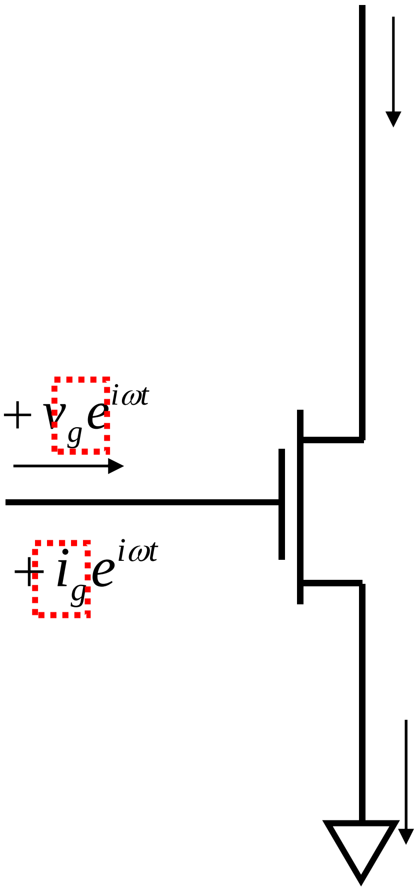
Lecture 10:
HEMT AC properties

AC equivalent circuit:



$$i_D = I_D + i_d e^{i\omega t}$$

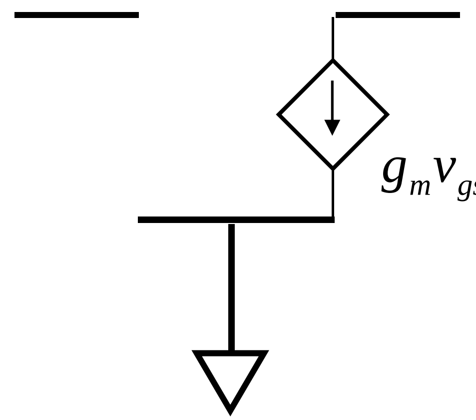
$$v_D = V_D + v_d e^{i\omega t}$$

$$v_G = V_G + v_g e^{i\omega t}$$

$$i_G = I_G + i_g e^{i\omega t}$$

$$i_S = I_S + i_s e^{i\omega t}$$

Hybrid π model:



$$\begin{pmatrix} i_g \\ i_d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ g_m & 0 \end{pmatrix} \begin{pmatrix} v_g \\ v_d \end{pmatrix}$$

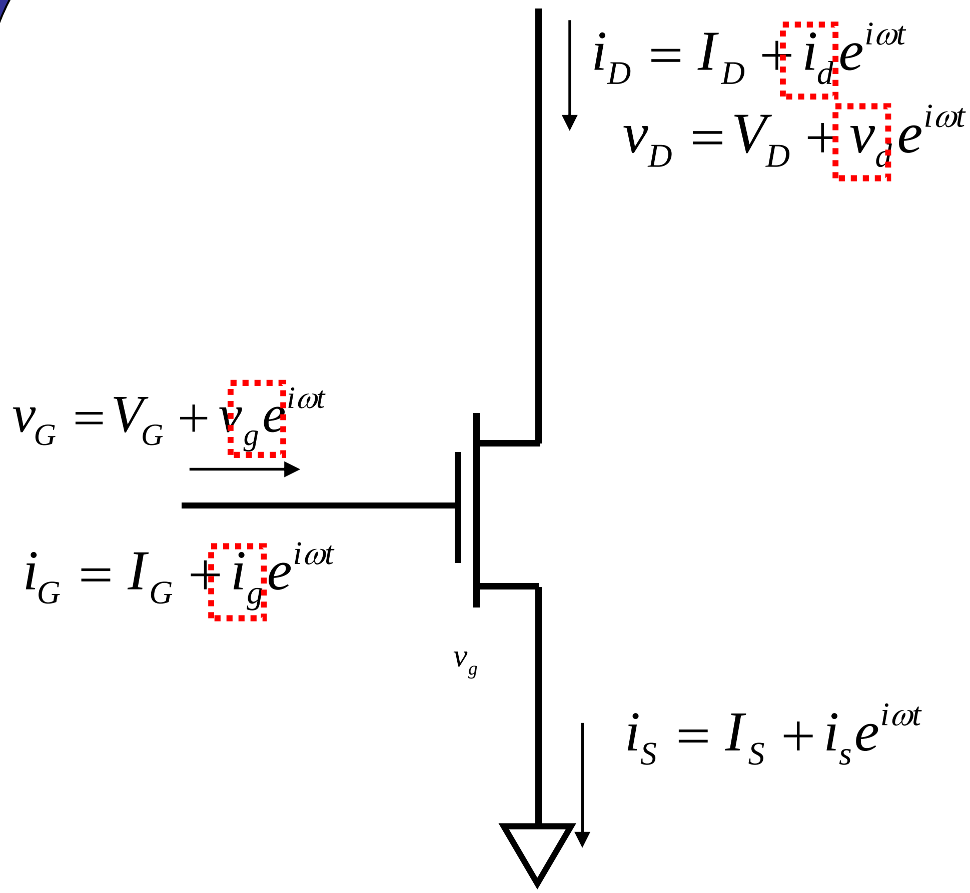
This is the common-source Y-matrix. You can get all the matrices from it.

What about capacitances?

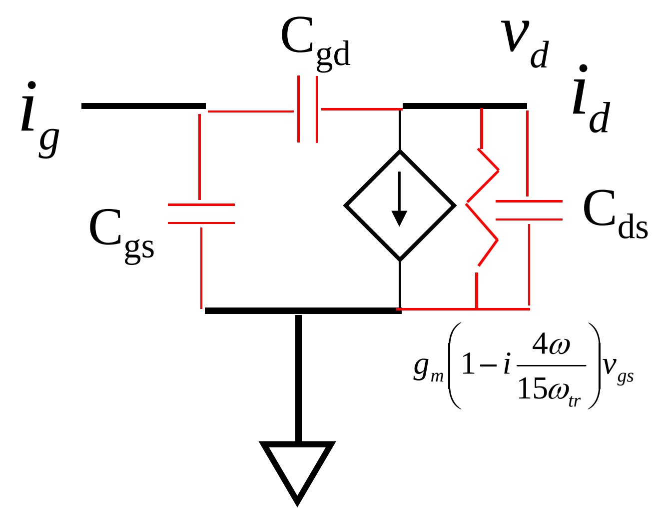
Book sticks to long channel devices, well below transit time frequency.

Modern devices are short and f_T *is* the transit time frequency.

