EECS70A Spring 2009 **Midterm Exam #2** Name:____________________________

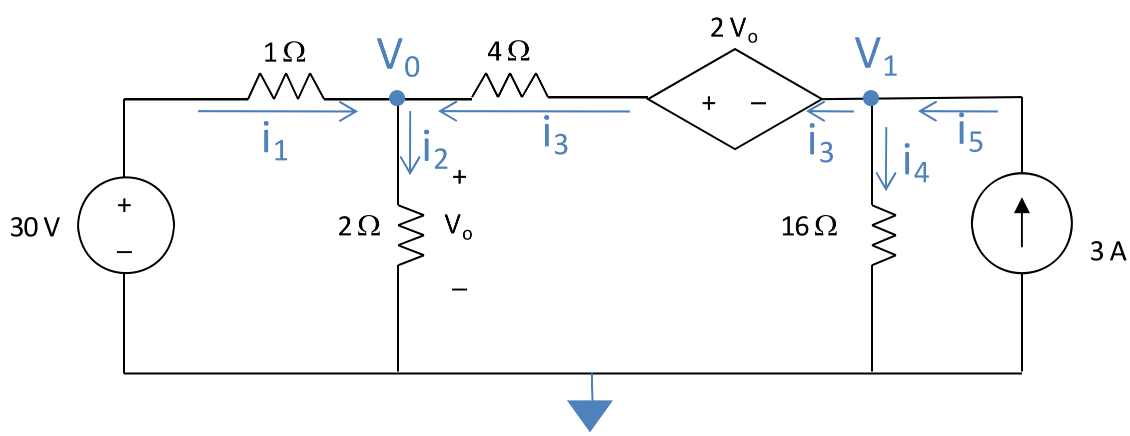
5/14/2009 11:00 to 12:20 pm ID no.:____________________________

Professor Peter Burke

SOLUTIONS: Midterm #2

PROBLEM ONE: (40 points)

Use nodal analysis to find all the currents and voltages in this circuit.
Label your current and voltage definitions clearly!!!



At node 1 we get the equation,

$$i_1 + i_3 = i_2 \qquad (1)$$

$$\left(\frac{30V - V_0}{1\Omega}\right) + \left(\frac{(V_1 + 2V_0) - V_0}{4\Omega}\right) = \left(\frac{V_0}{2\Omega}\right)$$

$$\left(\frac{30V}{1\Omega}\right) + \left(\frac{V_1}{4\Omega}\right) = \left(\frac{5V_0}{4\Omega}\right) \qquad (2)$$

At node 2 we get the equation,

$$i_4 + i_3 = i_5 \qquad (3)$$

$$\left(\frac{V_1}{16\Omega}\right) + \left(\frac{(V_1 + 2V_0) - V_0}{4\Omega}\right) = 3A$$

$$\left(\frac{5V_1}{16\Omega}\right) + \left(\frac{V_0}{4\Omega}\right) = 3A \qquad (4)$$

(2)

With two equations, (1) and (2), and two variables, V_0 and V_1, we can solve for their values and get V_0=648/29=22.3V and V_1=-240/29=-8.28V. Solving for the currents we get

$$i_1 = \left(\frac{30V - V_0}{1\Omega}\right) = \frac{222}{29} = 7.655A$$
$$i_2 = \left(\frac{V_0}{2\Omega}\right) = \frac{324}{29} = 11.15A$$
$$i_3 = \left(\frac{(V_1 + 2V_0) - V_0}{4\Omega}\right) = \frac{102}{29} = 3.45A$$
$$i_4 = \left(\frac{V_1}{16\Omega}\right) = -\frac{15}{29} = -0.53A$$
$$i_5 = 3A \tag{3}$$

Problem 1 : Grading Criteria:

Maximum points:
6pts for correctly setting up the node equation (1)
6pts for correctly setting up the node equation (3)
4pts for i_1 (full credit if $7.0 < i_1 < 8.1$)
4pts for i_2 (full credit if $10.0 < i_2 < 12.2$)
4pts for i_3 (full credit if $3.1 < i_3 < 3.8$)
4pts for i_4 (full credit if $-0.58 < i_4 < -0.48$)
4pts for i_5

