Announcements:

1. HW will be due on Wednesday this week (check website for new version)
2. Graded midterms are for pickup from TAs

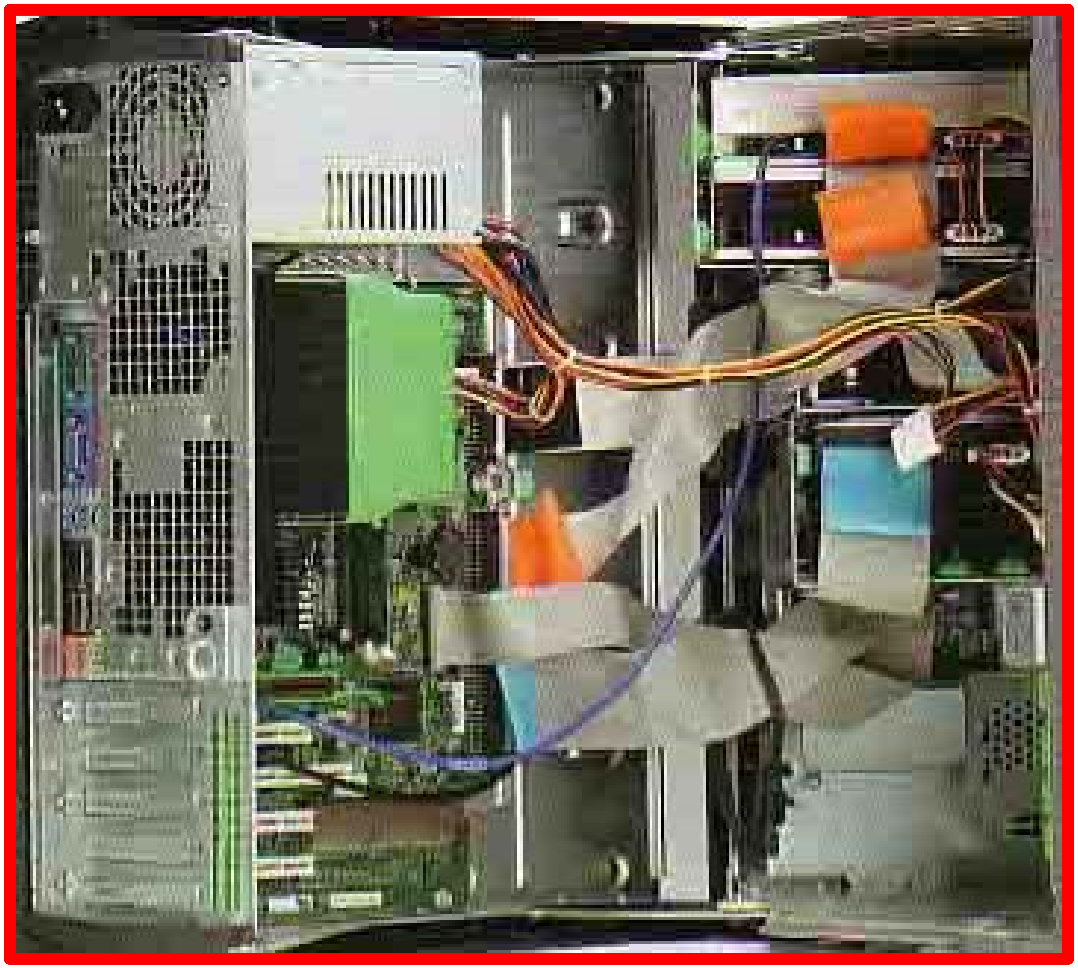
# EECS 70A: Network Analysis

## Lecture 9

# Today's Agenda

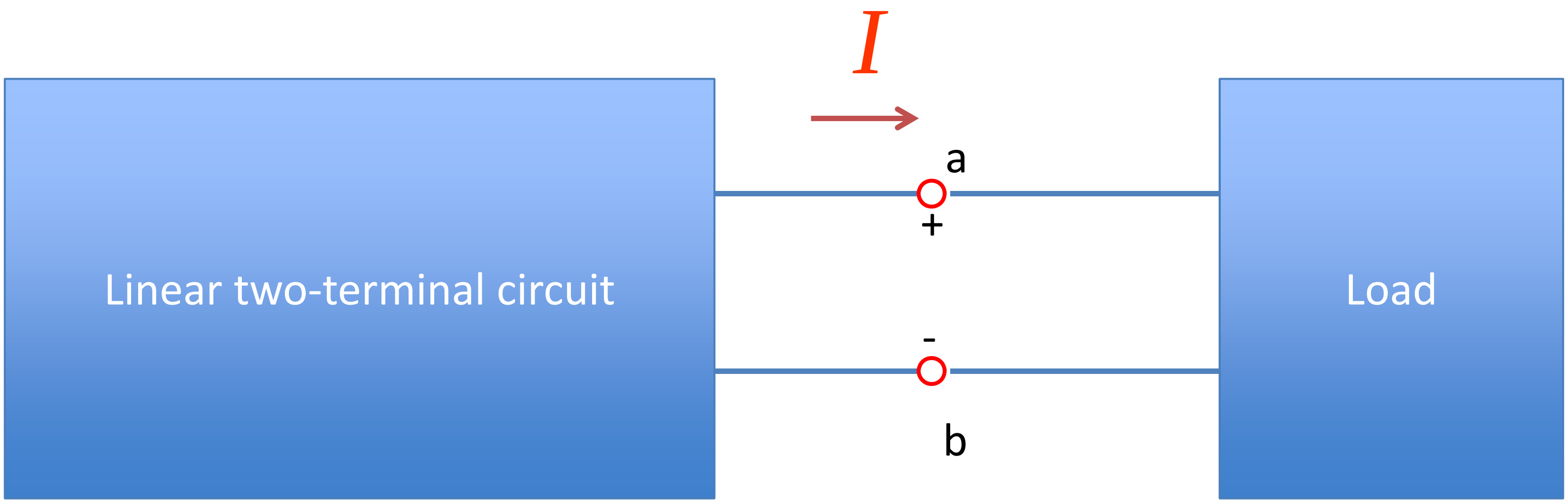
- Thevinin/Norton theorem
- Power transfer
- Capacitors
- Inductors

# Compartmentalization: Need for simplicity

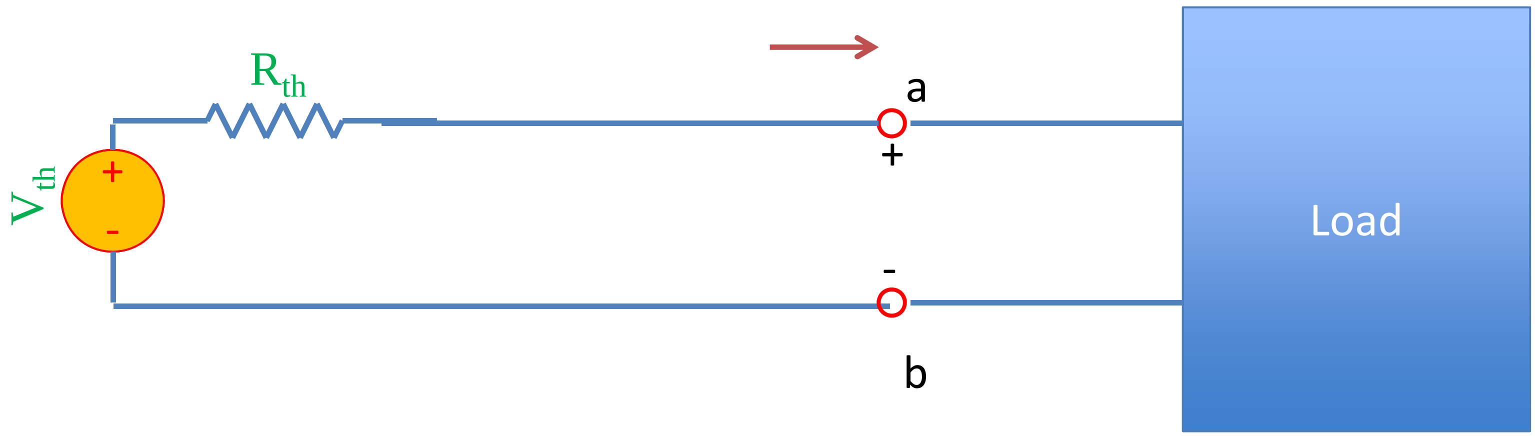


Power brick image.
And ask class to show their own…
Demo: Computer?

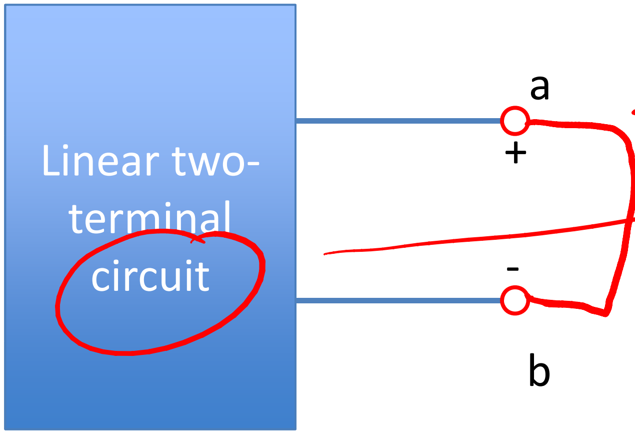
# Thevenin's Theorem




Equivalent to:



# Finding $V_{th}$, $R_{th}$




$I = \frac{V_{th}}{R_{th}}$

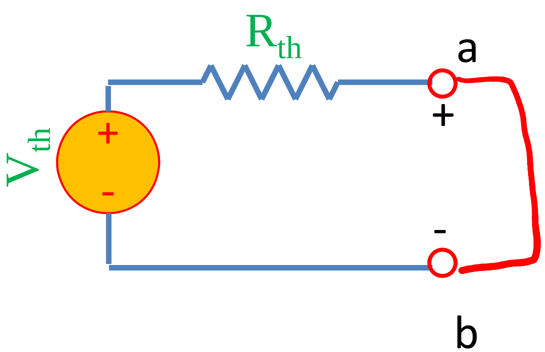
**Goal:** Find $V_{th}$, $R_{th}$:

Given: circuit

1) Find $V_{th}$:

Calculate $V_{ab}$ (when no external circuit connected)

Equivalent to:




2) Find $I_{short\ circuit}$

Then $R_{th} = \frac{V_{th}}{I_{short\ circuit}}$

$I = \frac{V_{th}}{R_{th}}$

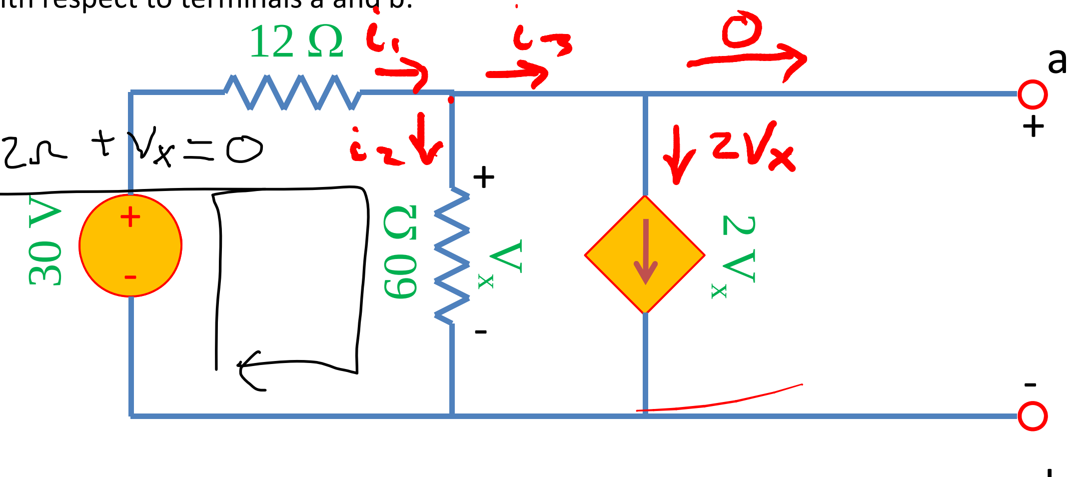
$\Leftrightarrow R_{th} = \frac{V_{th}}{I_{short\ circuit}}$

# Example

Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:

KVL

$-30V + i_1 12\Omega + V_x = 0$

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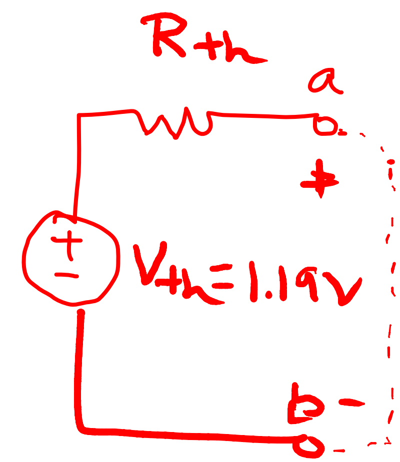



1) Find $V_{ab}$ complete circuit w/ nothing connected to a, b.

$V_{ab} = V_x$ KCL $\Rightarrow i_1 = i_2 + i_3$ (*)

$i_1 = \frac{30V - V_x}{12\Omega}$ $i_2 = \frac{V_x}{60}$ $i_3 = 2V_x$

(*) $\Rightarrow \frac{30V - V_x}{12\Omega} = \frac{V_x}{60} + 2V_x \Rightarrow V_x = 1.19\ V$ ✓

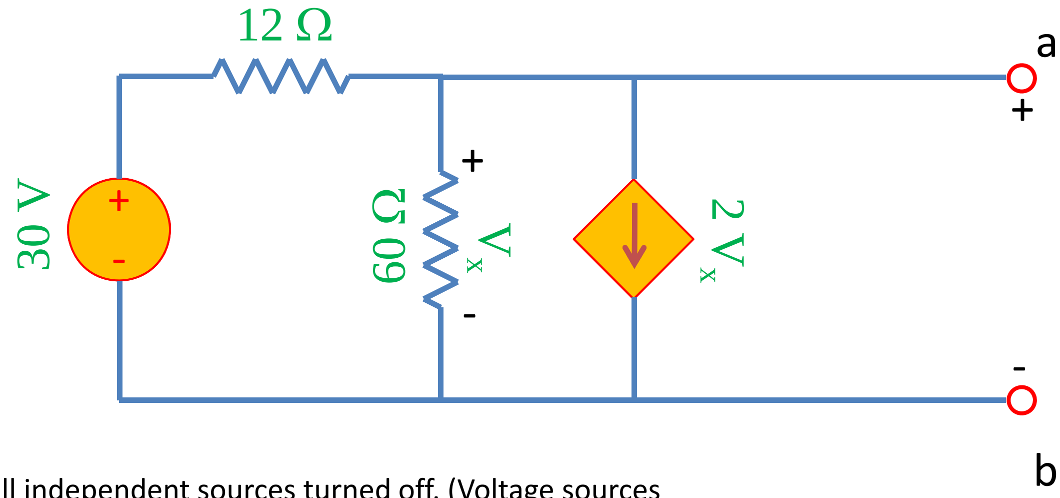
$= V_{th}$

→ 2) Find $I_{short\ circuit}$

$= \frac{30V}{12\Omega} = 2.5\ A \Rightarrow I_{short\ circuit} = \frac{V_{th}}{R_{th}}$

$\Rightarrow R_{th} = \frac{1.19V}{2.5A} = 0.476\Omega$

# Alternate method to find $R_{th}$:

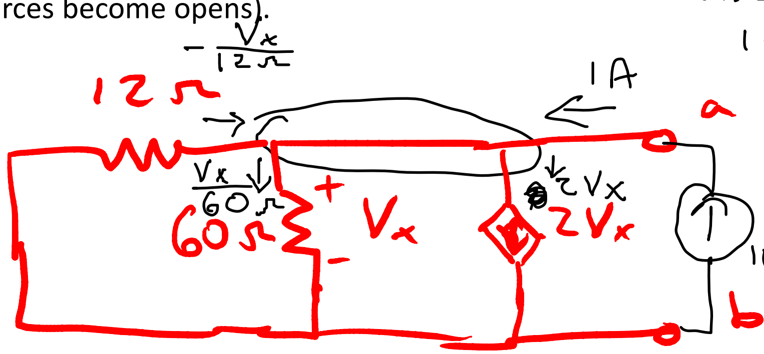



$R_{ab}$

Find $R_{ab}$ when all independent sources turned off. (Voltage sources become shorts, current sources become opens).

IN = OUT

$1A - \frac{V_x}{12\Omega} = \frac{V_x}{60\Omega} + 2V_x$




$R_{ab} = R_{th}$

TRICK: APPLY 1A CURRENT SOURCE TO a-b

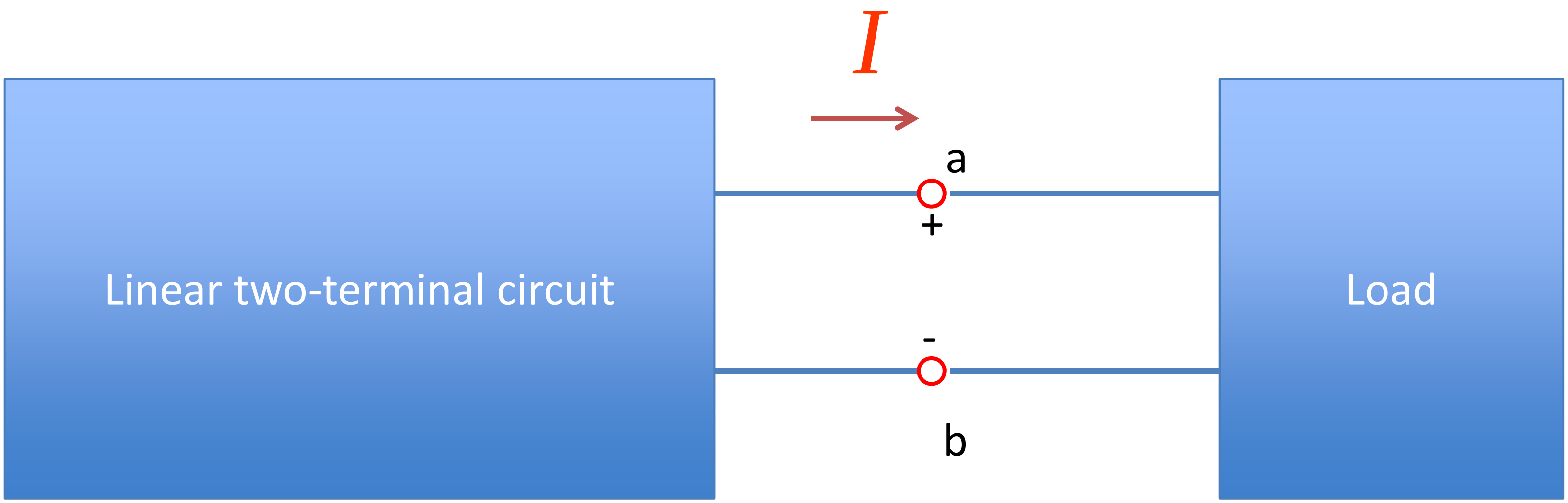
FIND $V_{ab}$ UNDER THAT CONDITION.