AC equivalent circuit:



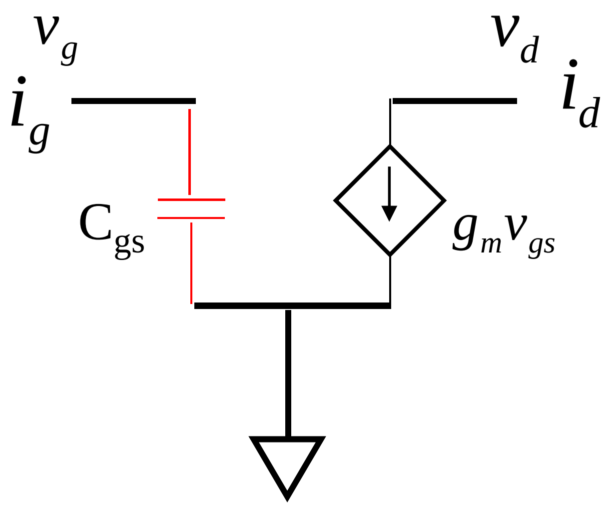
Hybrid π model:



C_{gs} dominates.

f_T

Hybrid π model:



When current flowing through capacitor is equal to $g_m v_{gs}$

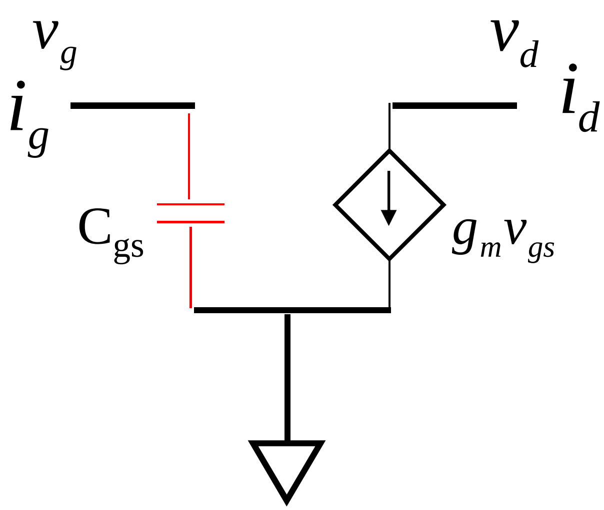
then the frequency is f_T.

$$i_g = v_{gs}\left(\omega C_{gs}\right) \qquad i_d = g_M v_{gs}$$

At f_T $\quad g_M v_{gs} = v_{gs}\left(\omega_T C_{gs}\right)$

$$\Rightarrow \omega_T = \frac{g_M}{C_{gs}} \Rightarrow f_T = \frac{g_M}{2\pi C_{gs}}$$

f_T



$$f_T = \frac{g_M}{2\pi C_{gs}}$$

In HW#6, you will prove for the long-channel device:

$$g_M = \frac{W\mu C'_{ox}}{L}\left(V_{GS} - V_T\right)$$

$$C'_{ox} \sim C_{gs}/(LW)$$

$$f_T \rightarrow \frac{1}{2\pi}\frac{\mu\left(V_{GS} - V_T\right)}{L^2}$$

For a short-channel device,

$$g_M = v_{sat} W C'_{ox}$$

$$f_T \rightarrow \frac{v_{sat}}{2\pi L} = \frac{1}{2\pi \tau_{tr}}$$

So book model is only good for frequencies much less than f_T.

Current

J is 2d, n_s is 2d. (Discuss).

$$J = e \cdot \mu \cdot n_s \cdot E$$

$$I_D = J \cdot (width) = e \cdot \mu \cdot n(x) \cdot E(x) \cdot W$$

$$I_D = \mu \cdot C_{ox}\left(V_{GS} - V_T - V_{CS}(x)\right) \cdot E(x) \cdot W$$

$$= \mu \cdot C_{ox}\underbrace{\left(V_{GS} - V_T - V_{CS}(x)\right)}_{u_{cs}(x)} \cdot \frac{\partial V_{CS}(x)}{\partial x} \cdot W$$

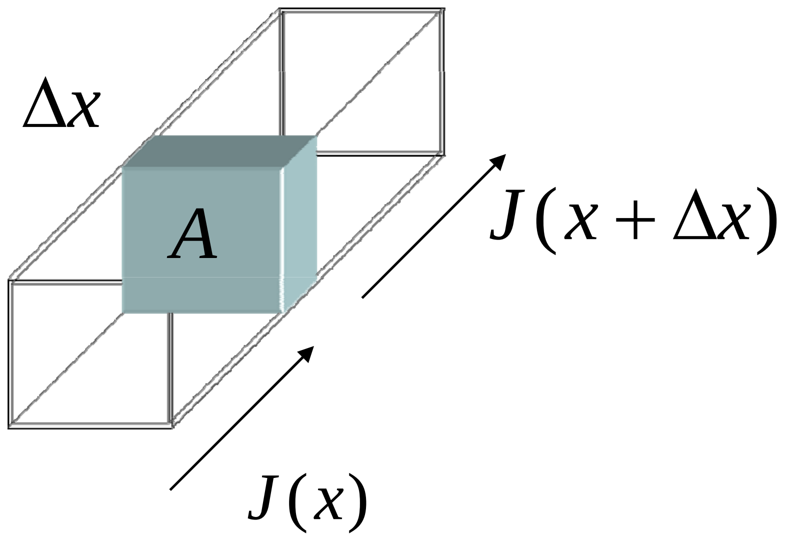
$$I_D = W \cdot \mu \cdot C_{ox} \cdot u_{cs}(x) \cdot \frac{\partial u_{cs}(x)}{\partial x}$$

Current

$$I_D = W \cdot \mu \cdot C_{ox} \cdot u_{cs}(x) \cdot \frac{\partial u_{cs}(x)}{\partial x}$$

$$\rightarrow I_D = W \cdot \mu \cdot C_{ox} \cdot u_{cs}(x,t) \cdot \frac{\partial u_{cs}(x,t)}{\partial x}$$

Continuity equation



How many electrons in green box after a time Δt?

$$\frac{1}{e} A J(x) \Delta t$$ enter

$$\frac{1}{e} A J(x+dx) \Delta t$$ leave

Add em up:

$$\Delta N = \frac{1}{e} A \left[J(x) - J(x+dx) \right] \Delta t$$

Divide by: $A \Delta x \Delta t$

$$\Rightarrow \frac{\partial n}{\partial t} = \frac{1}{e} \frac{\partial J(x)}{\partial x}$$

$$\frac{\partial n}{\partial t} = W \cdot C_{ox} \cdot \frac{\partial u_{cs}(x,t)}{\partial t}$$

Current

$$\frac{\partial I_D}{\partial x} = 0 \rightarrow \frac{\partial I_D}{\partial x} = W \cdot C_{ox} \cdot \frac{\partial u_{cs}(x,t)}{\partial t}$$

Current

$$\frac{\partial I_D}{\partial x} = W \cdot C_{ox} \cdot \frac{\partial u_{cs}(x,t)}{\partial t}$$

$$I_D = W \cdot \mu \cdot C_{ox} \cdot u_{cs}(x,t) \cdot \frac{\partial u_{cs}(x,t)}{\partial x}$$

$$\frac{\partial}{\partial x}\left(W \cdot \mu \cdot C_{ox} \cdot u_{cs}(x,t) \cdot \frac{\partial u_{cs}(x,t)}{\partial x} \right) = W \cdot C_{ox} \cdot \frac{\partial u_{cs}(x,t)}{\partial t}$$

$$\mu \cdot \frac{\partial}{\partial x}\left(u_{cs}(x,t) \cdot \frac{\partial u_{cs}(x,t)}{\partial x} \right) = \frac{\partial u_{cs}(x,t)}{\partial t}$$

Quasi-static vs full soln.

