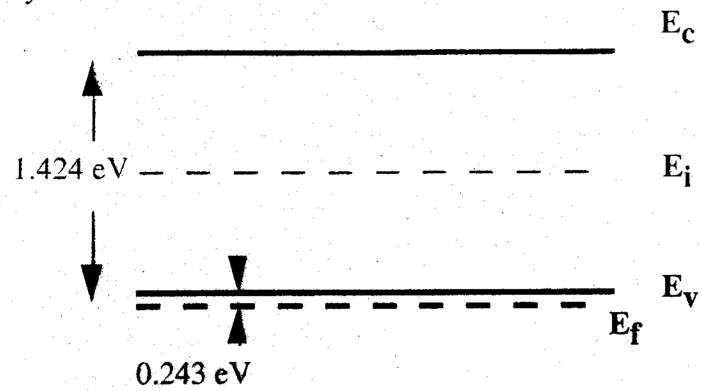
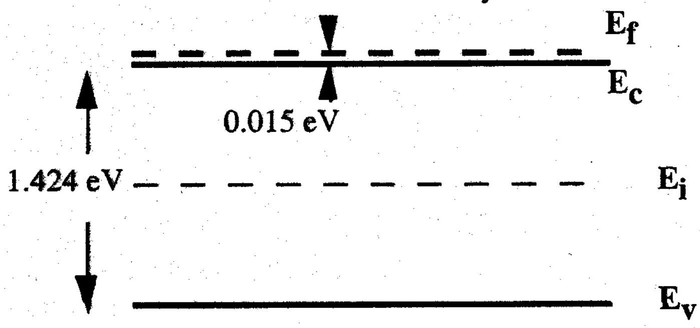
Solutions to Chapter 1 Problems
Fundamentals of III-V Devices, by W. Liu

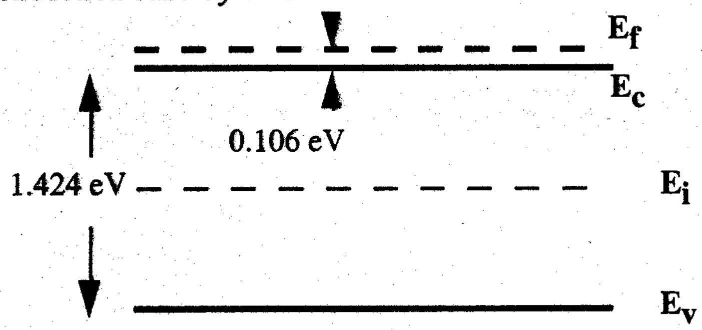
1. (a) For a p-type GaAs layer. $N_v = 4.7 \times 10^{18}$ cm^{-3}. $p/N_v = 1 \times 10^{20} / 4.7 \times 10^{18} = 21.3$. From Fig. 1-21, we locate 21.3 from the y-axis. The corresponding x-axis value is 9.4. Therefore, $E_v - E_f = 9.4 \times 0.0258 = 0.243$ eV. The Fermi-level is below the valence band by 0.243 eV.



(b) According to Example 1-3, N_c for $Al_{0.35}Ga_{0.65}As$ is 3.72×10^{17} cm^{-3}. $n/N_c = 5 \times 10^{17} / 3.72 \times 10^{17} = 1.34$. From Fig. 1-21, we locate 1.34 from the y-axis. The corresponding x-axis value is 0.6. Therefore, $E_f - E_c = 0.6 \times 0.0258 = 0.015$ eV. The Fermi-level is above the conduction band by 0.015 eV.



(c) For GaAs, $n_i = 1.79 \times 10^6$ cm^{-3} and $N_c = 4.7 \times 10^{17}$ cm^{-3}. The electron concentration is $n_i^2/p = (1.79 \times 10^6)^2 / 1 \times 10^{-6} = 3.2 \times 10^{18}$ cm^{-3}. $n/N_c = 3.2 \times 10^{18} / 4.7 \times 10^{17} = 6.8$. From Fig. 1-21, we locate 6.8 from the y-axis. The corresponding x-axis value is 4.1. Therefore, $E_f - E_c = 4.1 \times 0.0258 = 0.106$ eV. The Fermi-level is above the conduction band by 0.106 eV.



2. Let us suppose that the material is p-type. If region A is hotter, then the holes tend to

$$\delta p = \delta p\ (0)\, exp\left(-\frac{t}{\tau_p}\right).$$

The time constant is τ_p .

5. (a) At 10 V, the electric field is 10 V/cm. For the 5×10^{18} cm^{-3} sample, the mobility is given by Eq. (1-73), is $7200/(1 + 5.51 \times 10^{-17} \cdot 5 \times 10^{18})^{0.233} = 1943$ cm^2/V-s. We use Eq. (1-35) to solve for the current density. Since there is no concentration gradient, the diffusion component is zero. $J_n = 1.6 \times 10^{-19} \cdot 1943 \cdot 5 \times 10^{18} \cdot 10 = 1.55 \times 10^4$ A/cm^2. The current is 1.55×10^4 A/cm$^2 \cdot 100\ \mu$m$^2 = 0.0155$ A.