EQUATE $R_{ab} = \frac{V_{ab}}{1A}$

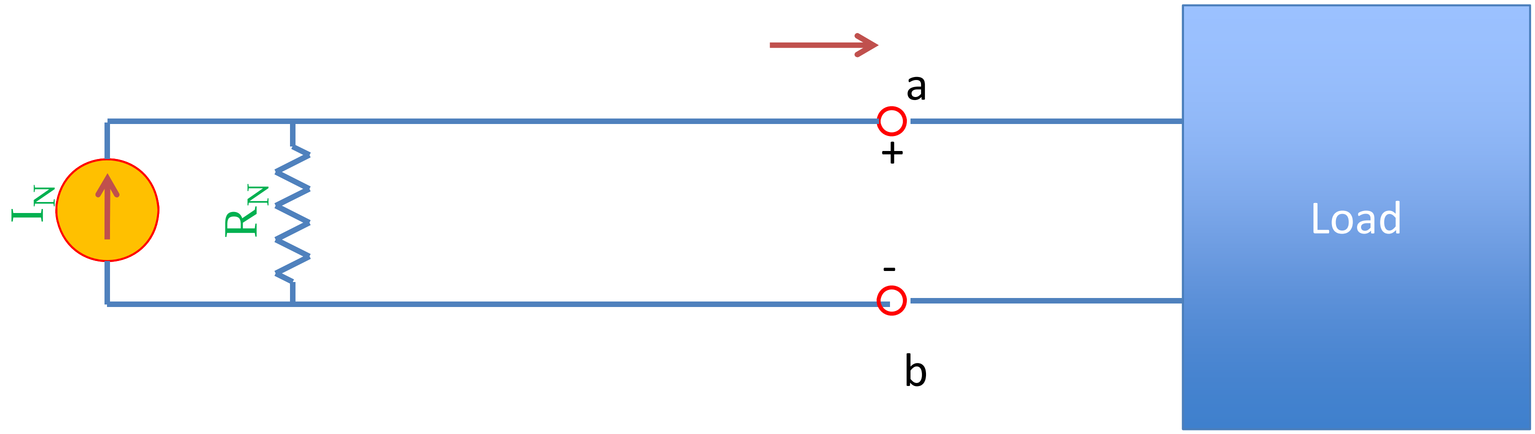
KCL $1A = 2V_x + \frac{V_x}{60\Omega} + \frac{V_x}{12\Omega} \Rightarrow V_x = 0.476V = V_{ab}$

$R_{ab} = \frac{0.476V}{1A} = 0.476\Omega = R_{th}$

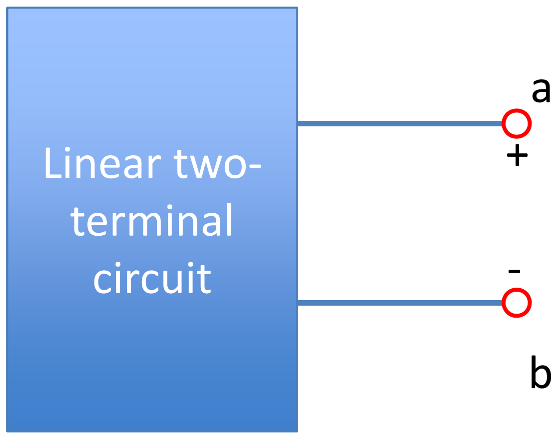
# Norton's Theorem




Equivalent to:



# Finding $V_{th}$, $R_{th}$




Once you Find $R_{th}$, $V_{th}$, easy to find $I_N$, $R_N$

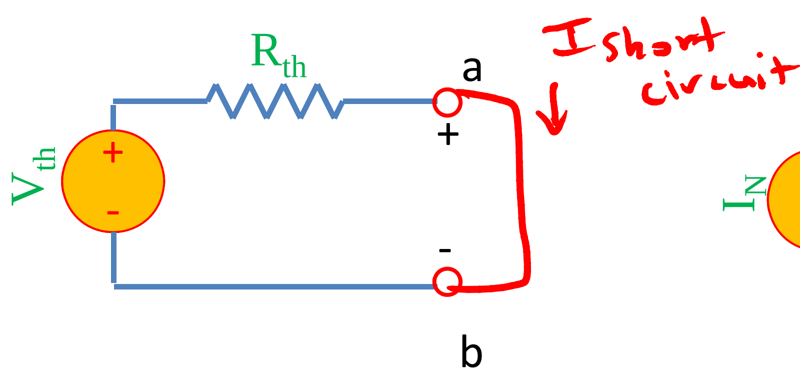
Open (means: nothing connected to ab)

$V_{ab} = V_{th} = I_N R_N$ (*)

Short (means: short a to b)

$I_{short\ circuit} = \frac{V_{th}}{R_{th}} = I_N$ (**)

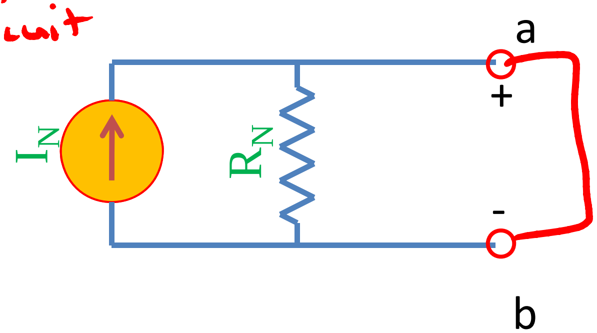
Equivalent to:




Equivalent to:

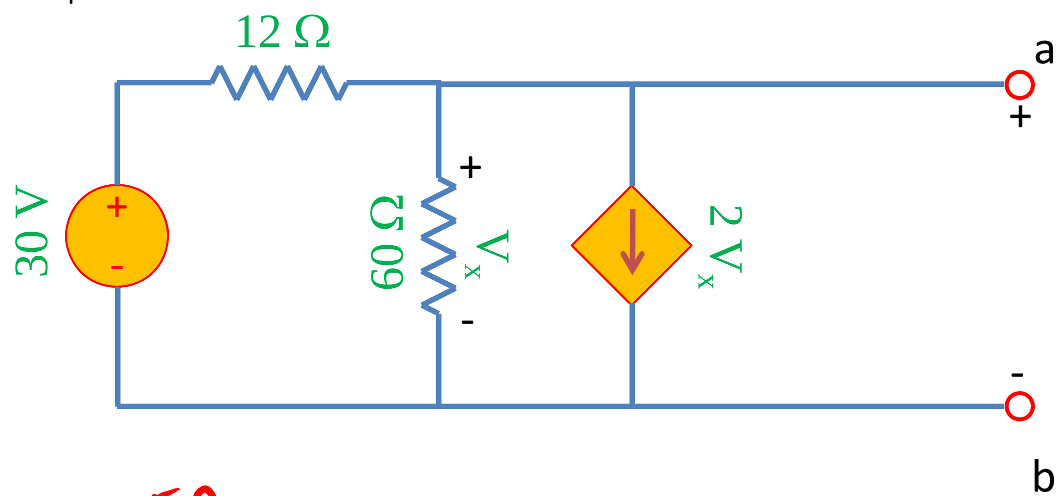
$R_N = R_{th}$

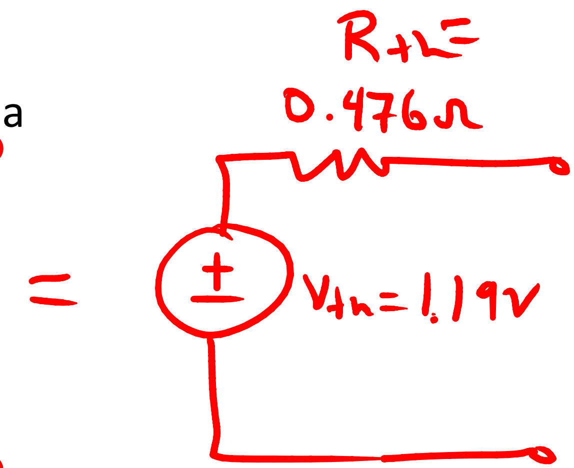
$I_N = V_{th}/R_{th}$

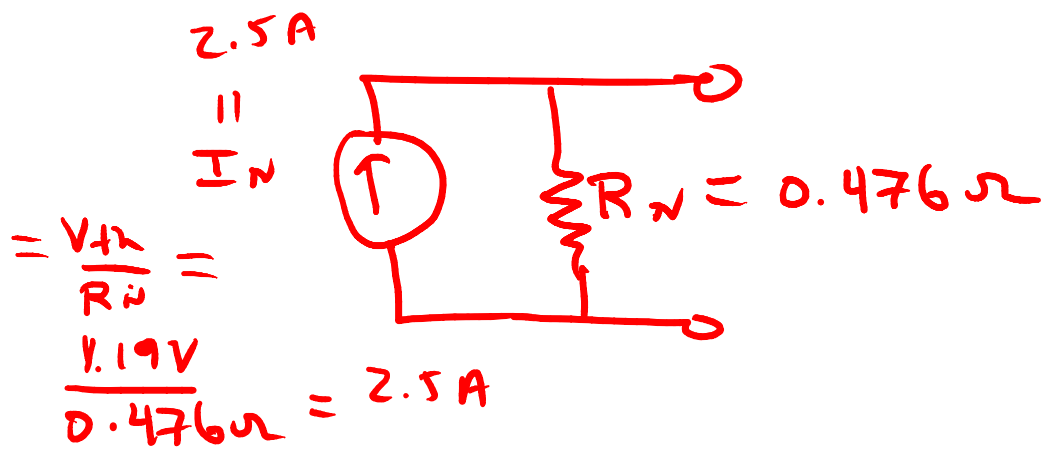


# Example

Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:

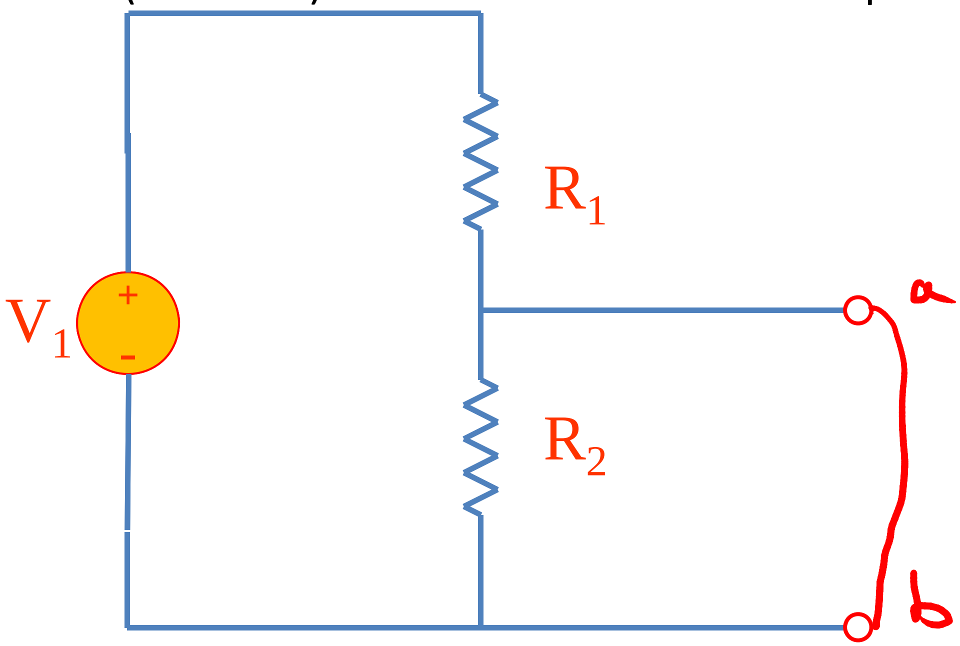


# IMPORTANT Example

(Students): Find Thevenin & Norton equivalent circuit: in terms of $V_1, R_1, R_2$