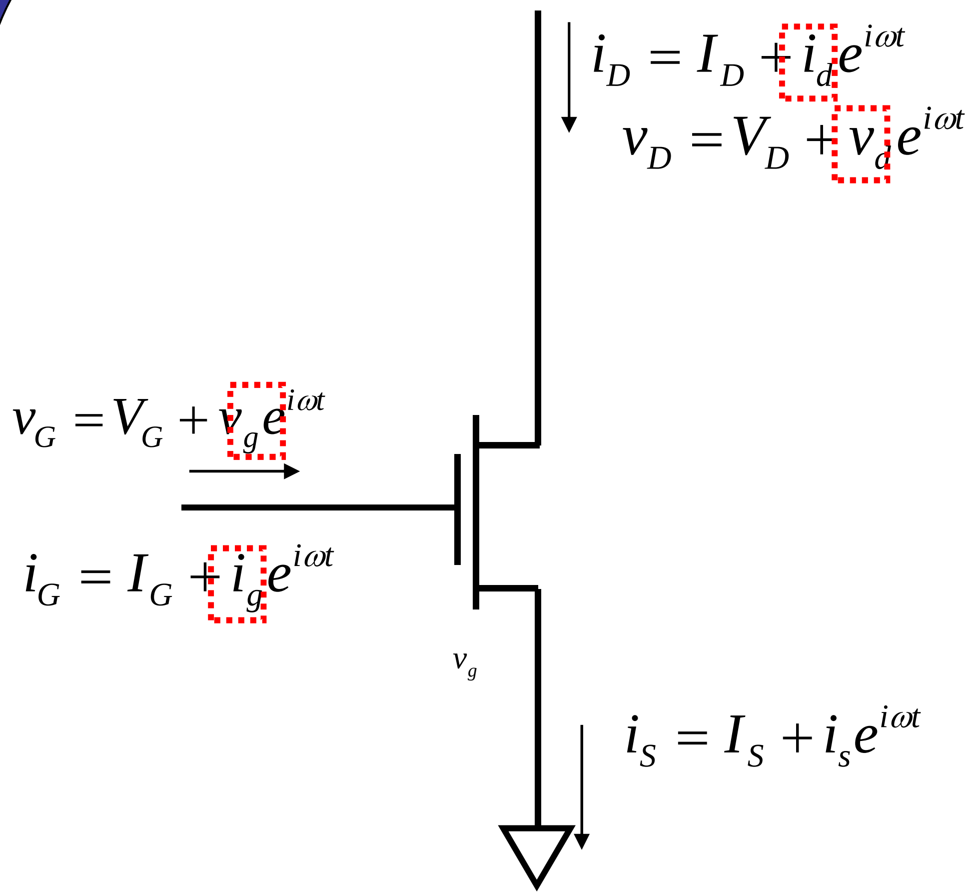
$$\mu \cdot \frac{\partial}{\partial x}\left(u_{cs}(x,t) \cdot \frac{\partial u_{cs}(x,t)}{\partial x} \right) = \frac{\partial u_{cs}(x,t)}{\partial t}$$

This differential equation in general must be solved by a computer.

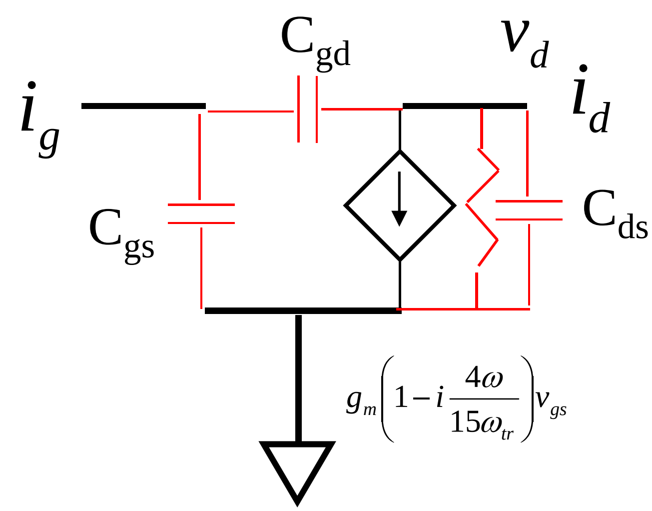
Quasi-static: Apply dc solution, only V_{GS} and V_{DC} changes with time.
This gives you y-parameters at frequencies well below transit frequency.

Full solution: Assume dc plus small ac voltages/currents. “Linearize” the differential equation: Taylor expand everything under the sun in terms of the ac voltages, currents. (Discuss on board.)
Solve the simplified, linearized differential equations.
Easy in principle but very tedious in practice. Book does it for constant mobility case. But modern FETs are in saturated velocity regime.
I don’t know of any analytical solutions in that regime, only numerical.

Quasi-static AC equivalent circuit:



Hybrid π model:

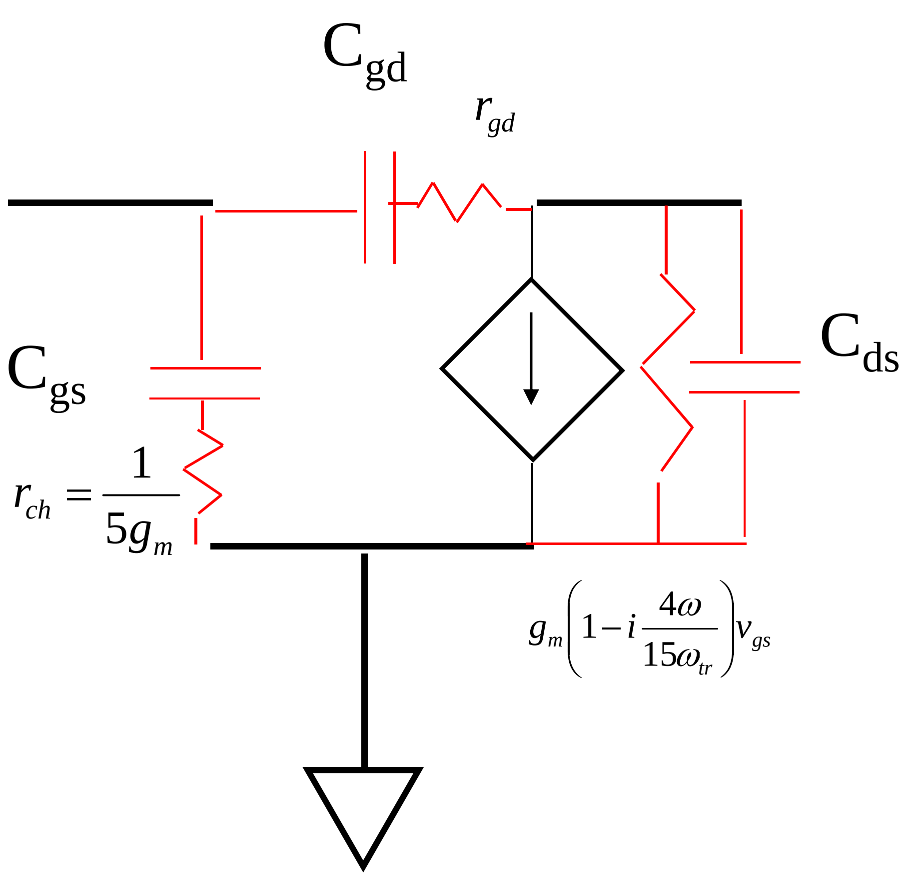


Non-quasi static model:

Hybrid π model:

v_g

i_g

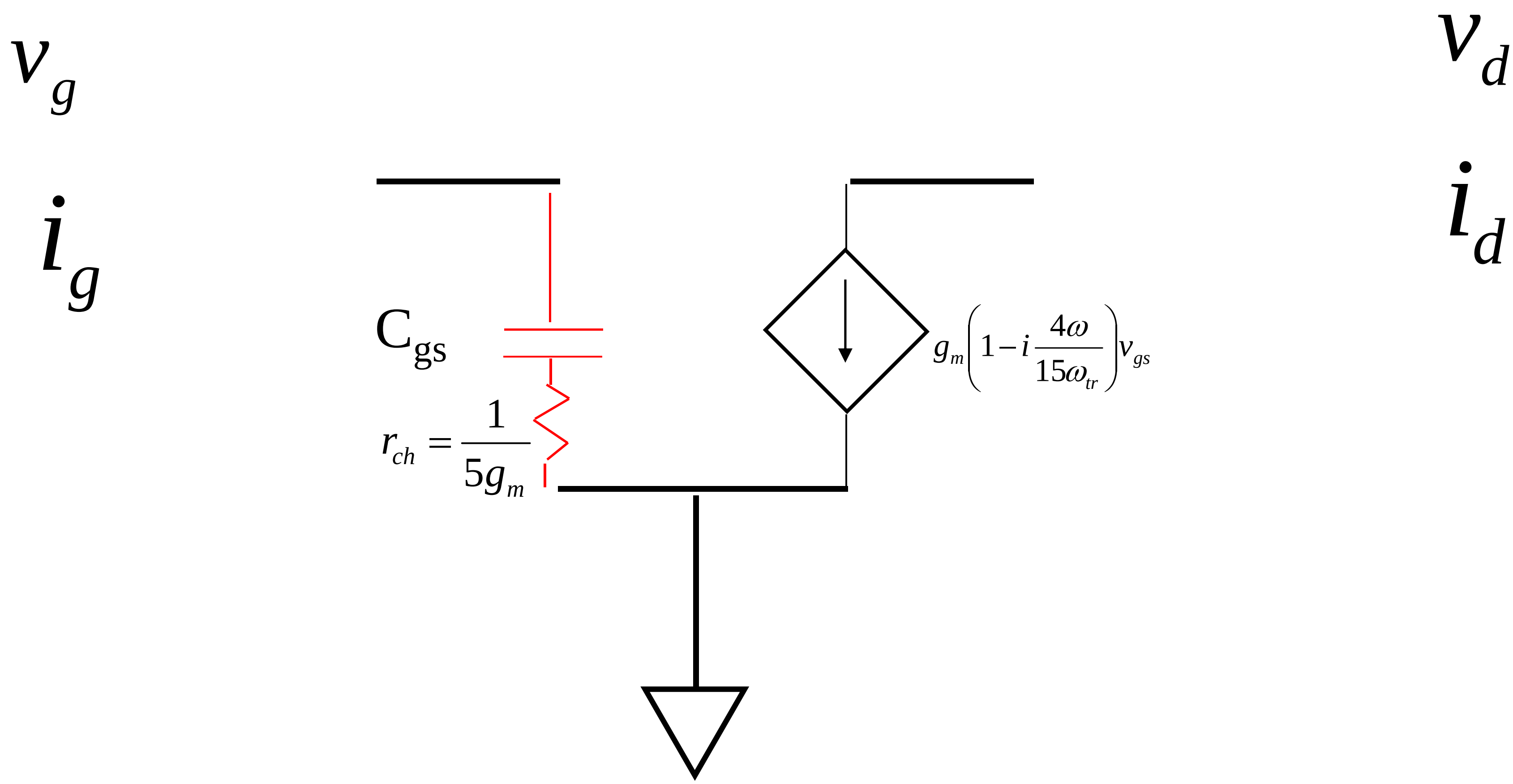


v_d

i_d

Non-quasi static model in saturation:

Hybrid π model:

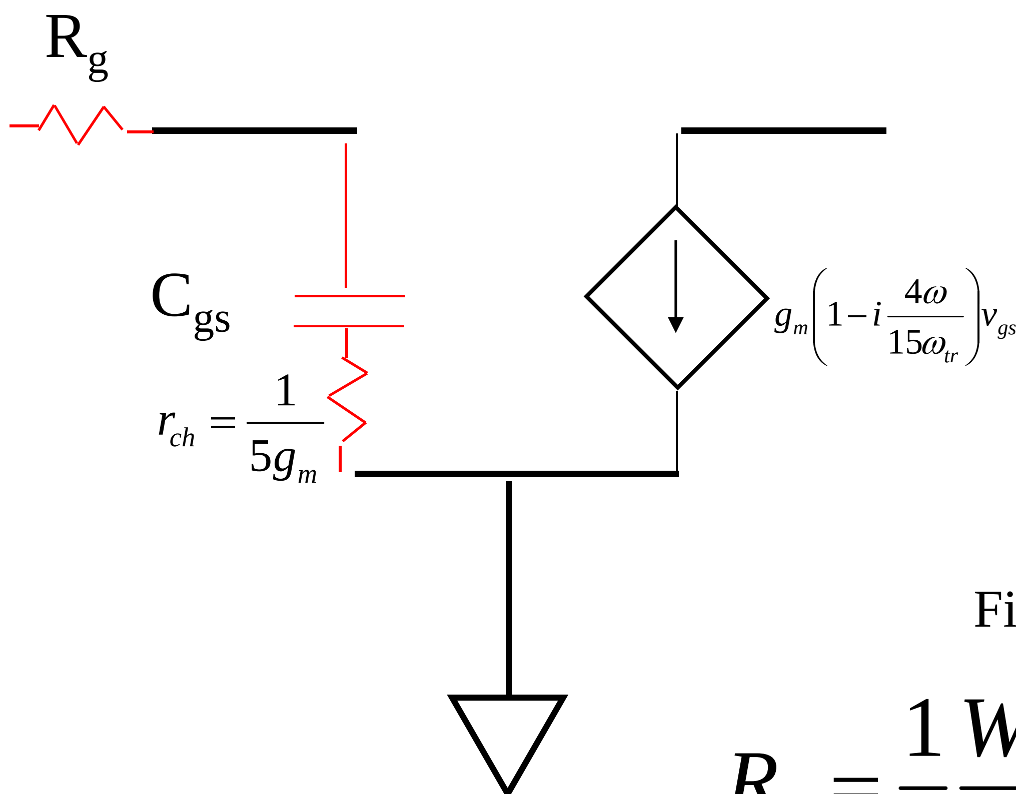


Parasitics: Gate resistance:

$$R_g = \frac{1}{3}\frac{W}{L}R_{square}$$

v_g $\qquad$ v_d

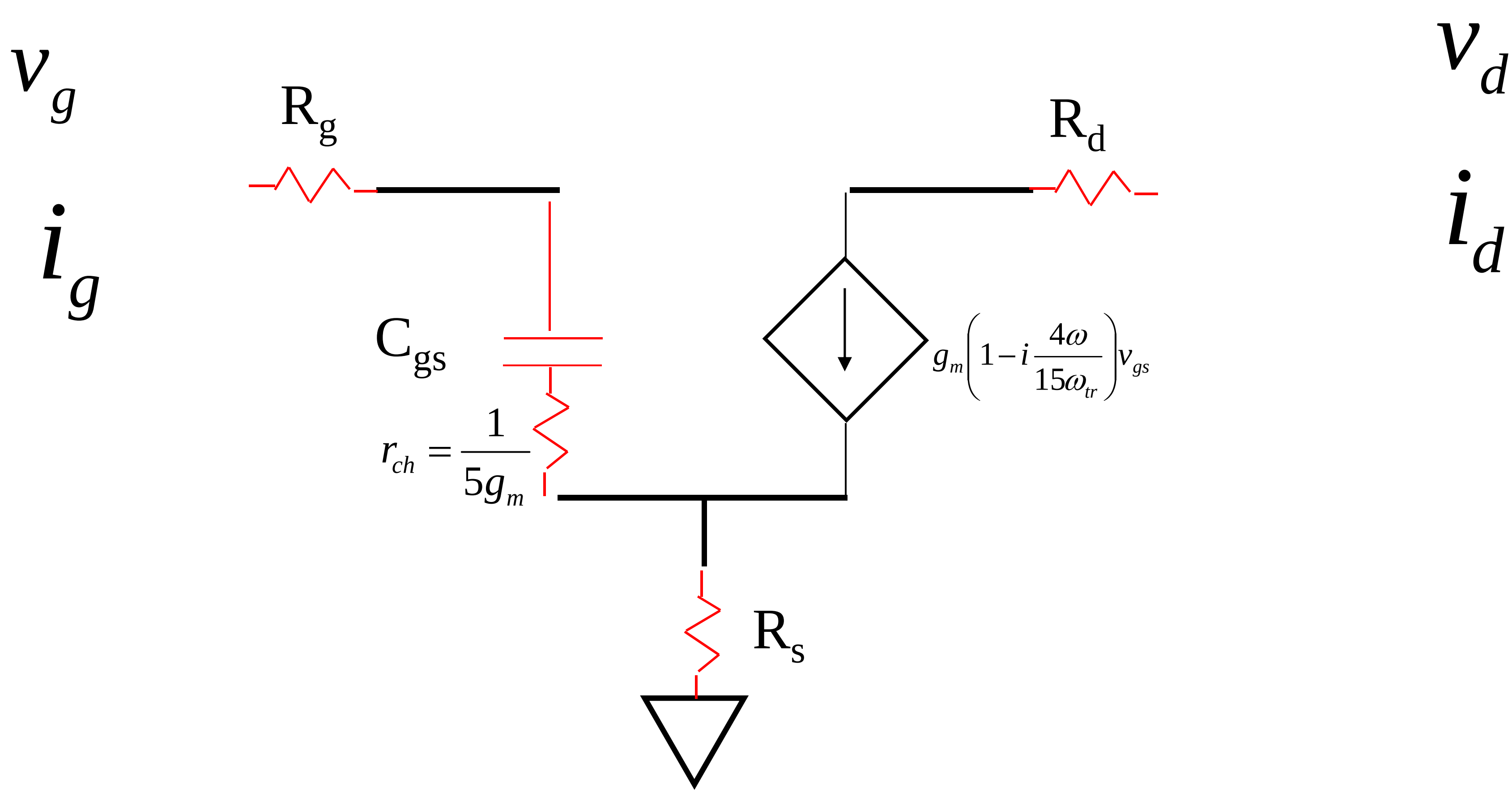
i_g $\qquad$ i_d



Fingers:

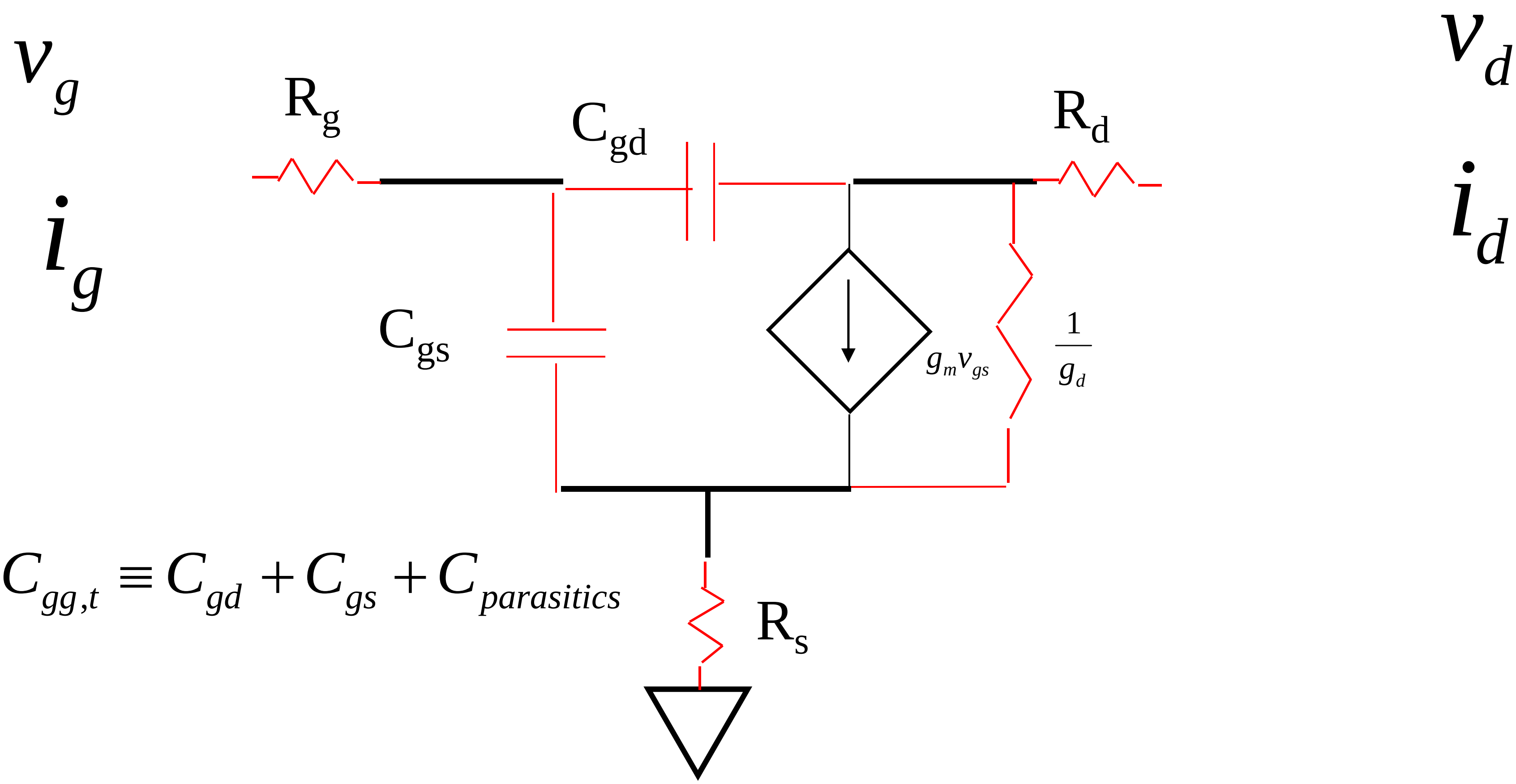
$$R_g = \frac{1}{3}\frac{W}{L}R_{square}\frac{1}{N}$$

Parasitics: Source/Draina resistance:



f_T

$$\frac{1}{2\pi f_T} = \frac{C_{gg,t}}{g_m} + \frac{C_{gg,t}}{g_m}(R_S + R_D)g_d + (R_S + R_D)C_{gd,t}$$



$$C_{gg,t} \equiv C_{gd} + C_{gs} + C_{parasitics}$$

f_{MAX}

$$f_{MAX} = \sqrt{\frac{f_T}{8\pi R_G C_{gd,t}\left[1+\left(\frac{2\pi f_T}{C_{gd,t}}\right)\Psi\right]}}$$

$$\Psi \equiv (R_S + R_D)\frac{{C_{gg,t}}^2 g_d^2}{g_m^2} + (R_S + R_D)\frac{C_{gd,t} C_{gg,t} g_d}{g_m} + \frac{{C_{gg,t}}^2 g_d}{g_m^2}$$

f_{max} helped by fingers.
f_T not helped by fingers.
fMax sometimes larger, sometimes smaller than f_T.

