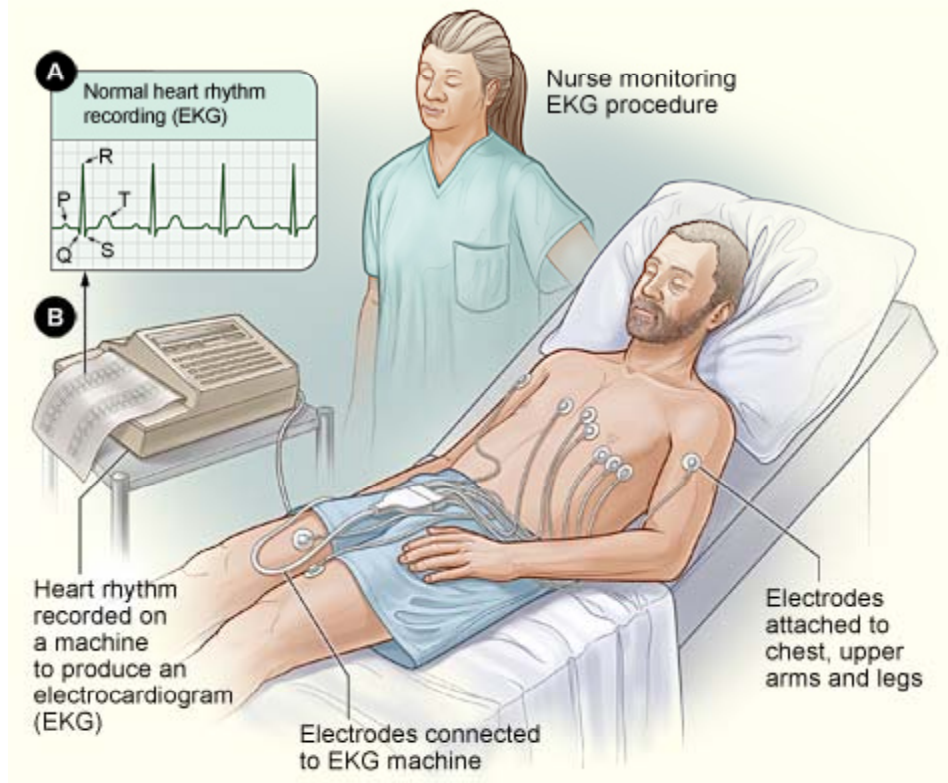
3 A (Monster HW problem)

