Given $u = \dfrac{A + jB}{C + jD}$ and A, B, C, D are all real.

1) Express u in the form $x + jy$

$re^{j\phi}$

$r\angle\phi$

2) Find $\mathrm{Re}(u)$

$\mathrm{Im}(u)$

3) Find $\mathrm{Re}\left(ue^{j\omega t}\right)$

Trick: Whenever denominator has complex #, multiply entire expression by the complex conjugate over itself. ②

$$u = \frac{A+jB}{C+jD}\,\frac{C-jD}{C-jD} = \frac{(A+jB)(C-jD)}{C^2 + \cancel{jCD} - \cancel{jCD} + (jD)(-jD)}$$

$$= \frac{(A+jB)(C-jD)}{C^2+D^2} = \frac{AC + jBC - jAD + (jB)(-jD)}{C^2+D^2}$$

$$= \frac{AC+BD}{C^2+D^2} + j\,\frac{BC-AD}{C^2+D^2}$$ ← form is $x+jy$

$$r = \sqrt{x^2+y^2} = \sqrt{\left(\frac{AC+BD}{C^2+D^2}\right)^2 + \left(\frac{BC-AD}{C^2+D^2}\right)^2}$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{BC-AD}{C^2+D^2}$$

$$u = \sqrt{\left(\frac{AC+BD}{C^2+D^2}\right)^2 + \left(\frac{BC-AD}{C^2+D^2}\right)^2}\; e^{j\tan^{-1}\left(\frac{BC-AD}{C^2+D^2}\right)}$$ ← form $re^{j\phi}$

$$u = \sqrt{\left(\frac{AC+BD}{C^2+D^2}\right)^2 + \left(\frac{BC-AD}{C^2+D^2}\right)^2}\;\angle\,\tan^{-1}\left(\frac{BC-AD}{C^2+D^2}\right)$$ ← form $r\angle\phi$

$$\mathrm{Re}(u) = \frac{AC+BD}{C^2+D^2}$$

$$\mathrm{Im}(u) = \frac{BC-AD}{\cancel{B}\,C^2+D^2}$$

$$\mathrm{Re}\left[u e^{j\omega t}\right] = \mathrm{Re}\left[r e^{j\phi} e^{j\omega t}\right] = r\,\mathrm{Re}\left[e^{j(\phi+\omega t)}\right]$$

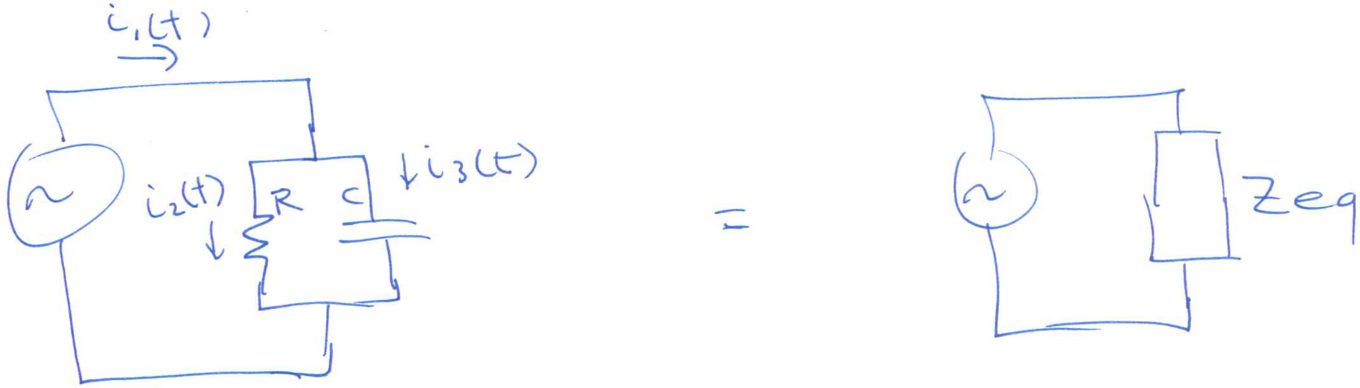
Euler

$$= r\,\mathrm{Re}\left[\cos(\omega t+\phi) + j\sin(\omega t+\phi)\right] = r\cos(\omega t+\phi)$$

$$= \sqrt{\left(\frac{AC+BD}{C^2+D^2}\right)^2 + \left(\frac{BC-AD}{C^2+D^2}\right)^2}\,\cos\left[\omega t + \tan^{-1}\left(\frac{BC-AD}{C^2+D2}\right)\right]$$

Find all currents, voltages in this circuit

$V(t) = V_0 \cos(3t + 45°)$



Soln: All voltages same. For $i_1(t)$:

$$\tau = RC$$

$$Z_{eq}^{-1} = Z_1^{-1} + Z_2^{-1} = \frac{1}{R} + j\omega C = \frac{1 + j\omega RC}{R} = \frac{1+j\omega\tau}{R}$$

$$\Rightarrow Z_{eq} = \frac{R}{1+j\omega\tau}$$

$i(t)$ $V(t) \rightarrow \mathbb{V} \rightarrow \mathbb{I} = \frac{\mathbb{V}}{Z_{eq}} \rightarrow i(t)$

$$\mathbb{V} = V_0 e^{j45°} = V_0 e^{j\frac{\pi}{4}}$$

$$\mathbb{I} = \frac{V_0 e^{j\pi/4}}{R/(1+j\omega\tau)} = \frac{V_0}{R} e^{j\frac{\pi}{4}}(1+j\omega\tau)$$

$$i(t) = \mathrm{Re}\left[\mathbb{I} e^{j\omega t}\right] = \mathrm{Re}\left[\frac{V_0}{R} e^{j\frac{\pi}{4}}(1+j\omega\tau)e^{j\omega t}\right] = \frac{V_0}{R}\mathrm{Re}\left[e^{j\frac{\pi}{4}}(1+j\omega\tau)e^{j\omega t}\right]$$

$=$

$$= \frac{V_0}{R} \, Re\left[e^{j\left(\frac{\pi}{4}+\omega t\right)} (1+j\omega\tau) \right]$$ Euler

$$= \frac{V_0}{R} Re\left[\left[\cos\left(\frac{\pi}{4}+\omega t\right) + j\sin\left(\frac{\pi}{4}+\omega t\right)\right]\left[1+j\omega\tau\right] \right]$$

$$= \frac{V_0}{R}\left[\cos\left(\frac{\pi}{4}+\omega t\right) - \omega\tau \sin\left(\frac{\pi}{4}+\omega t\right)\right] = i_1(t)$$