RF CMOS

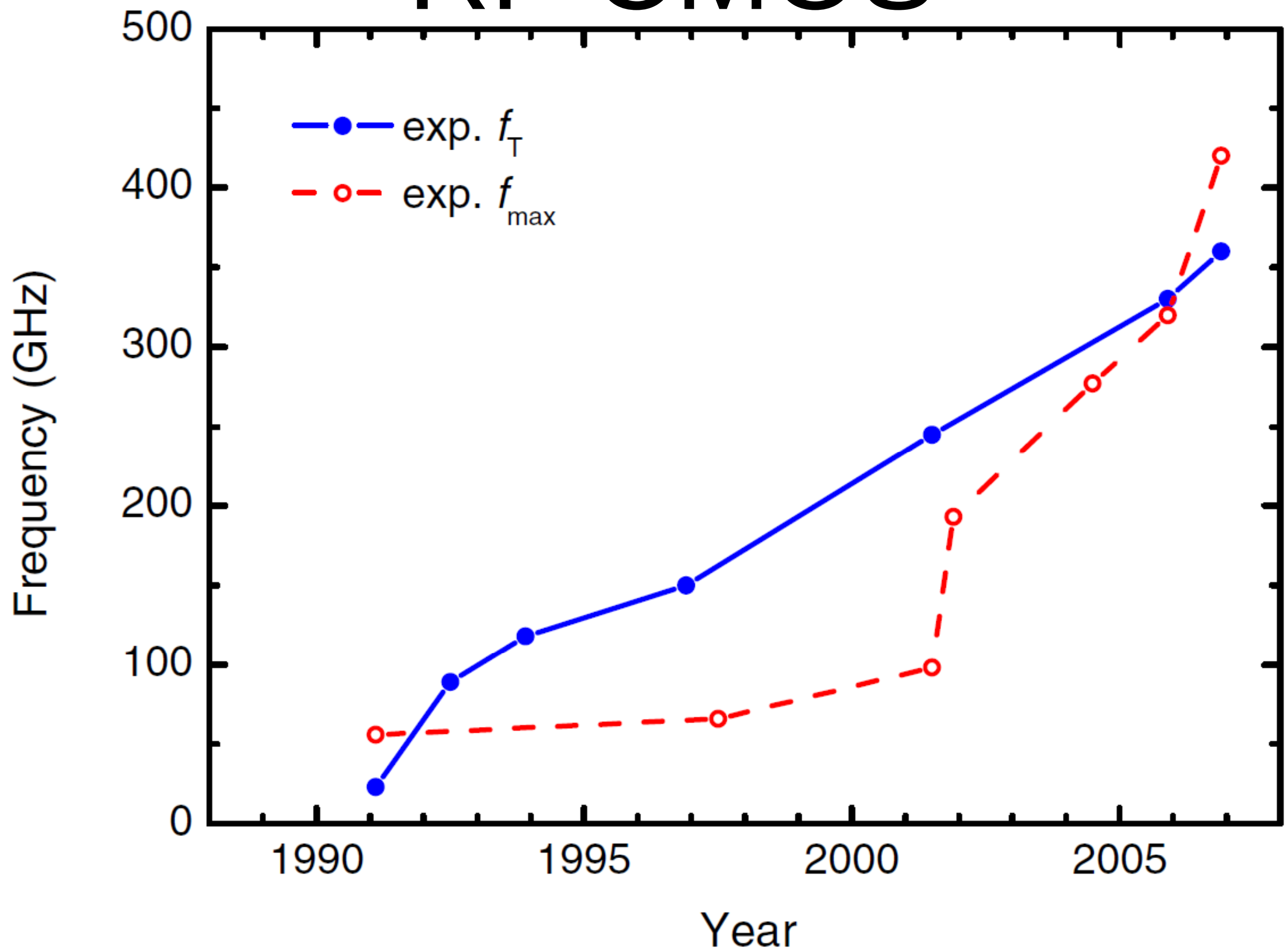
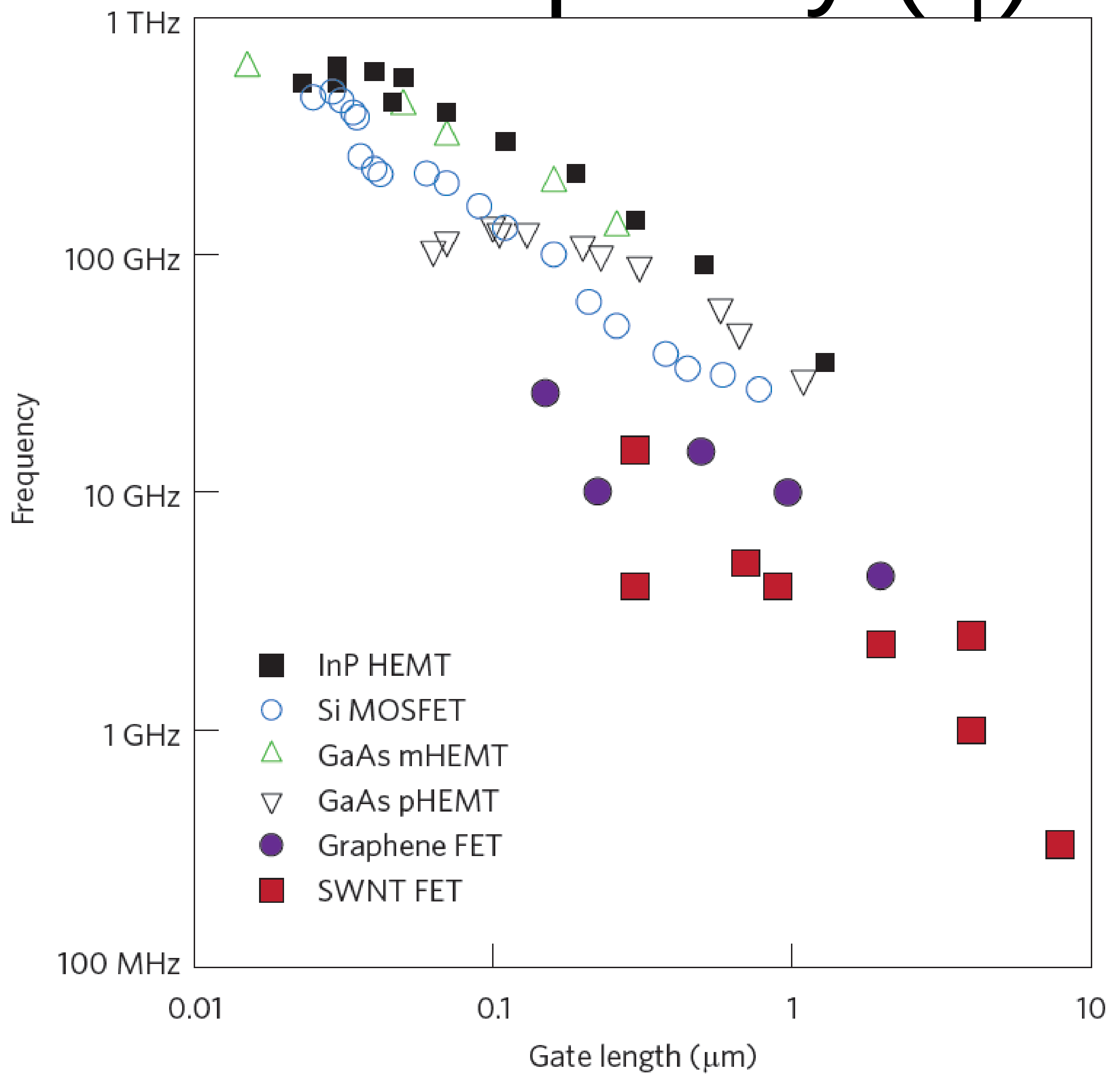


Fig. 1. Evolution of the record cutoff frequency f_T and the record maximum frequency of oscillation f_{max} of RF Si MOSFETs versus time.