(b) Let us assume that the current flow under this scenario is identical to that obtained from part (a). This is our initial guess, and we shall obtain the solution by iteration. The temperature rise is 200 K/W · 10 V · 0.0155 A = 31 °C. We shall approximate it as 30 °C; therefore, the mobility of the transistor is 1943 · 70 % = 1360 cm^2/V-s. With this new mobility value, the current density is 1.085×10^4 A/cm^2 and the current is 0.01085 A. If this were indeed the current, then the temperature rise would be 21.7 °C and the mobility would become 1521 cm^2/V-s. With this new mobility value, the current density is 1.21×10^4 A/cm^2 and the current is 0.0121 A. With this current, the temperature rise is 24.2 °C and the mobility becomes 1472 cm^2/V-s. With this new mobility value, the current density is 1.17×10^4 A/cm^2 and the current is 0.0117 A. We can repeat the iteration more if a more accurate solution is needed. However, at this point we note that 0.0117 A is not too far from our previous guess of 0.0121 A. Hence, we approximate the current flow to be 0.0117 A.

6. The electric field is [illegible] = 2×10^2 V/cm. From the equation in Example 1-7, $\mu_p = x/\varepsilon t = 100\ \mu$m / (5 [illegible] V/cm) / ([illegible] ns) = 100 cm^2/V-s.

7. (a) At $x \neq 0$, the generation rate is zero. Further, since there is no external electric field and the problem is static, $\varepsilon = \partial(\delta p)/\partial t = 0$. Therefore, Eq. (1-66) becomes:

$$D_p \frac{d^2(\delta p)}{dx^2} - \frac{\delta p}{\tau_p} = 0 .$$

The above equation applies to both $x > 0$ and $x < 0$. Let L_p be the square root of $D_p \cdot \tau_p$. Consider the case of $x > 0$ first, with a boundary condition that $\delta p(x = \infty) = 0$. Because the above differential equation has the solution of $A \cdot \exp(-x/L_p) + B\exp(+x/L_p)$, the boundary condition implies that $B = 0$. Therefore, the solution at $x > 0$ is $A \cdot \exp(-x/L_p)$. From symmetry, the solution at $x < 0$ is $A \cdot \exp(+x/L_p)$.

(b) At $x = 0$, we still have $\varepsilon = \partial(\delta p)/\partial t = 0$. However, the generation rate is no longer zero. Let the generation rate be g cm^{-3}·s^{-1}, then Eq. (1-66) at $x = 0$ is:

$$D_p \frac{d^2(\delta p)}{dx^2} + g - \frac{\delta p}{\tau_p} = 0 .$$

Let us integrate the above differential equation. We obtain the following identity:

$$D_p \int_{-\Delta}^{\Delta} \frac{d^2(\delta p)}{dx^2}\, dx + \int_{-\Delta}^{\Delta} g\, dx - \int_{-\Delta}^{\Delta} \frac{\delta p}{\tau_p}\, dx = 0 \qquad \text{(a)}$$

From elementary calculus, we find that if $f(x)$ is a continuous function, then

2-D Density of States

In two dimensionsal structures such as the quantum well, the procedure is much the same but this time one of the k-space components is fixed. Instead of a finding the number of k-states enclosed within a sphere. The problem is to calculate the number of k-states lying in an annulus of radius **k** to**k** +d **k** . k-space would be completely filled if each state occupied an area of

$$V_{2D} = \left(\frac{2\pi}{L}\right)^2 \tag{13}$$

And the 'volume' of the annulus is given by

$$v_{s2D}\,\mathrm{d}\mathbf{k} = 2\pi|\mathbf{k}|\,\mathrm{d}\mathbf{k} \tag{14}$$

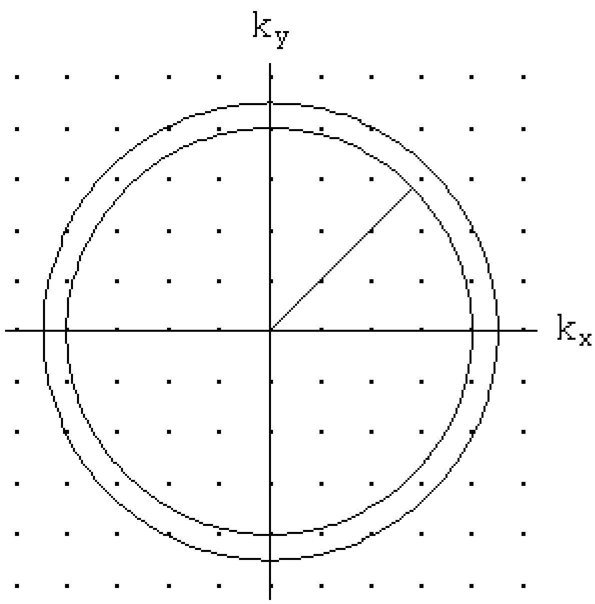


Figure 2 k-space in 2D. The density of states at an energy E is the number of k-states per unit volume contained with the annulus of radius k and thickness dk.

Dividing the 'volume' of the k-state by the area of the annulus gives and remembering to multiply by 2 to account for the electron spin states we get:

$$\mathrm{g}(\mathbf{k})_{2D}\,\mathrm{d}\mathbf{k} = 2\times\frac{v_{s2D}}{V_{2D}} = \frac{|\mathbf{k}|\,\mathrm{d}\mathbf{k}L^2}{\pi} \tag{15}$$

Or in terms of energy per unit volume at an energy *E* .

$$\mathrm{g}(E)_{2D}\,\mathrm{d}E = \frac{k\,\mathrm{d}k}{\pi} = \sqrt{\frac{2mE}{\hbar^2}}\left(\frac{2mE}{\hbar^2}\right)^{-1/2}\frac{m}{\hbar^2}\,\mathrm{d}E = \frac{m}{\pi\hbar^2}\,\mathrm{d}E \tag{16}$$

It is significant that the 2-d density of states does not depend on energy.