1) Find $V_{ab}$ (open)

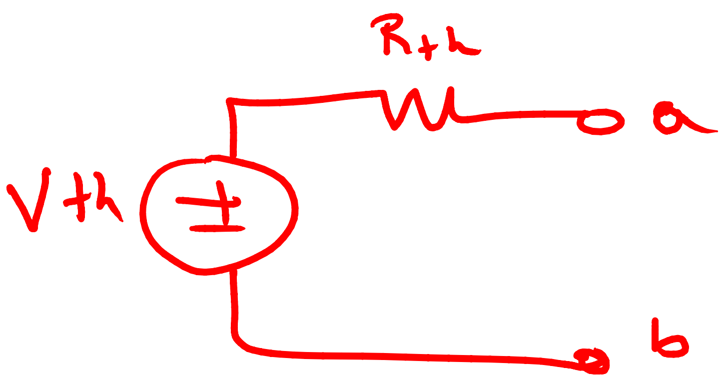
2) Find $I_{\text{short circuit}} = \frac{V_1}{R_1}$

$$V_{ab} = \frac{R_2 V_1}{R_1 + R_2} = V_{th}$$

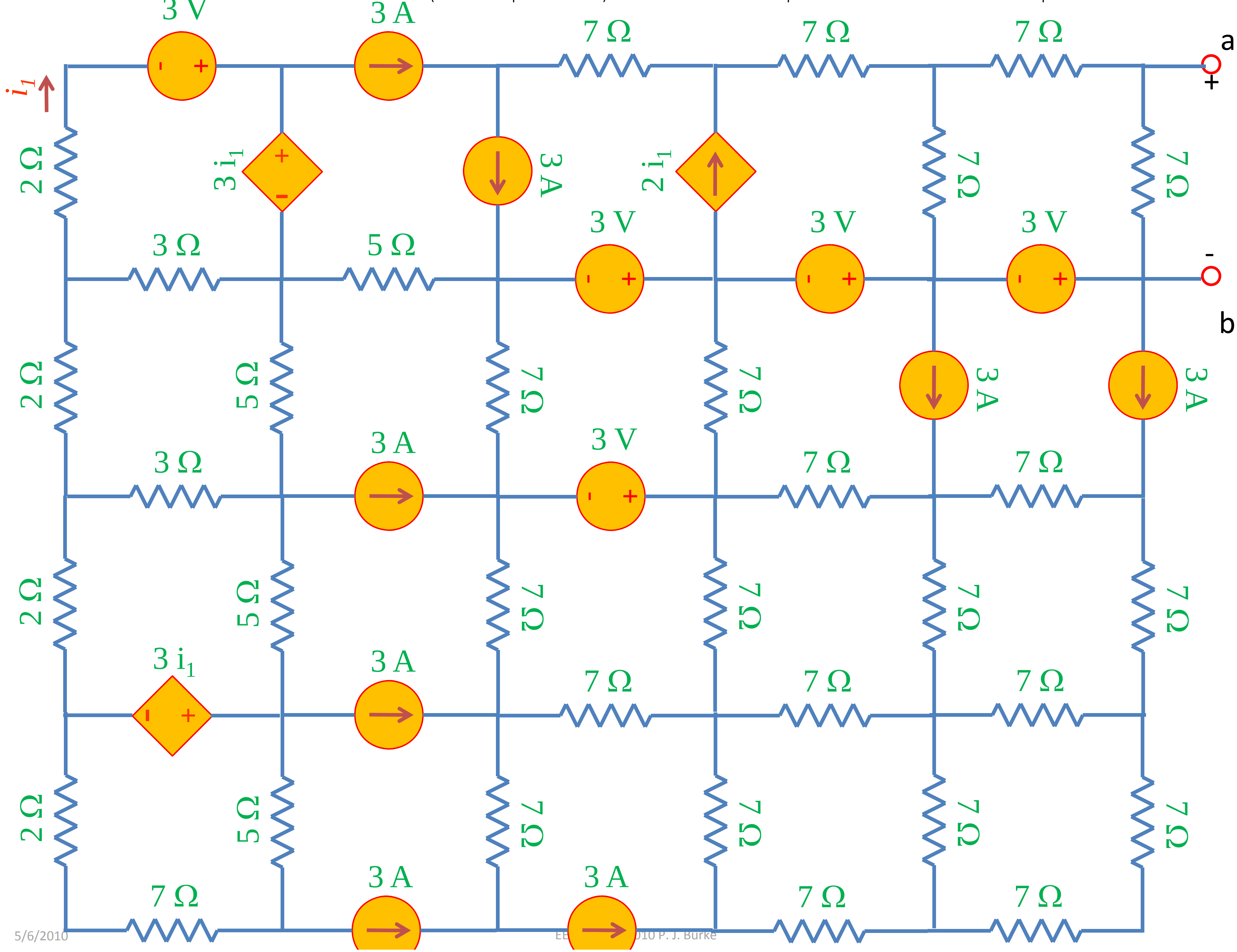
$$R_{th} = \frac{V_{th}}{I_{\text{short circuit}}} = \frac{\frac{R_2 V_1}{R_1 + R_2}}{\frac{V_1}{R_1}}$$

$$= \frac{R_1 R_2}{R_1 + R_2} = R_1 \| R_2$$

$$= R_{th}$$

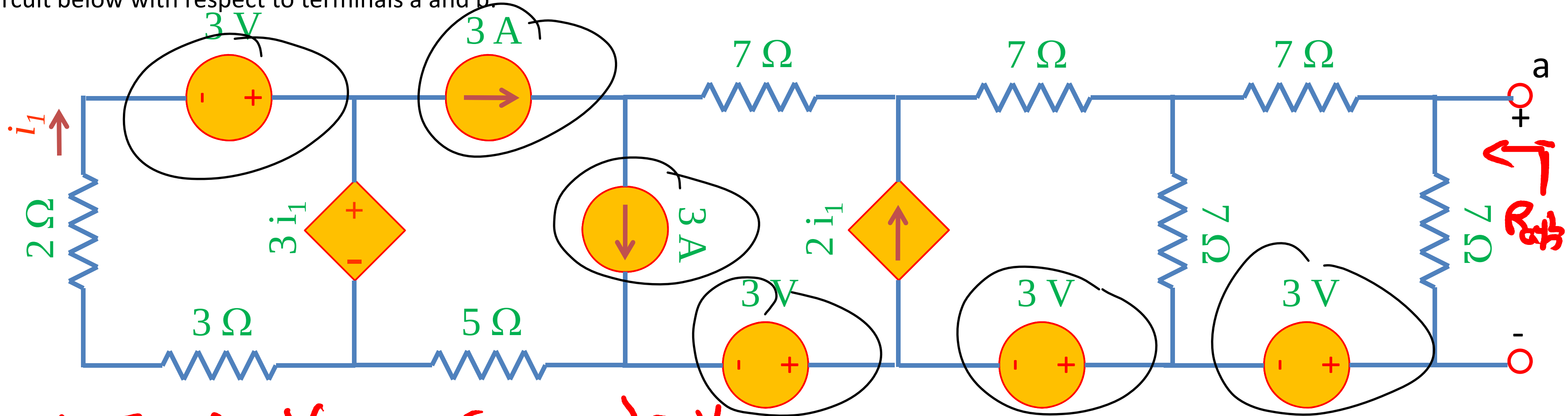


(Monster HW problem: Part 2):Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:



# "Baby" monster problem

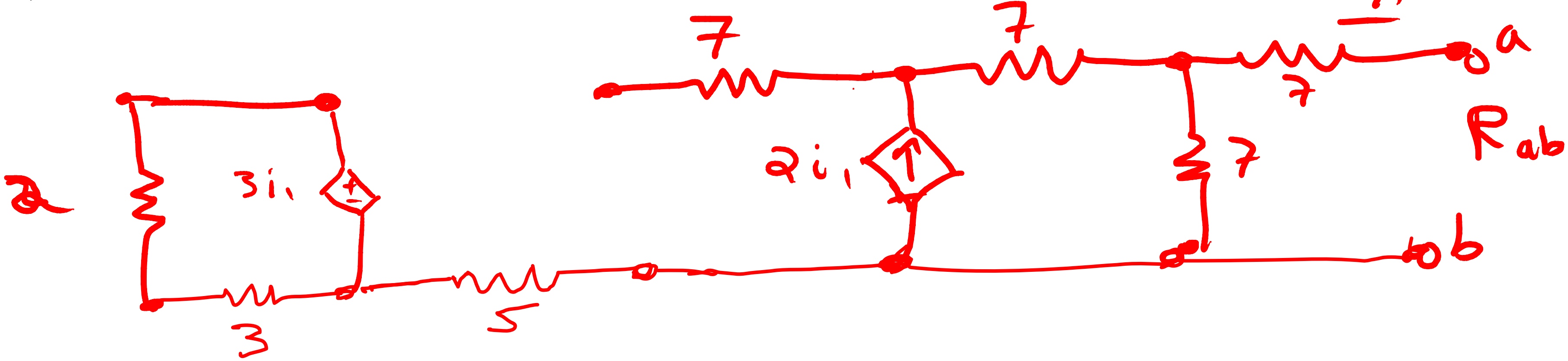
Find the Thevenin & Norton equivalent circuit of the circuit below with respect to terminals a and b:



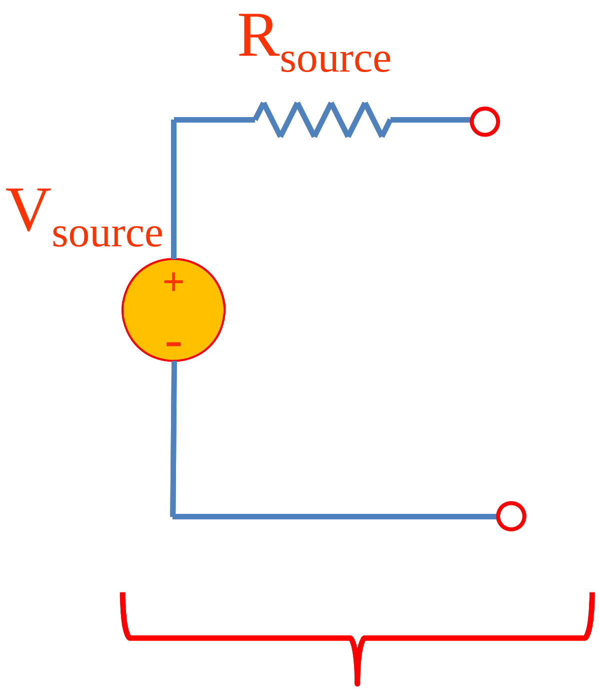

1) Find $V_{ab}$ (open) = $V_{th}$

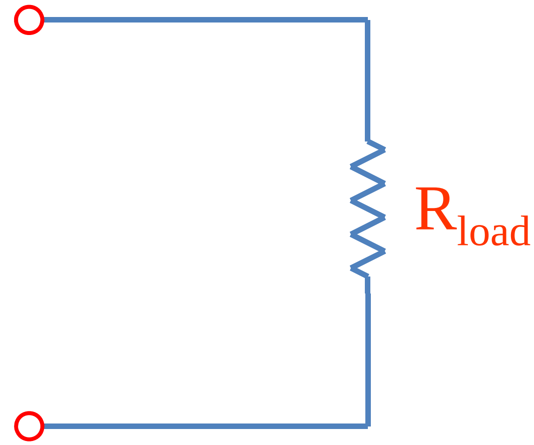
2) Find $I_{short\text{-}circuit}$ $\quad R_{th} = \frac{V_{th}}{I_{short\ circuit}}$

OR Find $R_{ab}$ when all independent sources off.



# Source/load




$$V_{load} = \frac{R_{load}}{R_{load} + R_{source}} V_{source}$$

*Derivation:*

Thevenin Thm:
*<u>Any</u>* circuit can be represented by this equivalent circuit.

Case 1:

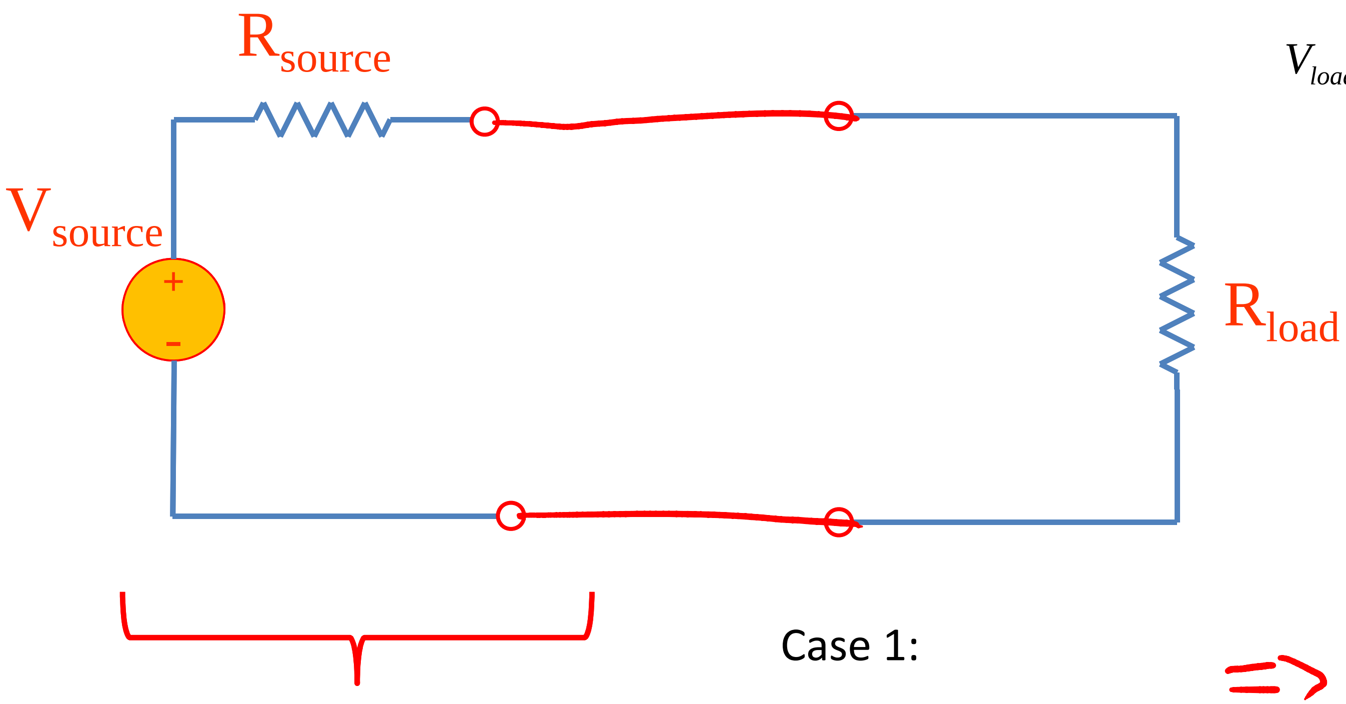
$$R_{load} >> R_{source}$$

Case 2:

$$R_{source} >> R_{load}$$

We say $R_{load}$ "*loads down*" the source.

# Source/load




$$V_{load} = \frac{R_{load}}{R_{load} + R_{source}} V_{source}$$

*Derivation:*

Thevenin Thm:
*Any* circuit can be represented by this equivalent circuit.

Case 1:

$$R_{load} >> R_{source}$$

$\Rightarrow V_{load} \approx V_{source}$

Case 2:

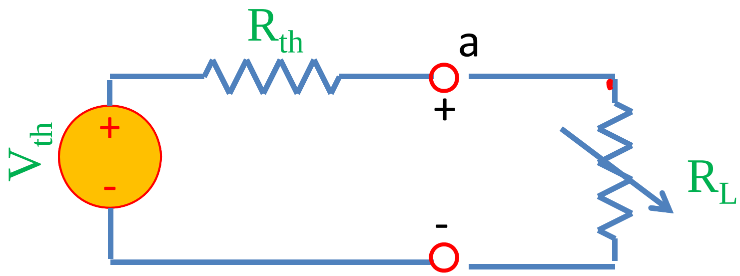
$$R_{source} >> R_{load}$$

$V_{load} \to 0$

$\approx \frac{R_{load}}{R_{source}} V_{source}$

We say $R_{load}$ *"loads down"* the source.

# Power



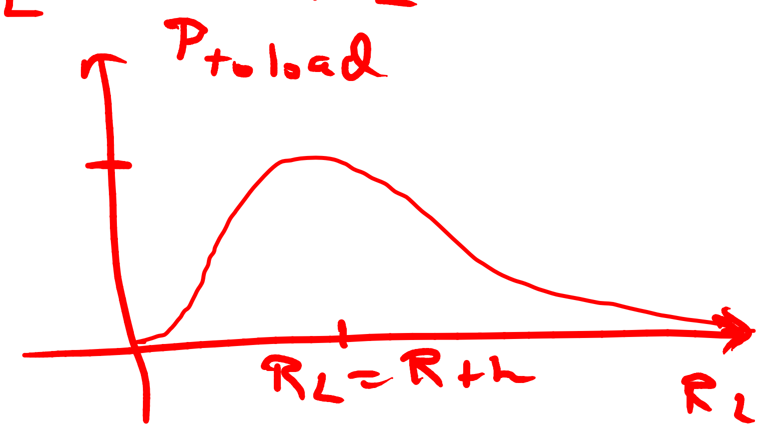

Arrow means $R_L$ variable (e.g. by a knob)

Power delivered to load = ?

$$\frac{R_L}{R_{th}+R_L} V_{th} = \text{voltage across } R_L$$

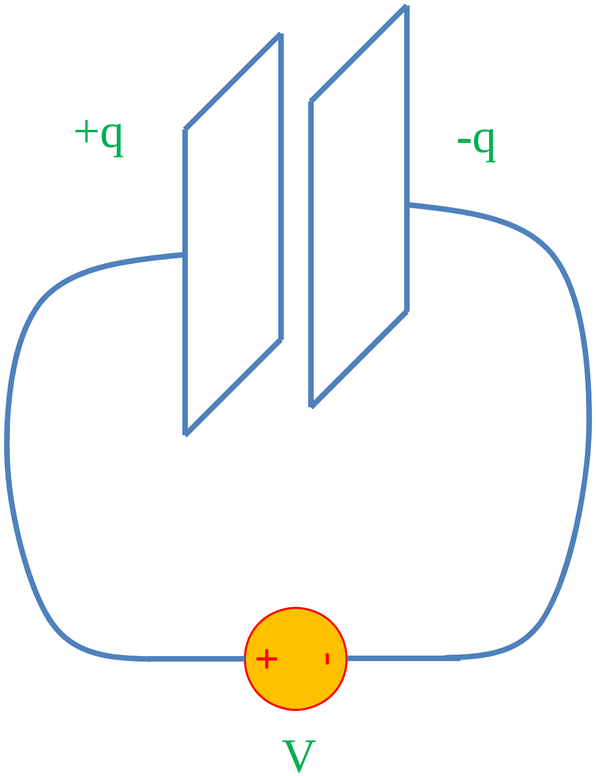
Power delivered to $R_L$ = $i^2 R_L$

$$= \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L$$



# Questions?

# Capacitors




$$q = CV$$

$$C = \frac{\varepsilon A}{d}$$

A=area
d=plate separation

Farads[F] = Coulombs/Volt [C]/[V]