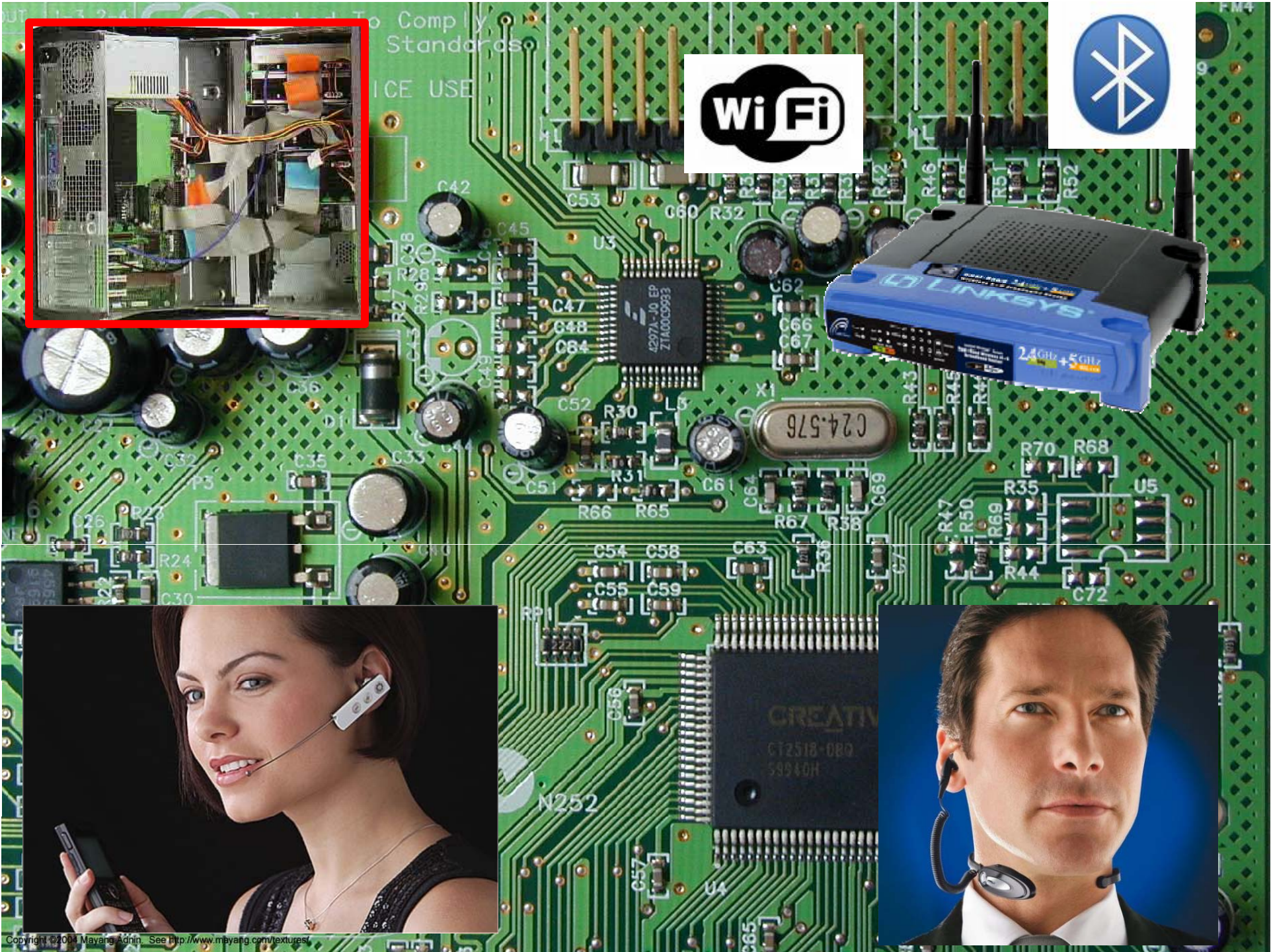




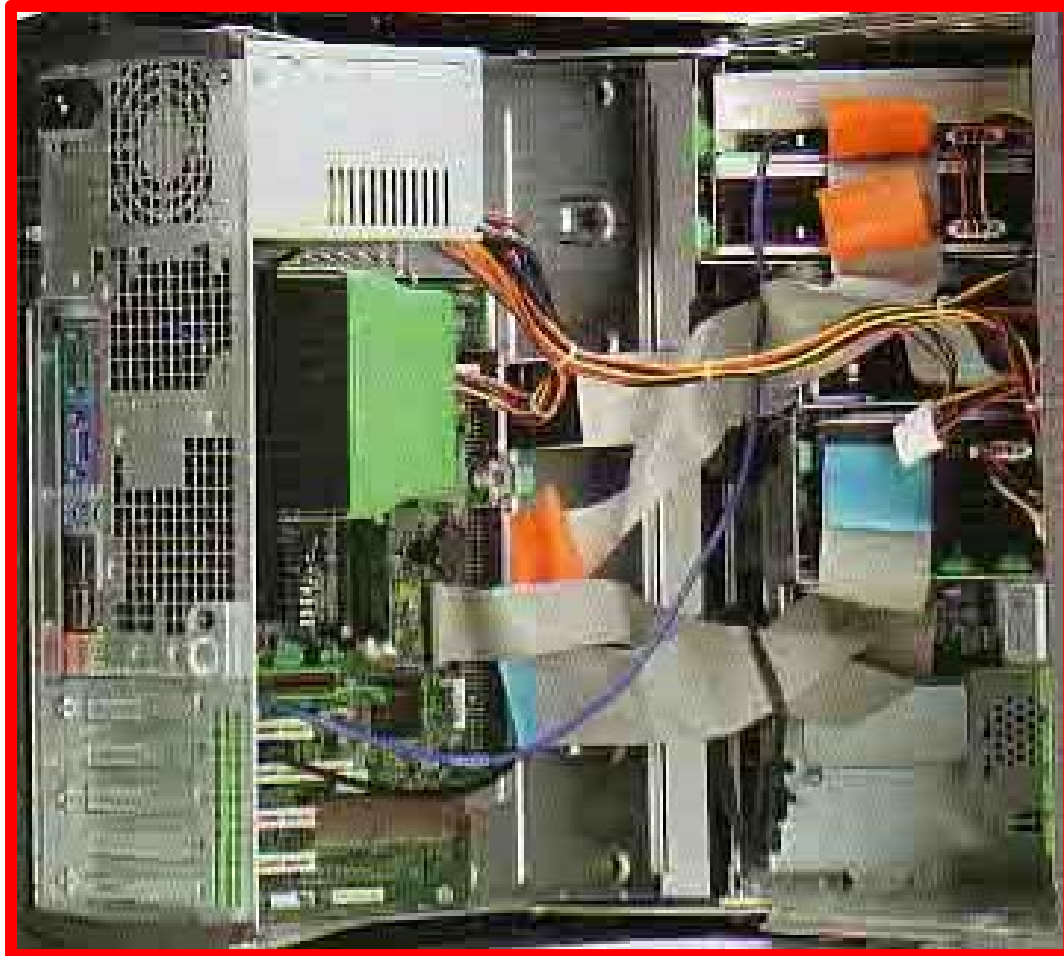






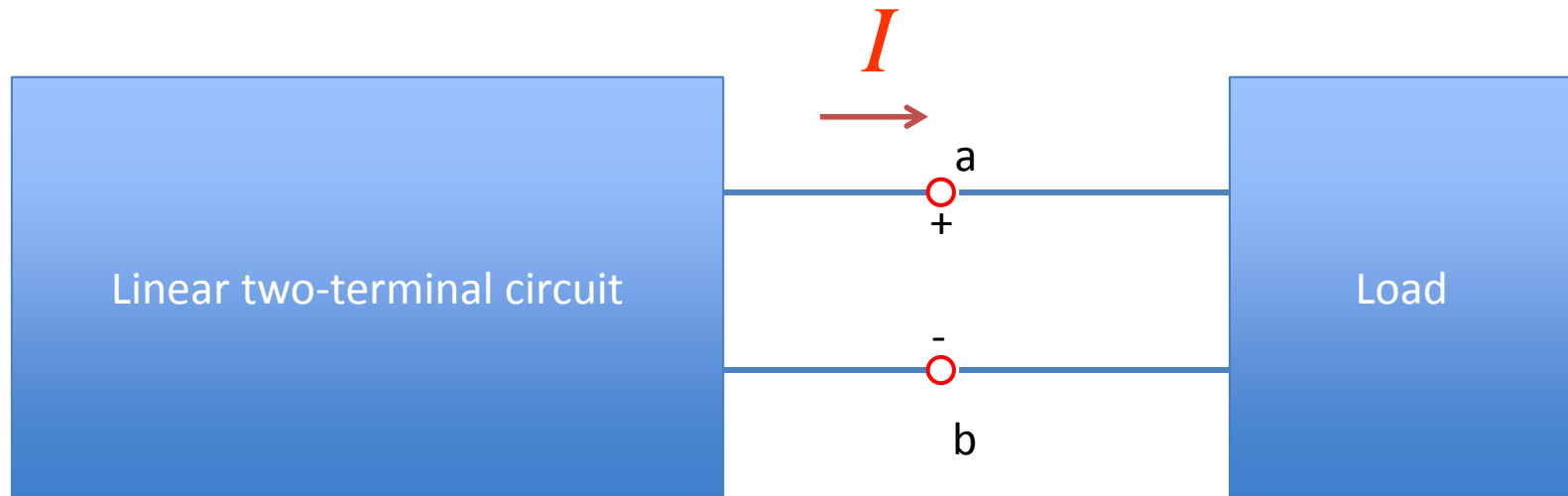


Compartmentalization: Need for simplicity

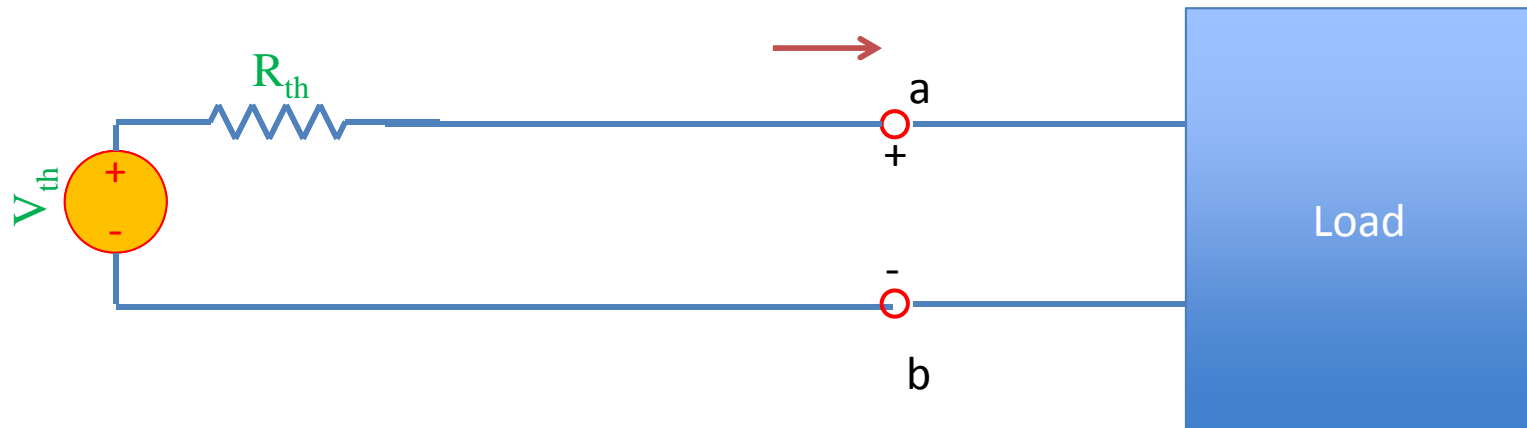


Power brick image.
And ask class to show their own...
Demo: Computer?

