EECS70A Spring 2008 Final Exam
6/10/2008 10:30 to 12:30 pm
Professor Peter Burke

| 1 | 2 | 3 | 4 | 5 | 6 | Total |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | $/ 10$ |  | 10 | $/ 20$ | $/ 20$ |  |

## DO NOT BEGIN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

EECS70A Spring 2008 Final Exam
6/10/2008 10:30 to 12:30 pm
Professor Peter Burke
PROBLEM ONE: (10 points)
A) State and describe Thevenin's Theorem.
B) State and describe Norton's Theorem.

EECS70A Spring 2008 Final Exam
6/10/2008 10:30 to 12:30 pm
Professor Peter Burke
PROBLEM TWO(10 points):

Describe the concepts of complete response and transient response of linear RLC circuits.

EECS70A Spring 2008 Final Exam
6/10/2008 10:30 to 12:30 pm
Professor Peter Burke
PROBLEM THREE(20 points):

A branch voltage in an $R L C$ circuit is described by

$$
\frac{d^{2} v}{d t^{2}}+4 \frac{d v}{d t}+8 v=24
$$

If the initial conditions are $v(0)=0=d v(0) / d t$, find $v(t)$.

EECS70A Spring 2008 Final Exam
6/10/2008 10:30 to $12: 30 \mathrm{pm}$
Professor Peter Burke

## PROBLEM FOUR(20 points):

The switch in the circuit below has been closed for a long time but is opened at $t=0$ Determine $i(t)$ for $t>0$.


EECS70A Spring 2008 Final Exam
6/10/2008 10:30 to 12:30 pm
Name: $\qquad$

Professor Peter Burke
PROBLEM FIVE (20 points):

An industrial load is modeled as a series combination of a capacitance and a resistance as shown below. Calculate the value of an inductance $L$ across the series combination so that the net impedance is resistive at a frequency of 50 kHz .


EECS70A Spring 2008 Final Exam
6/10/2008 10:30 to 12:30 pm
Professor Peter Burke

## PROBLEM SIX(20 points):

Obtain the transfer function $\mathbf{V}_{o} / \mathbf{V}_{i}$ of the circuit below.


# EECS70A / CSE 70A Network Analysis I <br> Prof. Peter Burke 

Final solution

## Problem 1:

A) A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source $\mathrm{V}_{\mathrm{Th}}$ in series with a resistor $\mathrm{R}_{\mathrm{Th}}$, where $\mathrm{V}_{\mathrm{Th}}$ is the open-circuit voltage at the terminals and $\mathrm{R}_{\mathrm{Th}}$ is the input or equivalent resistance at the terminals when the independent sources are turned off.
B) A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source $\mathrm{I}_{\mathrm{N}}$ in parallel with a resistor $\mathrm{R}_{\mathrm{N}}$, where $\mathrm{I}_{\mathrm{N}}$ is the sort-circuit current through the terminals and $\mathrm{R}_{\mathrm{N}}$ is the input or equivalent resistance at the terminals when the terminals when the independent sources are turned off.

## Grading criteria: 3 pts for equivalent circuit with a resistor and a source <br> 5 pts for mentioning $\mathrm{R}_{\mathrm{Th}}$ and $\mathrm{V}_{\mathrm{Th}}$ in series <br> 5 pts for mentioning $\mathrm{R}_{\mathrm{N}}$ and $\mathrm{I}_{\mathrm{N}}$ in parallel

## Problem 2:

Complete response $=$ transient response + steady-state response

A complete response consists of a transient response (temporary response) and a steadystate response (permanent response). The transient response is the circuit's temporary response that will die out with time. And the steady-state response is the behavior of the circuit a long time after an external excitation is applied.

Grading criteria: 5 pts for mentioning transient and steady-state responses for the complete response

## Problem 3:

$$
\begin{gathered}
s^{2}+4 s+8=0 \text { leads to } s=\frac{-4 \pm \sqrt{16-32}}{2}=-2 \pm j 2 \\
v(t)=V_{s}+\left(A_{1} \cos 2 t+A_{2} \sin 2 t\right) e^{-2 t} \\
8 V_{s}=24 \text { means that } V_{s}=3 \\
v(0)=0=3+A_{1} \text { leads to } A_{1}=-3 \\
d v / d t=-2\left(A_{1} \cos 2 t+A_{2} \sin 2 t\right) e^{-2 t}+\left(-2 A_{1} \sin 2 t+2 A_{2} \cos 2 t\right) e^{-2 t} \\
0=d v(0) / d t=-2 A_{1}+2 A_{2} \text { or } A_{2}=A_{1}=-3 \\
\left.v(t)=\underline{\left[3-3(\cos 2 t+\sin 2 t) e^{-2 t}\right.}\right] \text { volts }
\end{gathered}
$$

Grading criteria: 5 pts for correct characteristic eq. with correct roots
5 pts for choosing a correct type of solution
15 pts showing correct $2^{\text {nd }}$ order circuit process without including Vs

## Problem 4:

For $\mathrm{t}<0$, the equivalent circuit is as shown below.