Partial Credit for i's above
3pts partial credit for correct equation but wrong numerical answer
1pts partial credit for a mistake in the equation

4pts for V_0 (full credit if $20 < V_0 < 24$ and correct equation)

2pts partial credit fair attempt at solving

4pts for V_1 (full credit if $7.5 < V_1 < 9.1$ and correct equation)

2pts partial credit fair attempt at solving

PROBLEM ONE SPACE FOR WORK:

PROBLEM TWO (30 points):

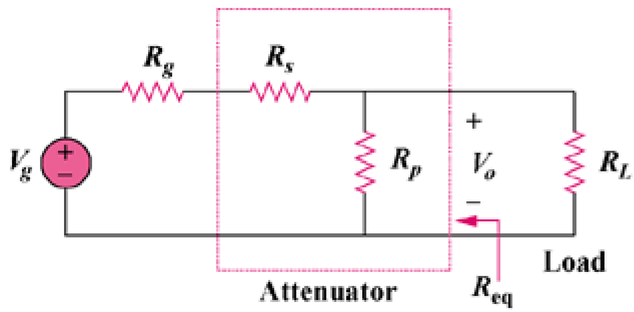
An attenuator is an interface circuit that reduces the voltage level without changing the output resistance.

(a) By specifying R_s and R_p of the interface circuit in the figure below, design an attenuator that will meet the following requirements:

$\frac{V_o}{V_g} = 0.125, \quad R_{eq} = R_{Th} = R_g = 100\Omega$. (Note: 0.125 = 1/8).

(b) Using the interface designed in part (a), calculate the current through a load of R_L = 50 Ω when V_g = 12 V.

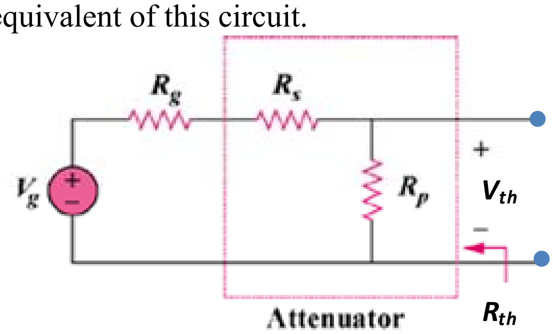
(c) What value of R_L achieves maximum power delivered to the load? Express your answer in Ω. In this case, what is the power (in Watts) delivered to the load?



Solution:

(a)

Finding the Thevenin equivalent of this circuit.



The equation for R_{th} is the equivalent resistance seen looking into the load-port when the voltage source, V_g, is set to zero. Also, it is given that R_{th}=100Ω thus giving

$$R_{th} = \frac{(R_g + R_s)R_p}{(R_g + R_s) + R_p} = \frac{(100\Omega + R_s)R_p}{(100\Omega + R_s) + R_p} = 100\Omega \quad (1)$$

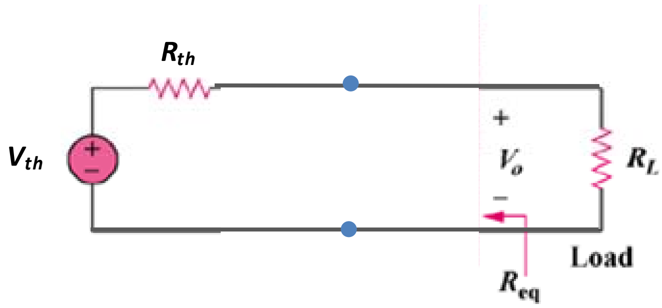
The Thevenin equivalent voltage can be shown to be a voltage divider

$$V_{th} = V_g \frac{R_p}{(R_g + R_s) + R_p} = V_g \frac{R_p}{(100\Omega + R_s) + R_p}$$

$$\frac{V_{th}}{V_g} = 0.125 = \frac{R_p}{(100\Omega + R_s) + R_p} \quad (2)$$

Now we have two equations, (1) and (2), and two unknown variables, R_s and R_p which we can solve for to get R_p=800/7=114.28Ω and R_s=700Ω.