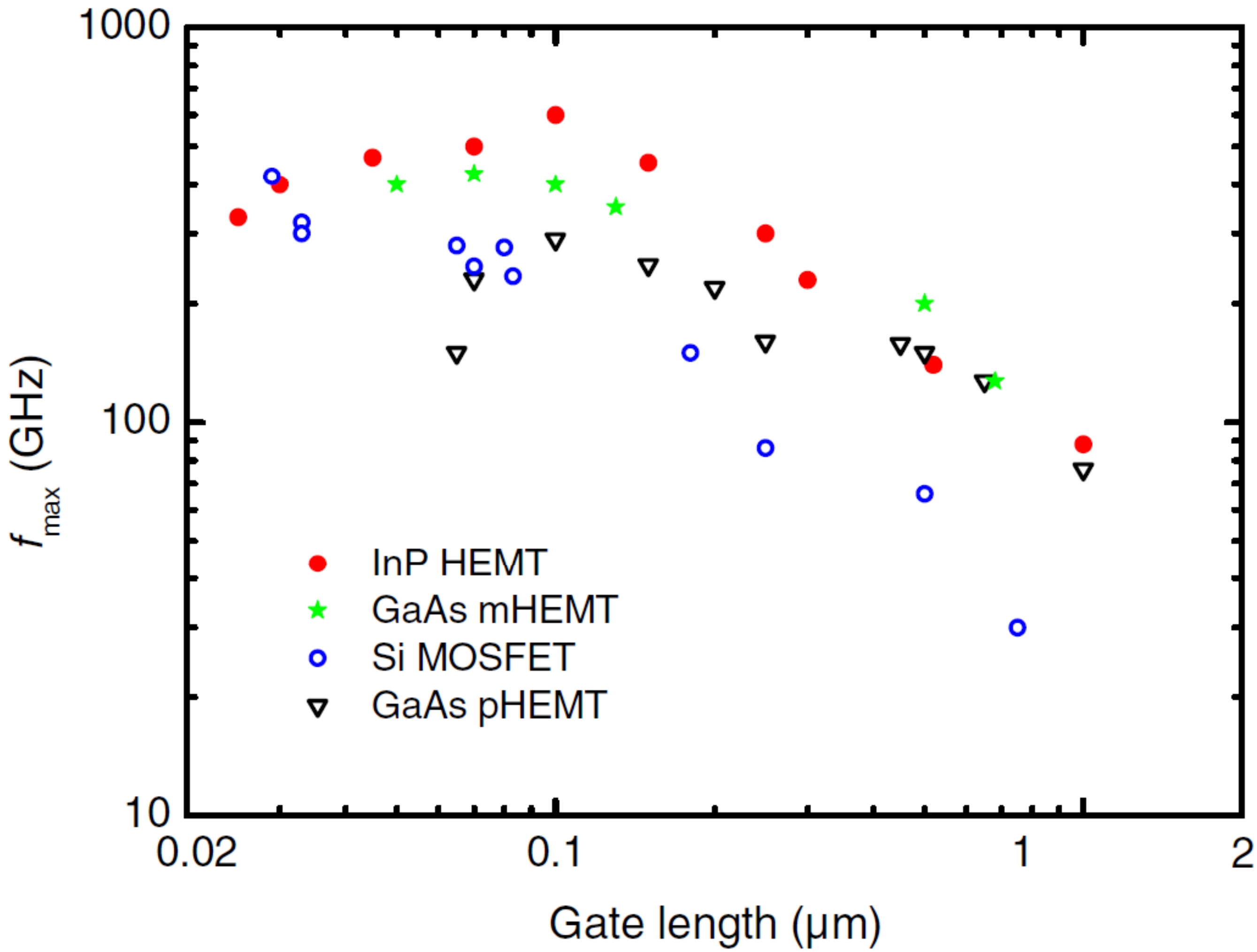
F. Schwierz and J. J. Liou, "RF Transistors: Recent Developments and Roadmap toward Terahertz Applications", *Solid-State Electronics*, ***51, 1079-1091, (2007).***

Cutoff frequency (f_T)



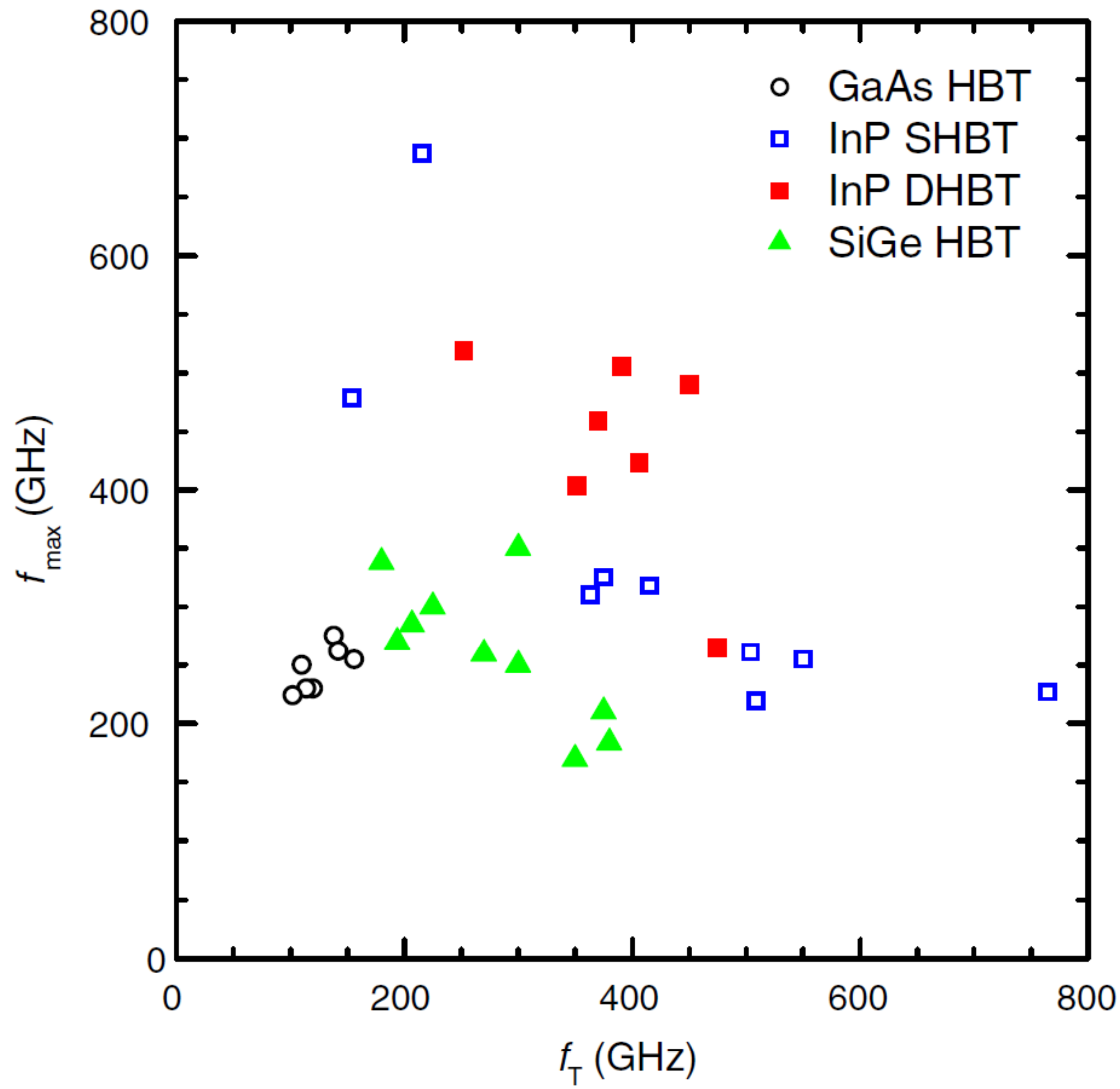
C. Rutherglen, D. Jain and P. Burke, "Nanotube Electronics for Radiofrequency Applications", *Nature Nanotechnology*, ***4, 811-819, (2009).***

III-V f_{Max}



F. Schwierz and J. J. Liou, "RF Transistors: Recent Developments and Roadmap toward Terahertz Applications", *Solid-State Electronics*, ***51, 1079-1091, (2007).***

f_T vs f_{Max}



F. Schwierz and J. J. Liou, "RF Transistors: Recent Developments and Roadmap toward Terahertz Applications", *Solid-State Electronics*, ***51, 1079-1091, (2007).***