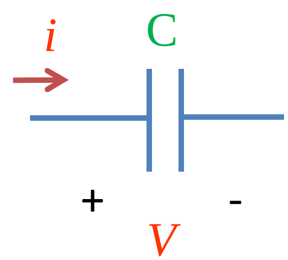
$$\varepsilon_0 = 8.85 \times 10^{-12}\, F/m$$

$$\varepsilon = \kappa \varepsilon_0$$

Dielectric constant:

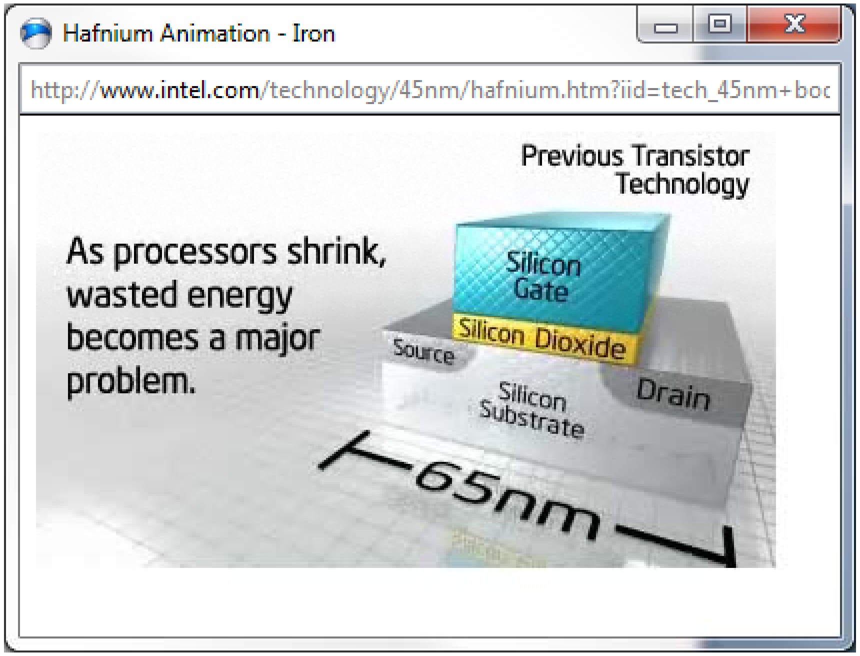
$\kappa = 3.9$ $SiO_2$

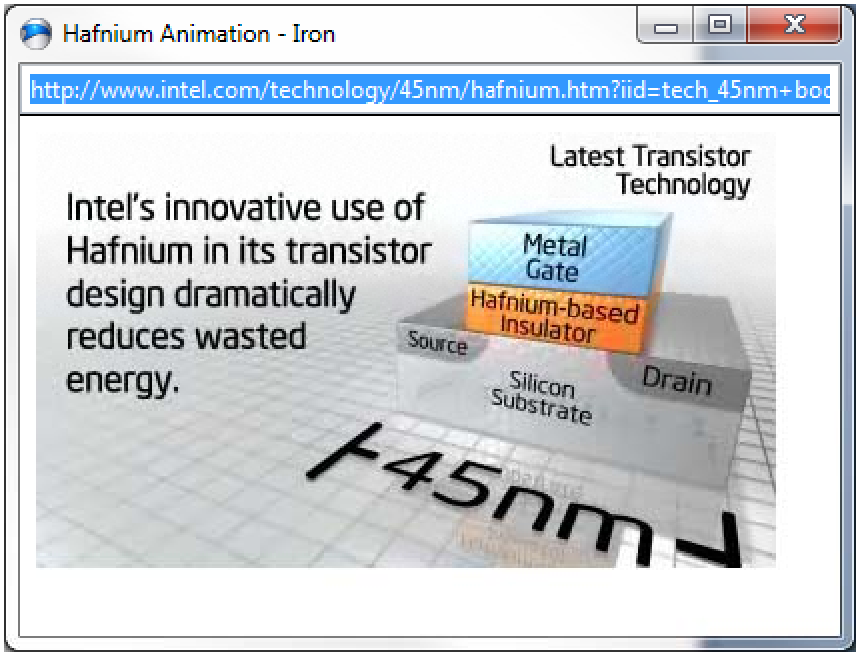
$\kappa = 25$ $HfO_2$



# "High-K Dielectric"

http://www.intel.com/technology/45nm/hafnium.htm?iid=tech_45nm+body_animation_hafnium



# Time dependence

$$q = CV \qquad i = \frac{dq}{dt} = C\frac{dV}{dt}$$

*q, V, i can depend on time !*

Implicit:

$$q(t) = CV(t) \qquad i(t) = \frac{dq(t)}{dt} = C\frac{dV(t)}{dt}$$

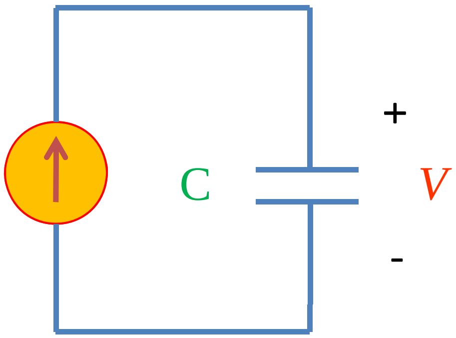
Will not always write (t), but it is assumed from now on.

$$i(t) = C\frac{dV(t)}{dt} \Rightarrow V(t) = \frac{1}{C}\int i(t)dt$$

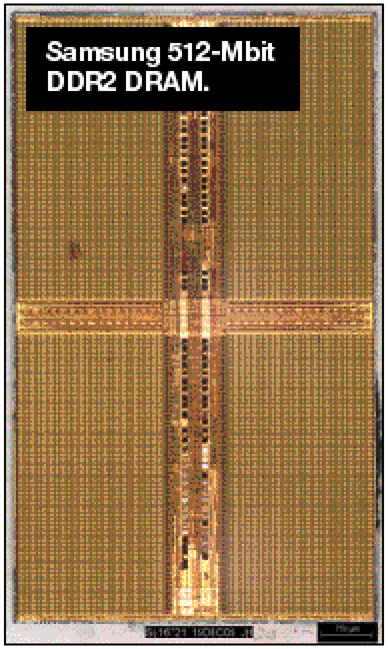
$$\Rightarrow q(t) = \int i(t)dt$$

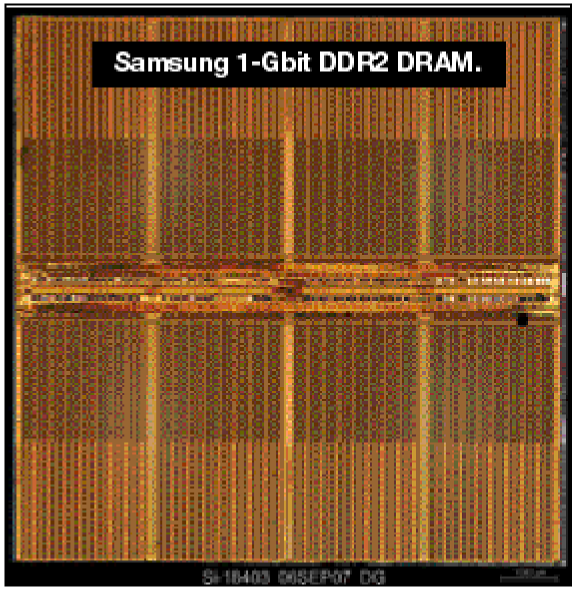
# Example Capacitor Problem

Find V(t), q(t)



# One-bit memory

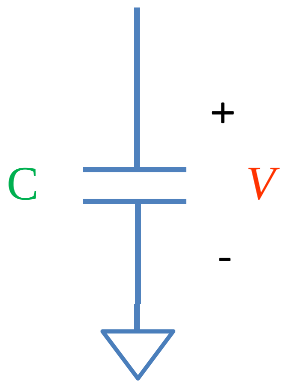



http://i.cmpnet.com/eet/news/07/11/DC1502_UTH_samsung.gif

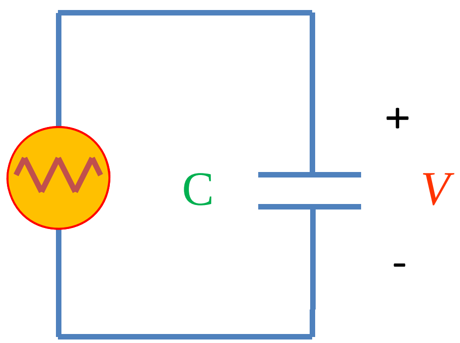
Typical dimensions:
0.1 micron x 0.1 micron area
10 nm thickness.
What is C?