(b)



As given, R_L=50, V_g=12V, and V_{th}=0.125V_g , R_{th}=100Ω we get

$$i = \frac{V_{th}}{R_{th} + R_L} = \frac{0.125(12V)}{100\Omega + 50\Omega} = 0.01\text{A} \quad (3)$$

(c)

For maximum power delivery to the load resister, $R_{L,max}$=R_{th}=100Ω, (see pg 151 in textbook) and the corresponding power delivered to the load is

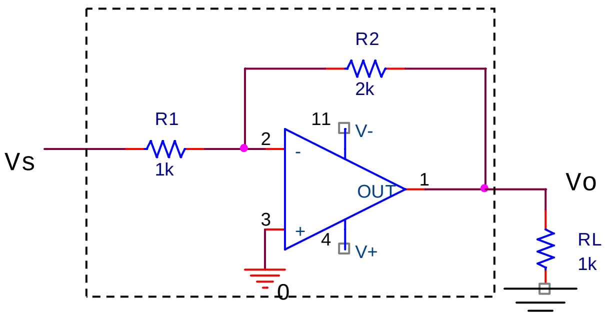
$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{\left(0.125(12V)\right)^2}{4(100\Omega)} = 0.005625\text{W} \quad (4)$$

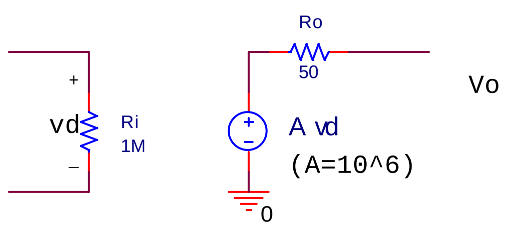
Problem 2: Grading Criteria:

a) 16pts: 5pts for eq (1) R_{th}, 5pts for eq (2) V_{th}, 3pts for R_s (full credit if $630 < R_s < 770$), and 3pts for R_p (full credit if $103 < R_p < 125$).
   a. 2pts credit for fair attempt at R_{th}, V_{th}
   b. 1pt partial credit for fair attempt at R_s and R_p
b) 6pts for the correct current though the load (full credit if $0.009 < i < 0.011$)
   a. 2pts-3pts partial credit for fair attempt at load current
c) 8pts: 4pts for correct $R_{L,max}$ and 4pts for correct P_{max} (full credit if $0.0051 < P_{max} < 0.0061$)
   a. 2pts partial credit for fair attempt at P_{max}
   b. 2pts partial credit for describing how to find $R_{L,max}$ for max power using derivates.

PROBLEM THREE (30 points):

Consider the following inverting Op Amp circuit. The circuit model for the non-ideal Op Amp is also included below. The boxed region is to be represented using Thevenin equivalent circuit. Find an algebraic expression for V_{th} and R_{th} in terms of A, V_s, R_1, R_2, R_i, R_o. Then find numerical result for V_{th} and R_{th} in terms of V_s. Hint: Replace the op-amp by its equivalent model, and then analyze the resultant circuit. Show your result becomes the value we had in class for the limit of an ideal op-amp. (What does ideal op-amp mean?)





(R_i = 1 MΩ, R_o = 50 Ω)

Solution:

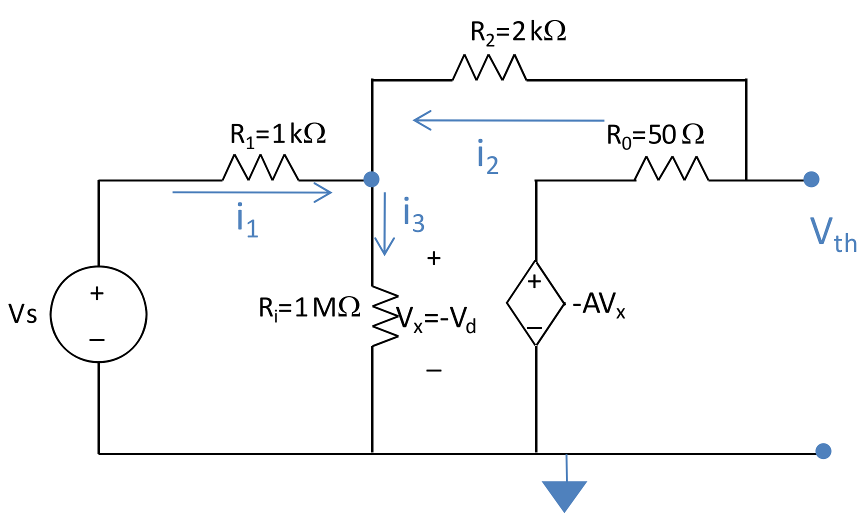


Figure 1: Circuit diagram for solving V_{th}

First, find V_{th}

$$i_1 + i_2 = i_3$$

$$\left(\frac{v_s - V_x}{R_1}\right) + \frac{((-AV_x) - V_x)}{R_0 + R_2} = \frac{V_x}{R_i}$$

$$\frac{V_s}{R_1} = V_x\left(\frac{1}{R_1} + \frac{A+1}{R_0 + R_2} + \frac{1}{R_i}\right)$$

$$\rightarrow V_x = \left(\frac{V_s}{\left(1 + \frac{R_1(A+1)}{R_0 + R_2} + \frac{R_1}{R_i}\right)}\right) \tag{1}$$

And we can write the equation,

$$V_{th} = -AV_x - i_2 R_0$$
$$V_{th} = -AV_x — AV_x - V_x R_0 + R_2 R_0$$
$$V_{th} = V_x\left(-A + \left(\frac{A+1}{R_0 + R_2}\right)R_0\right)$$
$$V_{th} = \left(\frac{V_s}{\left(1 + \frac{R_1(A+1)}{R_0 + R_2} + \frac{R_1}{R_i}\right)}\right)\left(-A + \left(\frac{A+1}{R_0 + R_2}\right)R_0\right)$$
$$V_{th} = V_s\left(\frac{\left(-A + \left(\frac{A+1}{R_0 + R_2}\right)R_0\right)}{\left(1 + \frac{R_1(A-1)}{R_0 + R_2} + \frac{R_1}{R_i}\right)}\right)$$
$$V_{th} \approx -\frac{R_2}{R_1}V_s = -2V_s \tag{2}$$

Finding R_{th}, since we have a dependent voltage source we apply an external voltage, v_0, and remove the independent voltage source to get $R_{th} = v_0/i_0$.

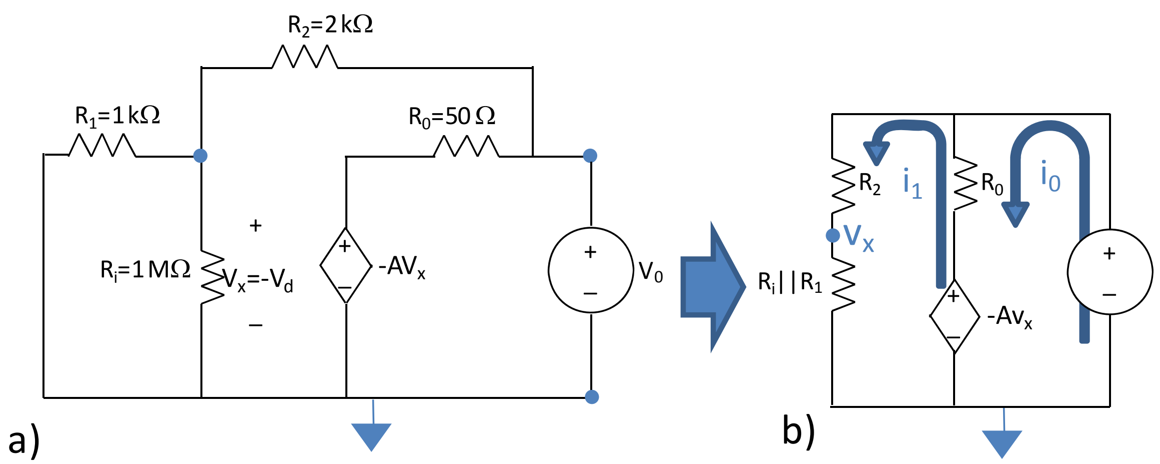


Figure 2 : Circuit diagram for solving for Rth using meth analysis. From a) to b) we have redrawn the circuit to improve the visualization of it.