$$
\mathrm{v}(0)=-12 \mathrm{~V} \quad \text { and } \quad \mathrm{i}(0)=12 / 2=6 \mathrm{~A}
$$

For $t>0$, we have a series RLC circuit.

$$
\begin{gathered}
\alpha=\mathrm{R} /(2 \mathrm{~L})=2 /(2 \times 0.5)=2 \\
\omega_{0}=1 / \sqrt{\mathrm{LC}}=1 / \sqrt{0.5 \times 1 / 4}=2 \sqrt{2}
\end{gathered}
$$

Since $\alpha$ is less than $\omega_{0}$, we have an under-damped response.

$$
\begin{gathered}
\omega_{d}=\sqrt{\omega_{o}^{2}-\alpha^{2}}=\sqrt{8-4}=2 \\
i(t)=(A \cos 2 t+B \sin 2 t) e^{-2 t} \\
i(0)=6=A
\end{gathered}
$$

$$
\begin{gathered}
d i / d t=-2(6 \cos 2 t+B \sin 2 t) e^{-2 t}+(-2 x 6 \sin 2 t+2 B \cos 2 t) e^{-\alpha t} \\
\operatorname{di}(0) / d t=-12+2 B=-(1 / L)\left[R i(0)+v_{C}(0)\right]=-2[12-12]=0
\end{gathered}
$$

Thus, $B=6 \quad$ and $\quad i(t)=\underline{(6 \cos 2 t+6 \sin 2 t)} e^{-2 t} \underline{A}$

Grading criteria: 5 pts for correct equivalent circuit and initial values for $\mathrm{t}<0$
10 pts for correct equivalent circuit and characteristic eq. and variables for $\mathrm{t}>0$

$$
3 \text { pts for correct solution for } \mathrm{i}(\mathrm{t})
$$

5 pts for correct process to get $\mathrm{i}(\mathrm{t})$ using initial value and derivative of i(t)
Problem 5:

$$
\begin{aligned}
& \mathbf{Z}_{\text {in }}=j \omega L \|\left(R+\frac{1}{j \omega C}\right) \\
& \mathbf{Z}_{\text {in }}=\frac{j \omega L\left(R+\frac{1}{j \omega C}\right)}{R+j \omega L+\frac{1}{j \omega C}}=\frac{\frac{L}{C}+j \omega L R}{R+j\left(\omega L-\frac{1}{\omega C}\right)} \\
& \mathbf{Z}_{\text {in }}=\frac{\left(\frac{L}{C}+j \omega L R\right)\left(R-j\left(\omega L-\frac{1}{\omega C}\right)\right)}{R^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}}
\end{aligned}
$$

To have a resistive impedance, $\operatorname{Im}\left(\mathbf{Z}_{\text {in }}\right)=0$. Hence,

$$
\omega \mathrm{LR}^{2}-\left(\frac{\mathrm{L}}{\mathrm{C}}\right)\left(\omega \mathrm{L}-\frac{1}{\omega \mathrm{C}}\right)=0
$$

$$
\begin{aligned}
& \omega R^{2} \mathrm{C}=\omega \mathrm{L}-\frac{1}{\omega \mathrm{C}} \\
& \omega^{2} \mathrm{R}^{2} \mathrm{C}^{2}=\omega^{2} \mathrm{LC}-1 \\
& \mathrm{~L}=\frac{\omega^{2} \mathrm{R}^{2} \mathrm{C}^{2}+1}{\omega^{2} \mathrm{C}}
\end{aligned}
$$

Now we can solve for L.

$$
\begin{aligned}
& \mathrm{L}=\mathrm{R}^{2} \mathrm{C}+1 /\left(\omega^{2} \mathrm{C}\right) \\
= & \left(200^{2}\right)\left(50 \times 10^{-9}\right)+1 /\left((2 \pi \times 50,000)^{2}\left(50 \times 10^{-9}\right)\right. \\
= & 2 \times 10^{-3}+0.2026 \times 10^{-3}=\underline{\mathbf{2 . 2 0 3} \mathbf{~ m H}} .
\end{aligned}
$$

Grading criteria: 3 pts for correct w using $\mathrm{w}=2 \pi \mathrm{f}$
10 pts for showing correct total impedance $\left(\mathrm{Z}_{\mathrm{T}}\right)$ or $\left(\mathrm{Z}_{\text {in }}\right)$ equation
10 pts for showing imaginary part of total impedance is zero

## Problem 6:

$$
\begin{aligned}
& H(s)=\frac{V_{o}}{V_{i}} \\
& V_{o}=\frac{2}{10+2+\frac{8}{j w}} V_{i}=\frac{2 j w}{12 j w+8} V_{i} \\
& \frac{V_{o}}{V_{i}}=\frac{2 j w}{12 j w+8}=\frac{2 s}{12 s+8}
\end{aligned}
$$

Grading criteria: 15 pts for correct voltage divider equation
10 pts for correct current for getting $\mathrm{V}_{\mathrm{o}}$ using Ohm's law or KVL