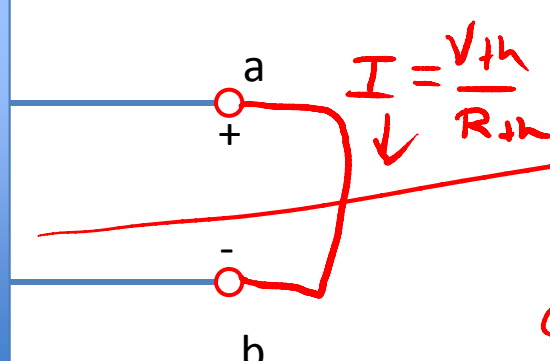
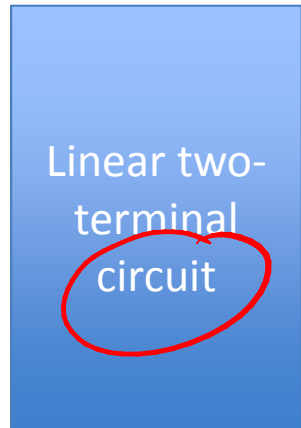
Thevenin's Theorem



Equivalent to:



Finding V_{th} , R_{th}

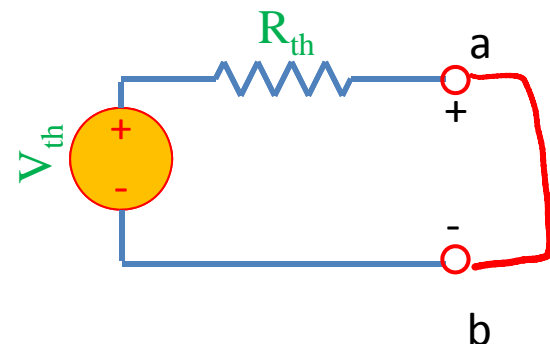


Goal: Find V_{th} , R_{th} :

Given: circuit

1) Find V_{th} :
Calculate V_{ab} (when no external circuit connected)

Equivalent to:

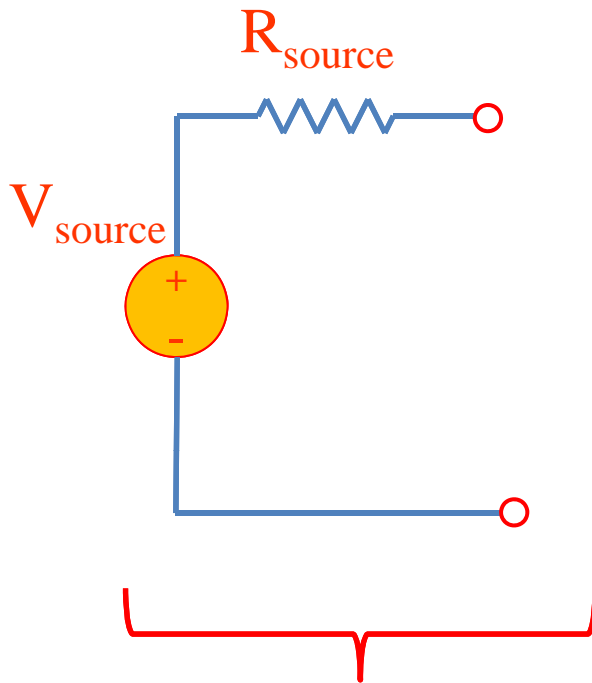


$I = \frac{V_{th}}{R_{th}}$

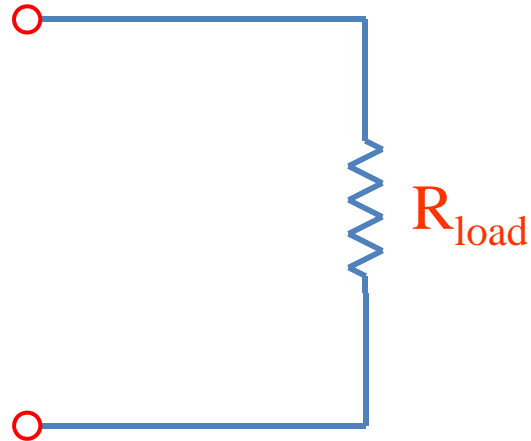
$\Leftrightarrow R_{th} = \frac{V_{th}}{I_{short\ circuit}}$

2) Find $I_{short\ circuit}$
Then $R_{th} = \frac{V_{th}}{I_{short\ circuit}}$

Source/load



Thevenin Thm:
Any circuit can be
represented by this
equivalent circuit.



$$V_{load} = \frac{R_{load}}{R_{load} + R_{source}} V_{source}$$

Derivation:

Case 1:

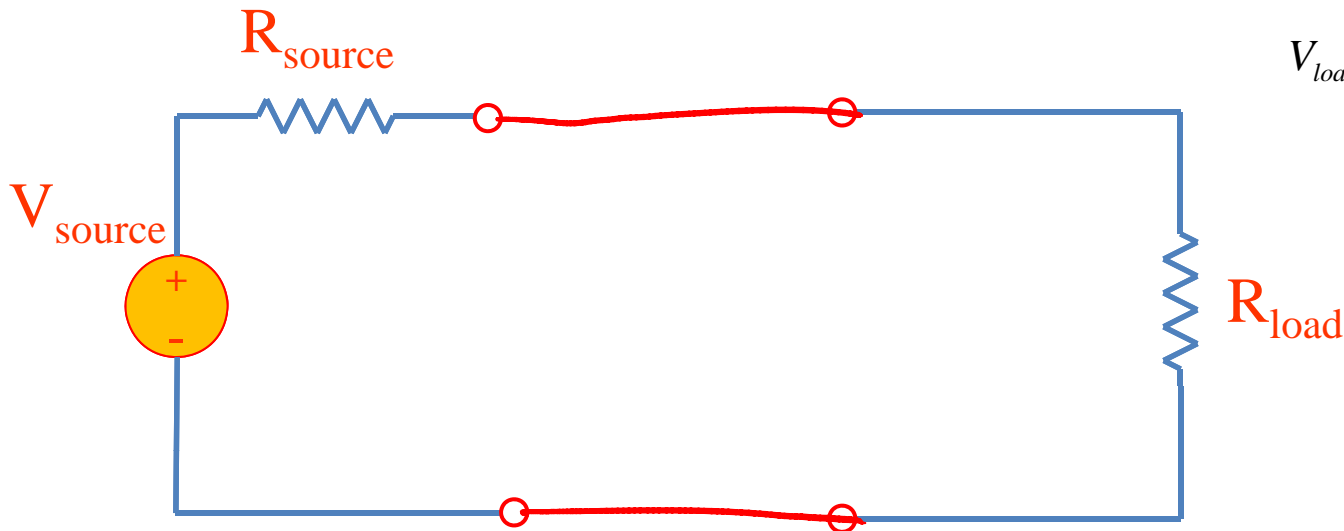
$$R_{load} \gg R_{source}$$

Case 2:

$$R_{source} \gg R_{load}$$

We say R_{load} “loads down” the source.

Source/load



$$V_{load} = \frac{R_{load}}{R_{load} + R_{source}} V_{source}$$

Derivation:



Thevenin Thm:
Any circuit can be represented by this equivalent circuit.

Case 1:

$$R_{load} \gg R_{source}$$

$$\Rightarrow V_{load} \approx V_{source}$$

Case 2:

$$R_{source} \gg R_{load}$$

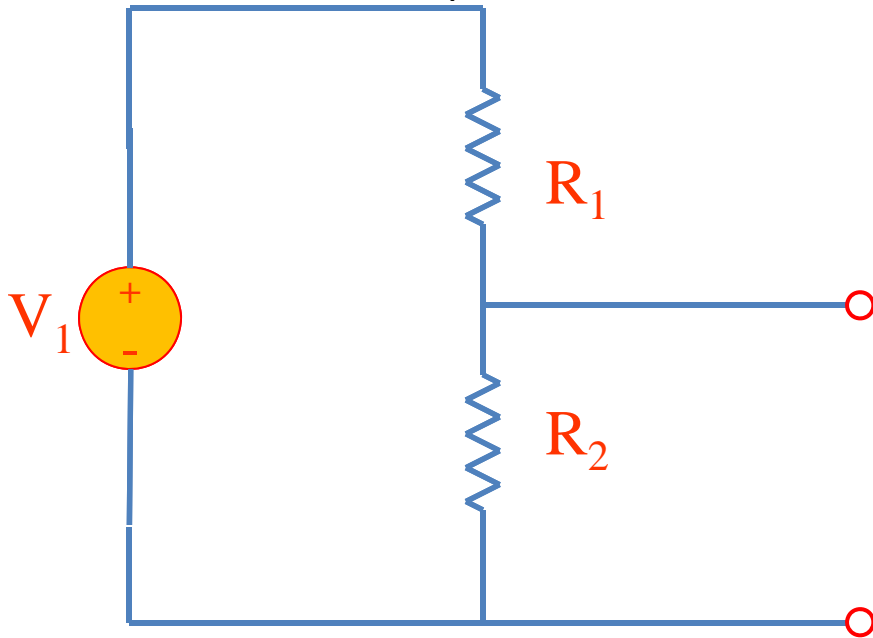
$$V_{load} \rightarrow 0$$

$$\approx \frac{R_{load}}{R_{source}} V_{source}$$

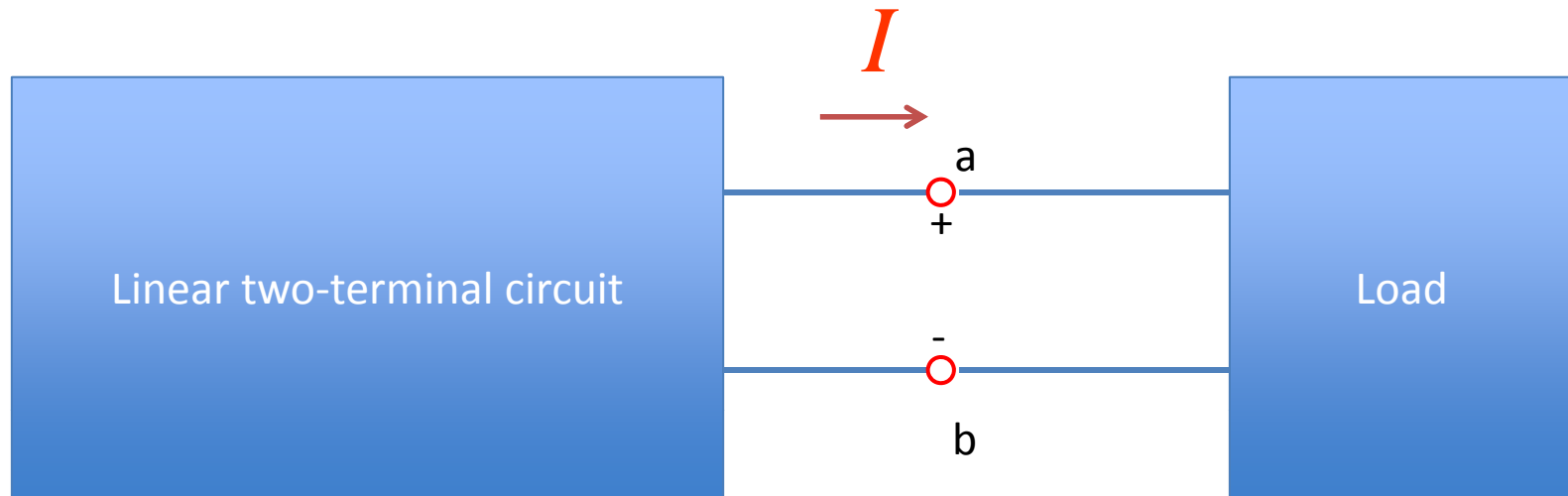
We say R_{load} "loads down" the source.

Example

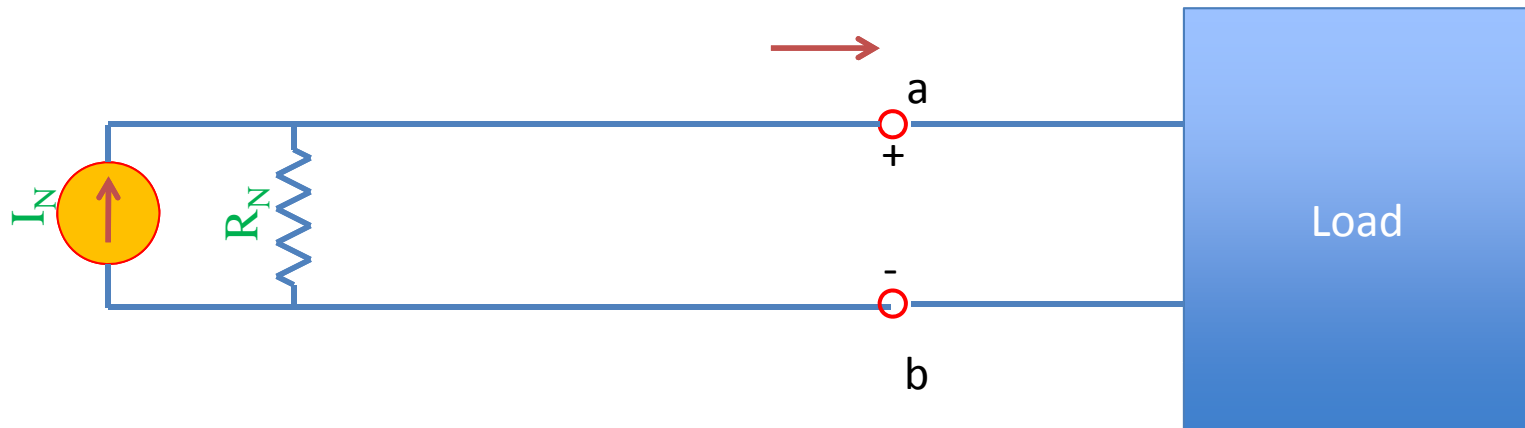
Find Thevenin equivalent circuit:



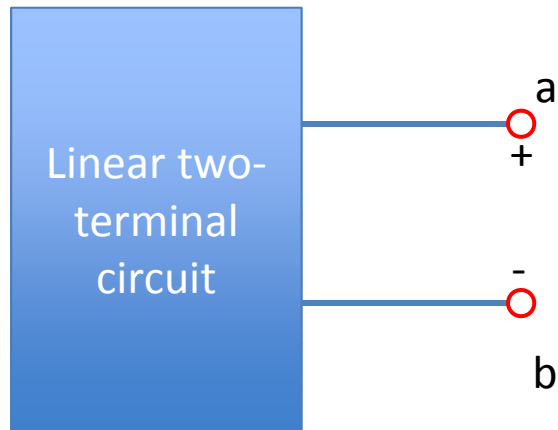
Norton's Theorem



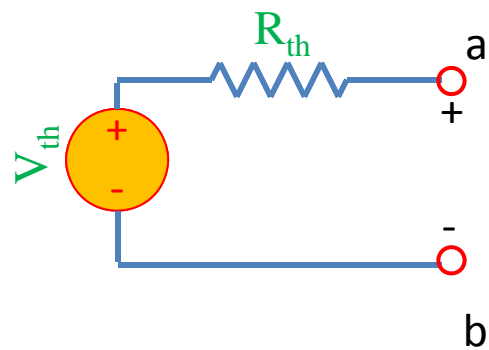
Equivalent to:



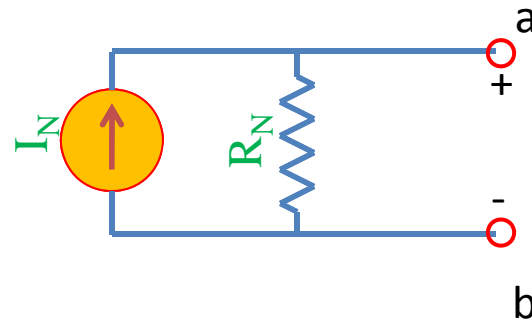
Finding V_{th} , R_{th}



Equivalent to:

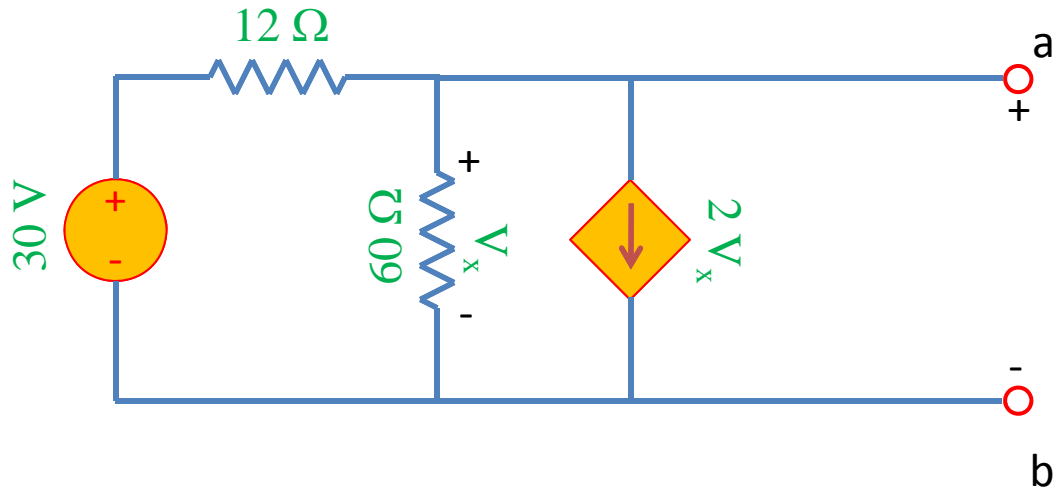


Equivalent to:

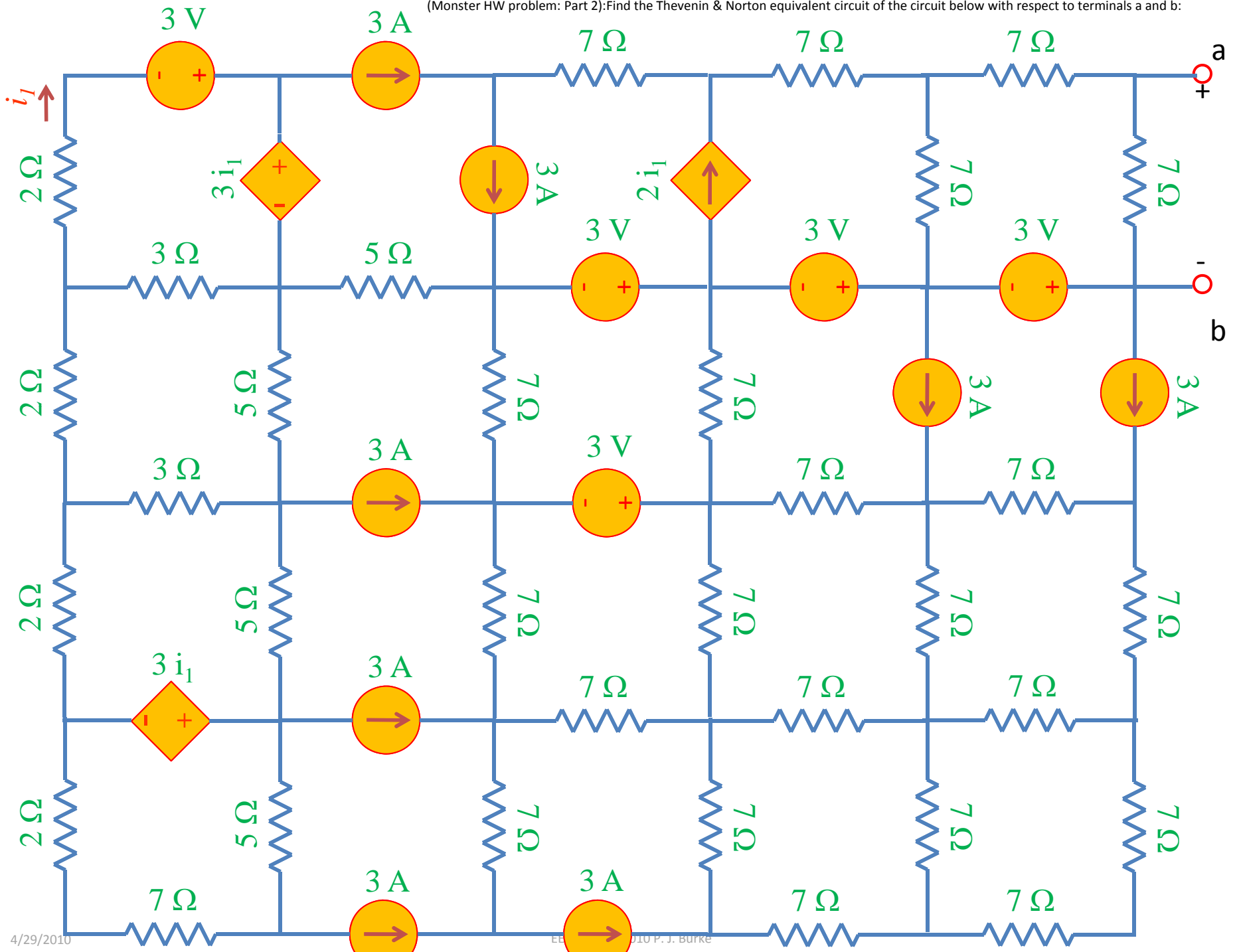


Example

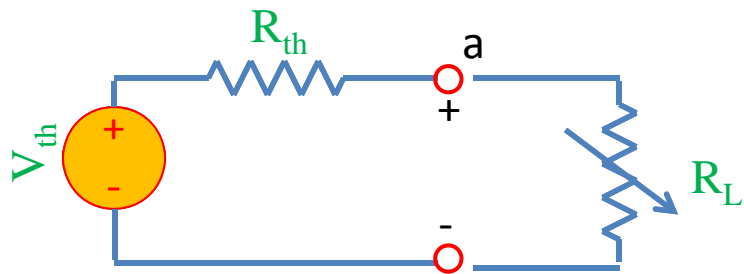
Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:



(Monster HW problem: Part 2): Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:



Power

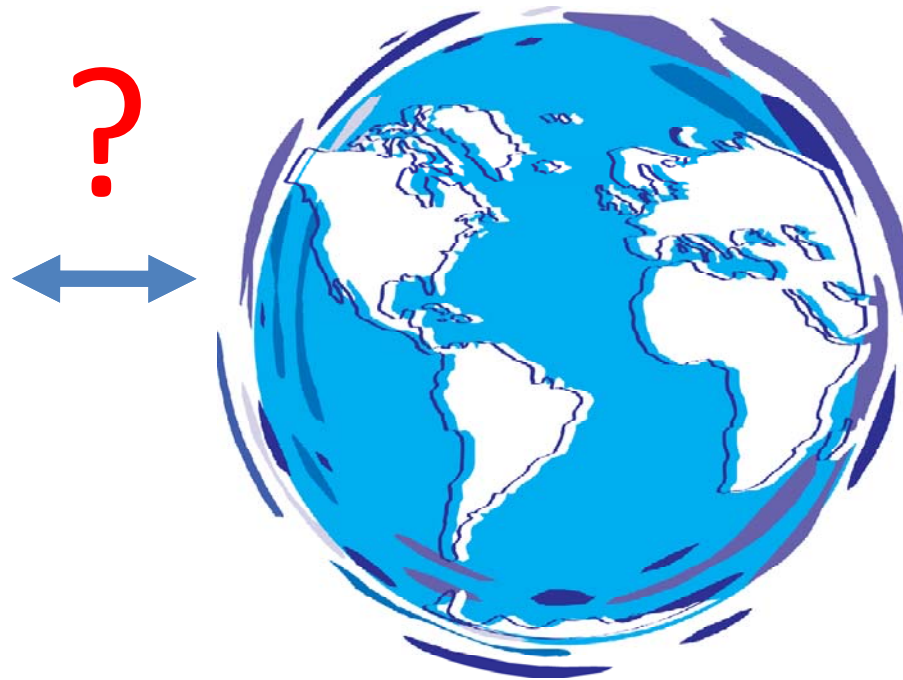


Arrow means R_L variable (e.g. by a knob)

Power delivered to load = ?

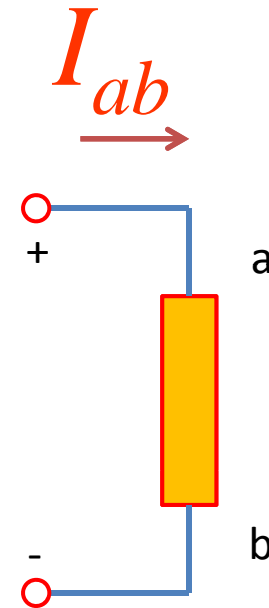
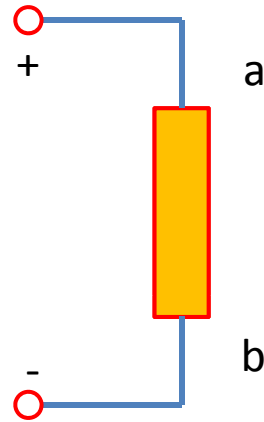
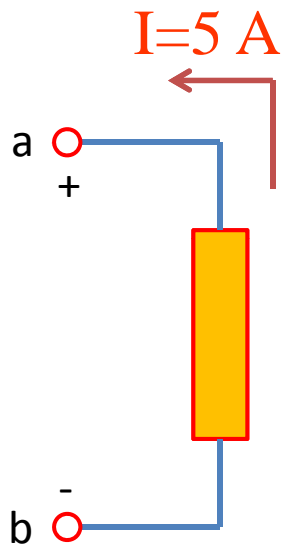
Questions?