First looking at loop i_0 and use $v_x = i_1(R_i \parallel R_1)$ to get,

$$v_0 - (i_0 - i_1)R_0 = -v_x A$$
$$v_0 - (i_0 - i_1)R_0 = -i_1(R_i \parallel R_1)A$$
$$\Rightarrow v_0 = i_0 R_0 - i_1(R_0 + (R_i \parallel R_1)A) \quad (3)$$

And for loop i_1 and again using $v_x = i_1(R_i \parallel R_1)$ we get,

$$-v_x A - (i_1 - i_0)R_0 - i_1 R_2 - i_1(R_i \parallel R_1) = 0$$
$$-v_x A + i_0(R_0) - i_1(R_0 + R_2 + R_i \parallel R_1) = 0$$
$$-\big(i_1(R_i \parallel R_1)\big)A + i_0(R_0) - i_1(R_0 + R_2 + R_i \parallel R_1) = 0$$
$$i_0(R_0) - i_1\big(R_0 + R_2 + R_i \parallel R_1(1 + A)\big) = 0 \quad (4)$$

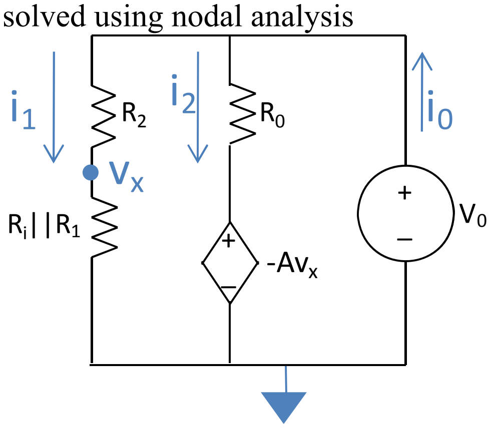
Using (4) to eliminate i_1 in (3) we get,

$$v_0 = i_0 R_0 - \left(\frac{i_0(R_0)}{\big(R_0 + R_2 + R_i \parallel R_1(1 + A)\big)}\right)(R_0 + (R_i \parallel R_1)A) \quad (5)$$

Thus, $R_{th} = v_0/i_0$,

$$R_{th} = \frac{v_0}{i_0} = R_0\left(1 - \left(\frac{(R_0 + (R_i \parallel R_1)A)}{\big(R_0 + R_2 + R_i \parallel R_1(1 + A)\big)}\right)\right) \quad (6)$$
$$R_{th} = R_0\left(\frac{R_2 + (R_i \parallel R_1)}{\big(R_0 + R_2 + R_i \parallel R_1(1 + A)\big)}\right) \approx \frac{R_0}{A}$$
$$\Longrightarrow R_{th} = 0.00015\Omega$$