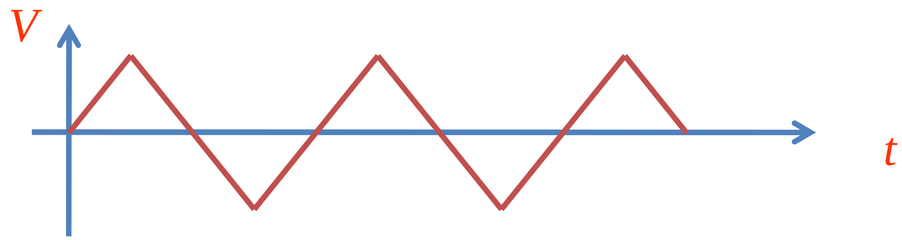
# 1 Bit Read/Write



# Example Problem #2

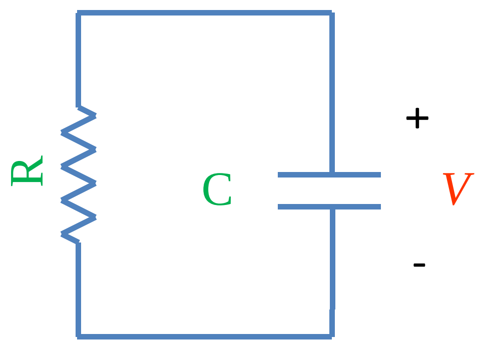
(Students): Find i(t), q(t)



# RC circuit

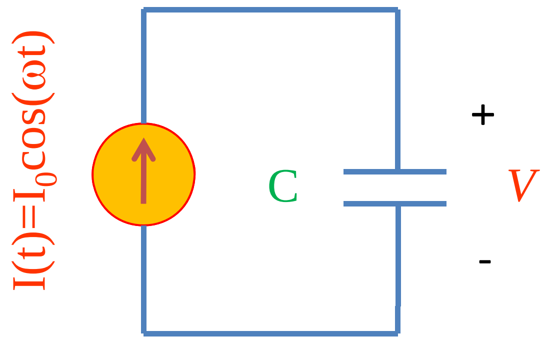
Find V(t), q(t), i(t)



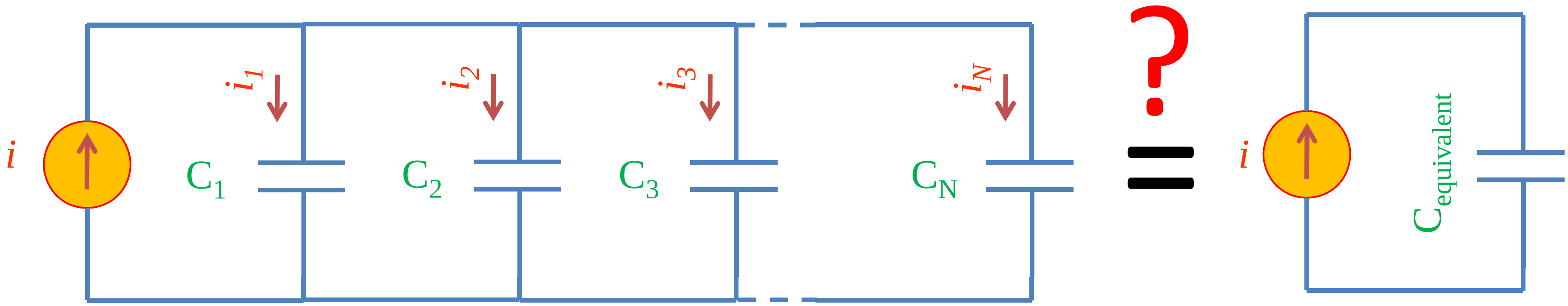
# DRAM vs. SRAM

# Example Capacitor Problem #2

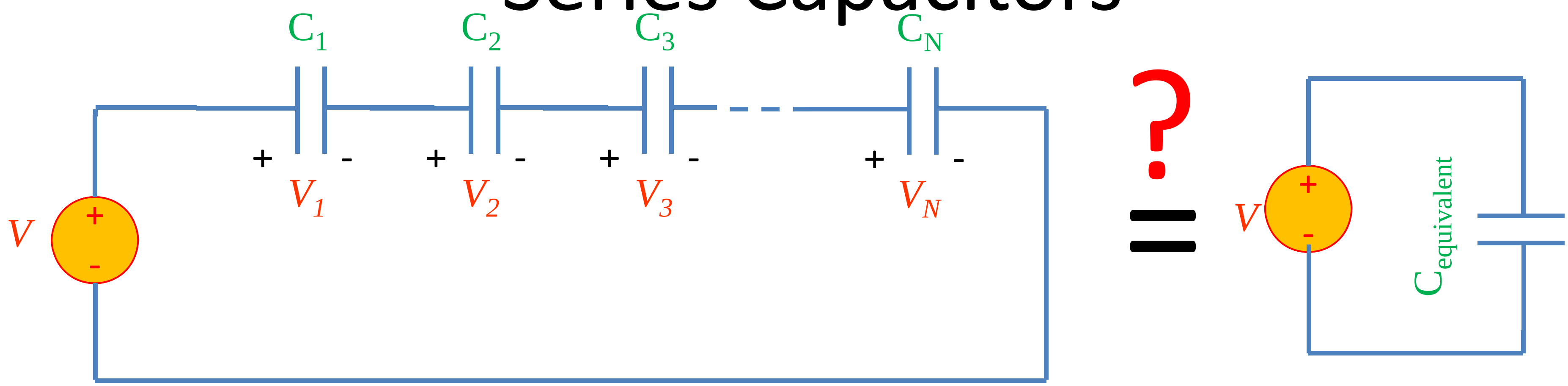
Find V(t), q(t)



# Parallel Capacitors

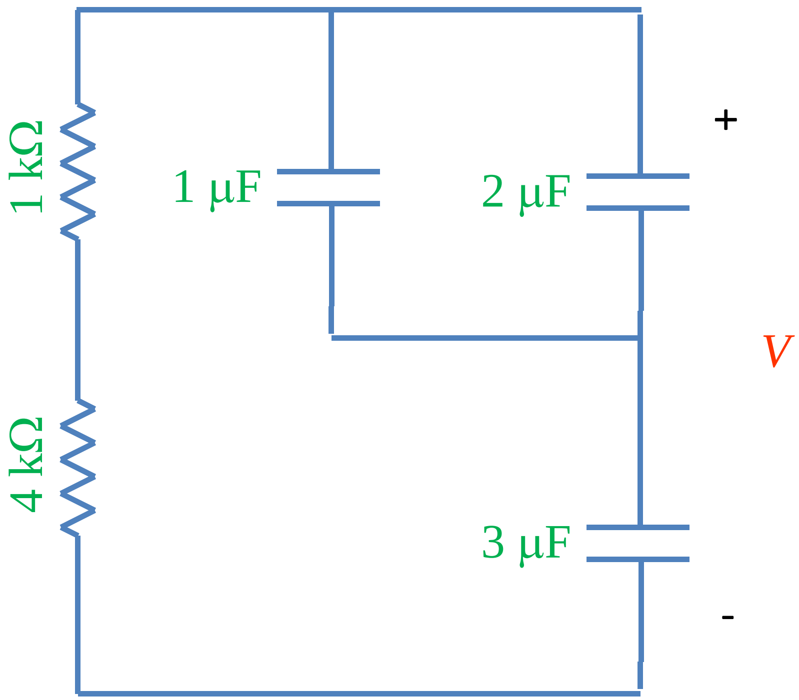


# Series Capacitors

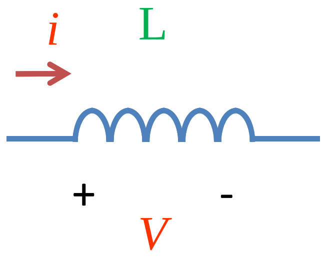


# Example problem #4

(Students) Find V(t), given that V(t=0) = 5 Volts



# Inductors




$$L = \frac{N^2 \mu A}{l}$$

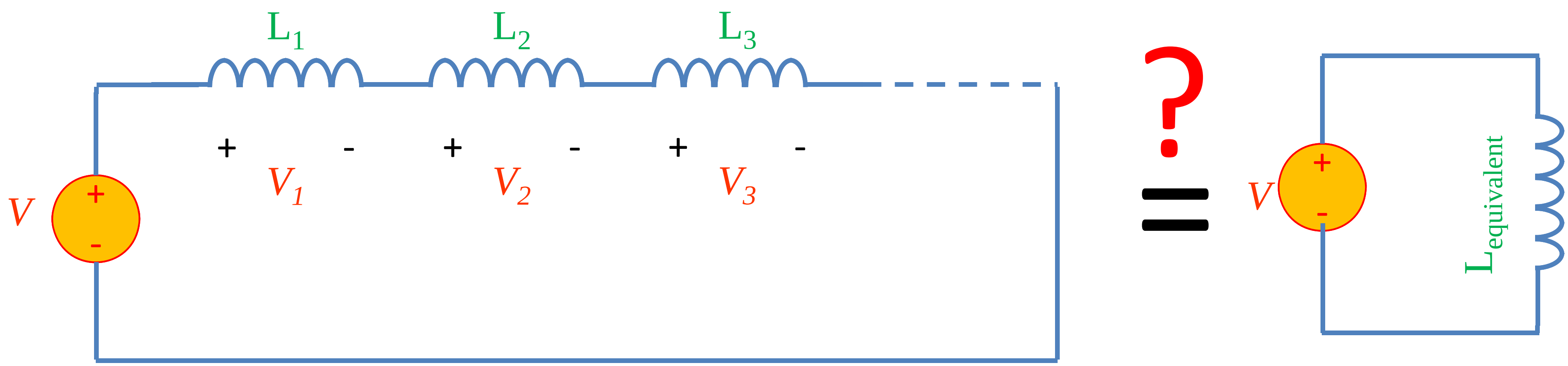
A=area
l=wire length
N = # of turns
$\mu = 4\,\pi\,10^{-6}$ H/m

$$V = L\frac{di}{dt}$$

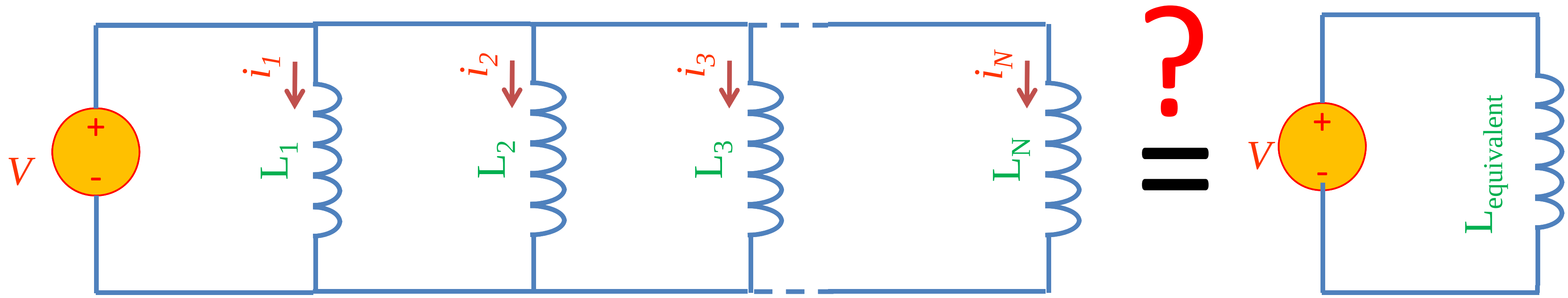
Henry[H]