Ground?

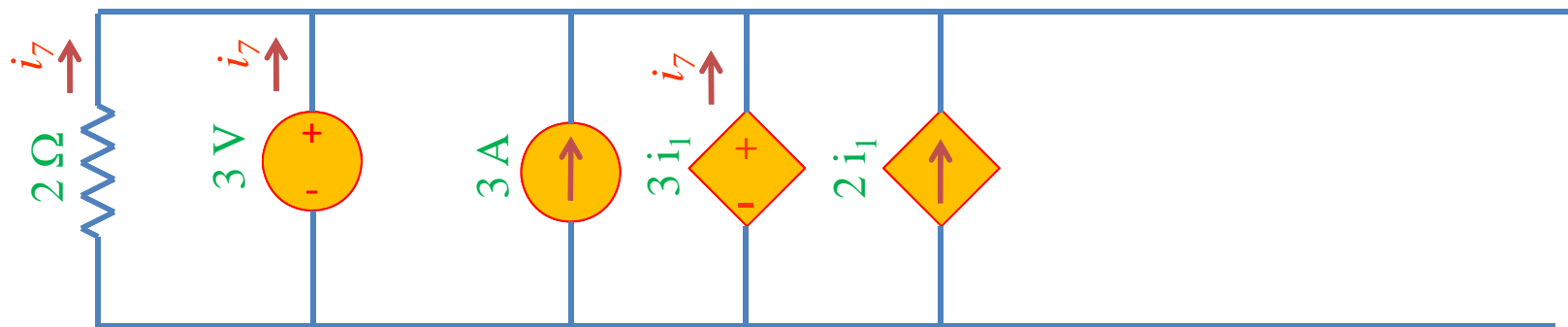
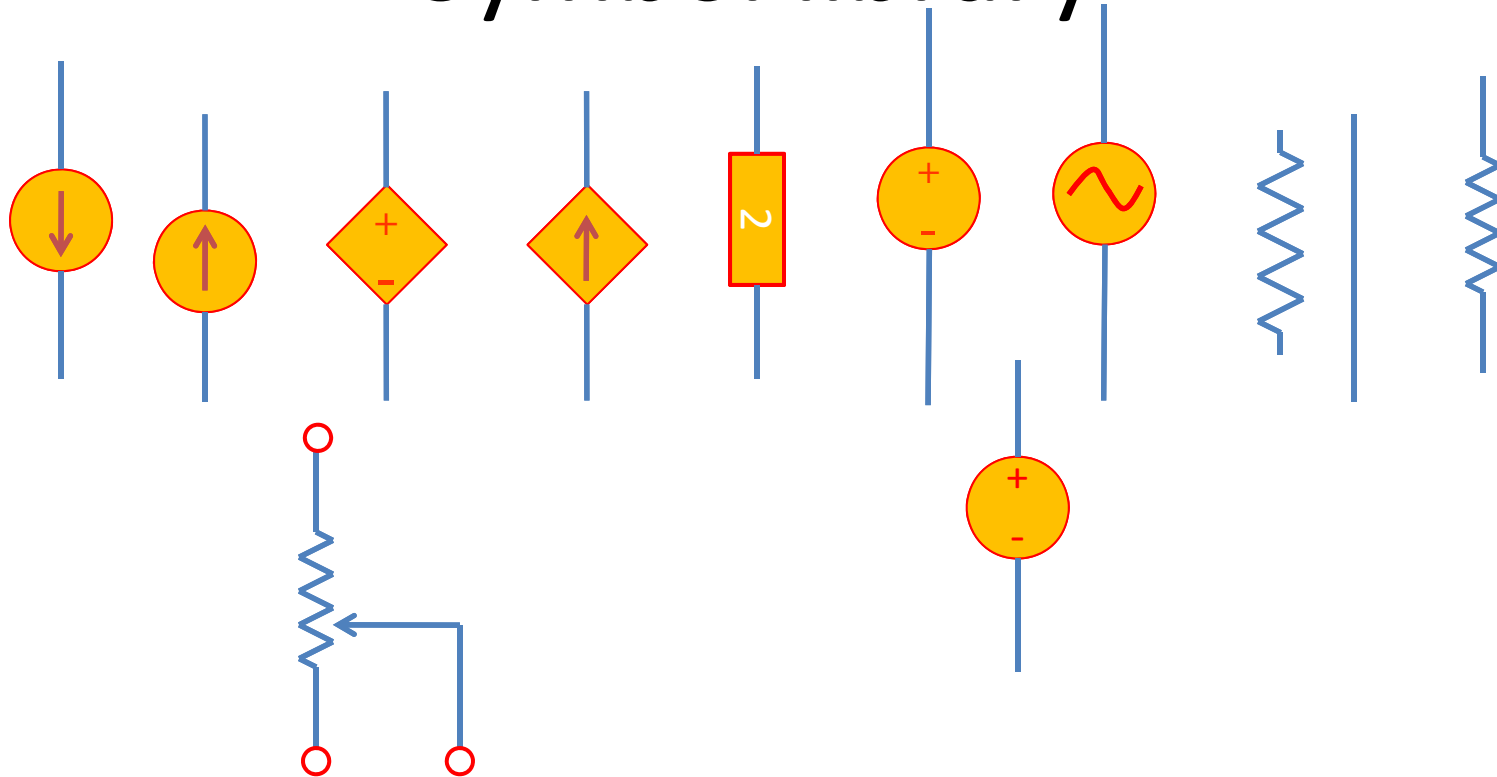


Ground
Reference
Earth

Symbol library



Symbol library



Symbol & circuit library