Alternatively, R_{th} can also be solved using nodal analysis

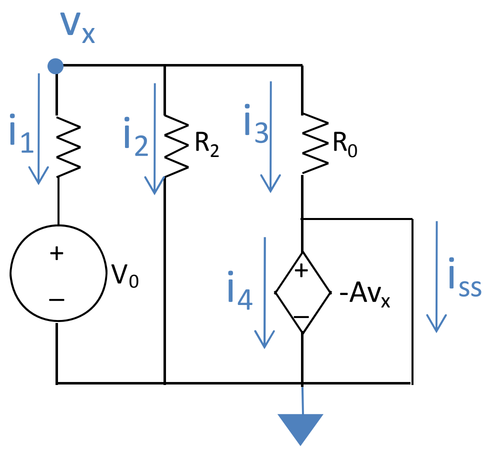


Using

$$i_0 = i_1 + i_2$$
$$i_0 = \frac{v_0}{R_2 + R_1 \parallel R_i} + \frac{v_0 - (-Av_x)}{R_0}$$
$$i_0 = v_0\left(\frac{1}{R_2 + R_1 \parallel R_i} + \frac{1}{R_0}\right) + \frac{A}{R_0}\left(v_0 \frac{R_1 \parallel R_i}{R_1 \parallel R_i + R_2}\right)$$
$$i_0 = v_0\left(\frac{1}{R_2 + R_1 \parallel R_i} + \frac{1}{R_0} + \frac{A}{R_0}\frac{R_1 \parallel R_i}{R_1 \parallel R_i + R_2}\right)$$
$$\Longrightarrow \frac{i_0}{v_0} = \left(\frac{R_0 + R_2 + R_1 \parallel R_i(1 + A)}{R_0(R_1 \parallel R_i + R_2)}\right)$$

$$\Rightarrow R_{th} = \frac{v_0}{i_0} = R_0\left(\frac{(R_1 \parallel R_i + R_2)}{R_0 + R_2 + R_1 \parallel R_i(1+A)}\right) \approx \frac{R_0}{A}$$

$$\Rightarrow R_{th} = 0.00015\Omega$$



Alternatively, you can short the load and find i_{short} and get $R_{th}=V_{th}/i_{short}$

$$i_1 + i_2 + i_3 = 0$$

$$\frac{v_x - v_s}{R_1} + \frac{v_x}{R_i} + \frac{v_x}{R_2} = 0$$

$$v_x\left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right) - \frac{v_s}{R_1} = 0$$

$$\Rightarrow v_x = \frac{v_s}{1 + \frac{R_1}{R_i} + \frac{R_1}{R_2}}$$

At the other node the current equation is,

$$i_4 + i_{ss} = i_3$$

$$\frac{0 - Av_x}{R_0} + i_{ss} = \frac{v_x}{R_2}$$

$$\Rightarrow i_{ss} = v_x\left(\frac{1}{R_2} + \frac{A}{R_0}\right)$$

$$\Rightarrow i_{ss} = \frac{v_s}{1 + \frac{R_1}{R_i} + \frac{R_1}{R_2}}\left(\frac{1}{R_2} + \frac{A}{R_0}\right)$$

Since $R_{th}=V_{th}/i_{ss}$ we get,

$$\Rightarrow R_{th} = \frac{v_{th}}{i_{ss}} = \frac{\left(\frac{\left(-A + \left(\frac{A+1}{R_0 + R_2}\right)R_0\right)}{\left(1 + \frac{R_1(A-1)}{R_0 + R_2} + \frac{R_1}{R_i}\right)}\right)}{\frac{\left(\frac{1}{R_2} + \frac{A}{R_0}\right)}{1 + \frac{R_1}{R_i} + \frac{R_1}{R_2}}} \approx \frac{R_0}{A}$$

$$\Rightarrow R_{th} = 0.00015\Omega$$

Assuming an ideal op-amp we get $V_{th}=-(R_2/R_1)V_s=-2V_s$ and $R_{th}=0\Omega$ which is approximately what was derived above. Other characteristics of an ideal op-amp are $R_0=0$, $R_i=\infty$, $A=\infty$.

Problem 3: Grading Criteria:

6pts for drawing the circuit diagram

8pts for R_{th} :

- 4pts partial credit for fair attempt at R_{th} and is given as a function of R_1, R_2, R_i, R_0
  OR
- 2pts partial credit for fair attempt at R_{th} and is given as a function of R_1, R_2

8pts for V_{th} :

- 4pts partial credit for fair attempt at V_{th} and is given as a function of R_1, R_2, R_i, R_0, A, V_s
  OR
- 2pts partial credit for fair attempt at V_{th} and is given as a function of R_1, R_2. V_s

5pts for comparing to ideal op-amp, $V_0=-R_2/R_1V_s=-2V_s$.

- 3pts if you write $V_0=2V_s$

3pts for saying $R_0=0$, $R_i=\infty$, $A=\infty$.