$$V = L\frac{di}{dt} \Rightarrow i(t) = \frac{1}{L}\int V(t)dt$$

# Series Inductors

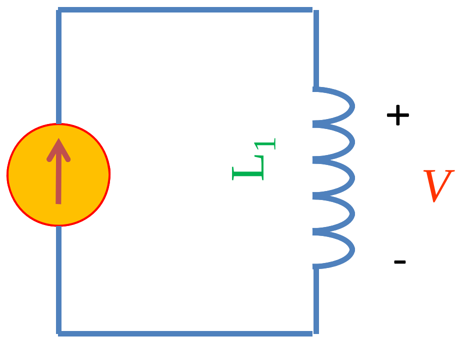


# Parallel Inductors



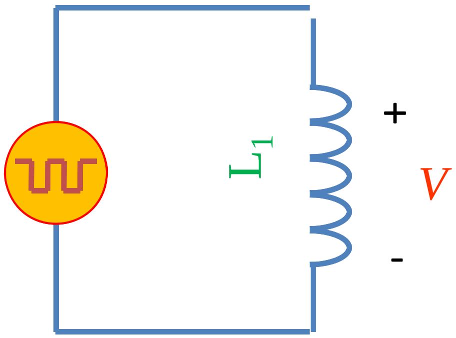
# Example Inductor Problem

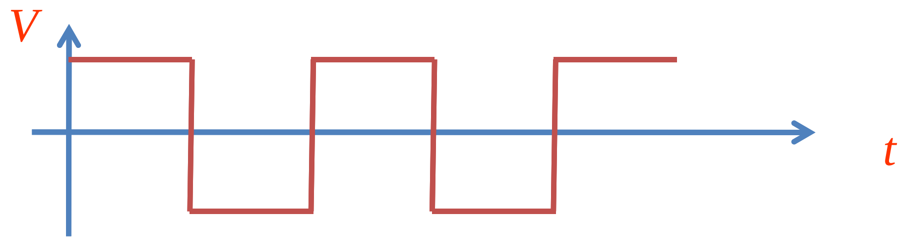
(Students): Find V(t).



# Example Inductor Problem #2

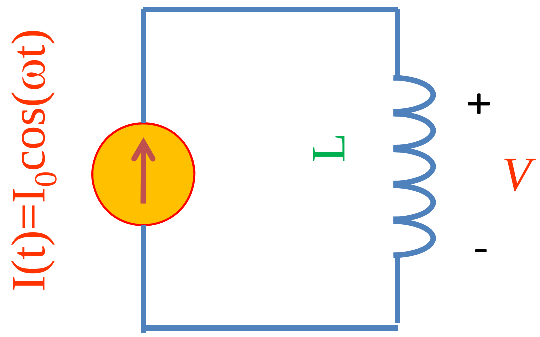
(Students): Find i(t)



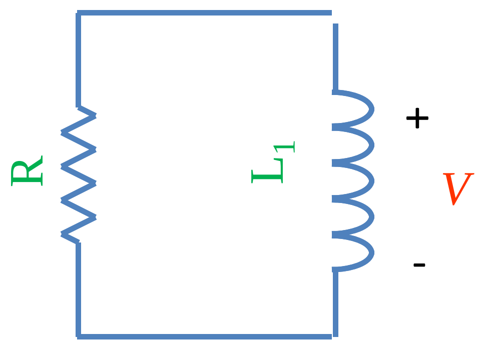
# Example Inductor Problem #3

Find V(t)



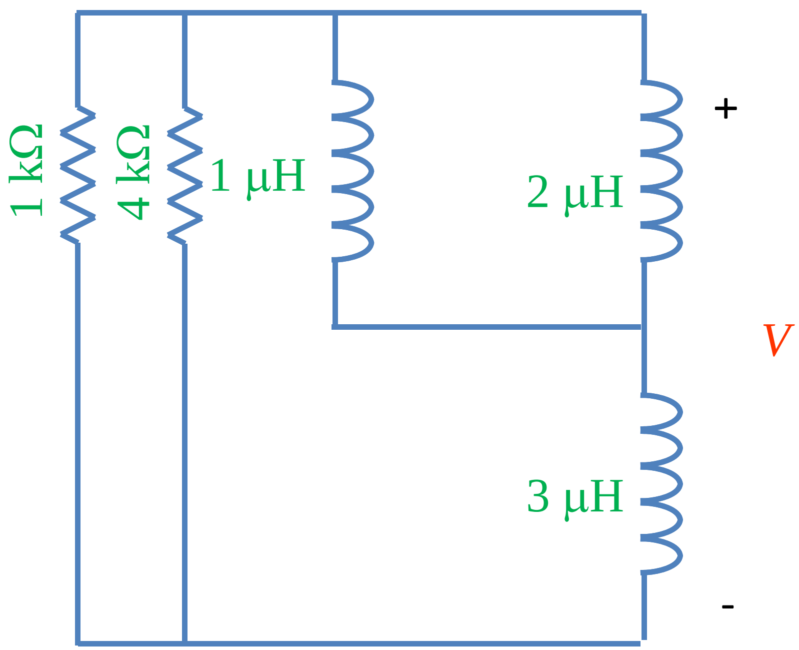
# LR circuit

Find V(t), i(t)

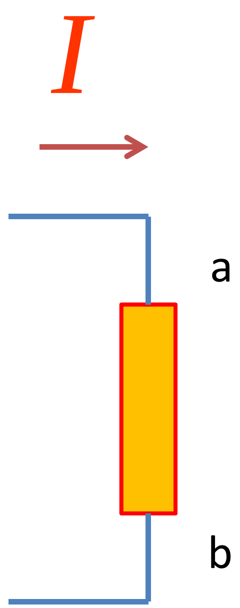


# Example LR problem

(Students) Find V(t), given that V(t=0) = 5 Volts



# Power




$$I \times V_{ab} = power$$

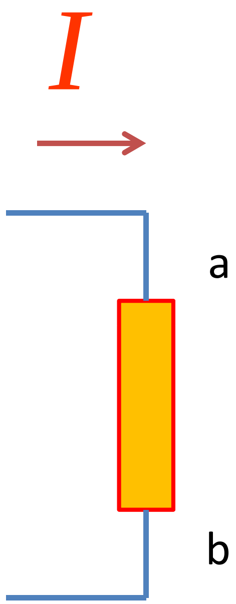
Watts [W] = Volt Amp [V-A]

Note: MKSA unit system:
*Meters Kilogram Second Amp*

Resistor:
Energy lost to heat…

Inductor or capacitor:
Energy *STORED* and can be recovered…

# Energy stored




$$I \times V_{ab} = power$$

Energy:

$$W = \int P dt = \int I \cdot V dt$$

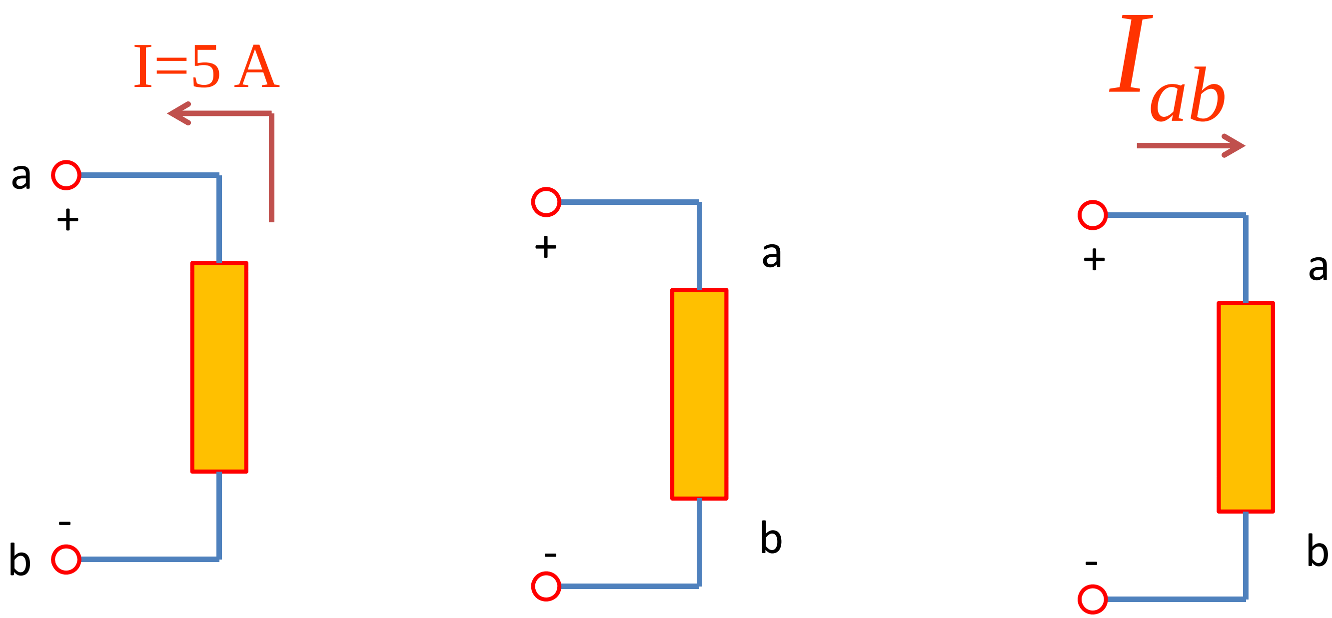
Capacitor stored energy:

$$\int I \cdot V dt = \int C \frac{dV}{dt} \cdot V dt = \frac{1}{2} C V^2 = \frac{Q^2}{2C}$$

Inductor stored energy:

$$\int I \cdot V dt = \int I \cdot L \frac{dI}{dt} dt = \frac{1}{2} L I^2$$

# Symbol library



# Symbol library