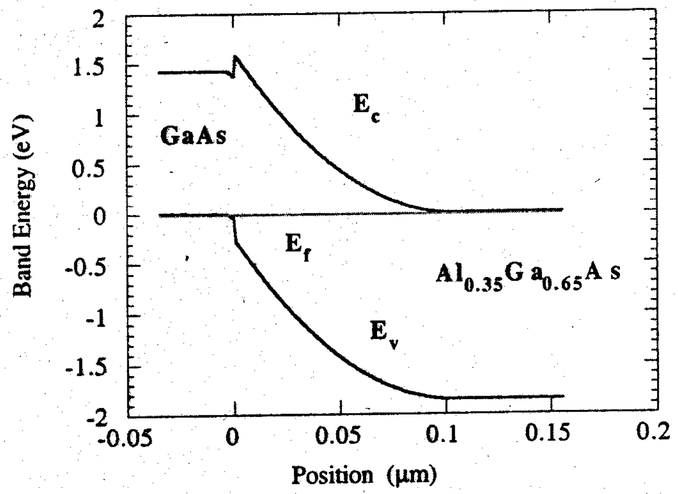
Solutions to Chapter 2 Problems
Fundamentals of III-V Devices, by W. Liu

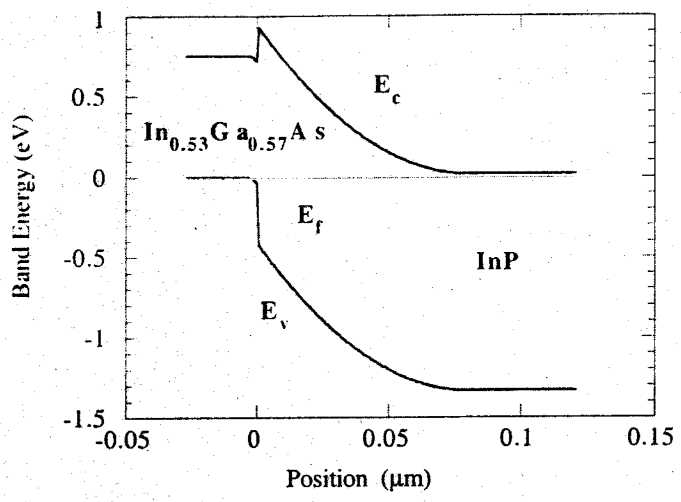
1. (a) For the (N) $Al_{0.35}Ga_{0.65}As$/ (p) GaAs heterojunction:

According to Example 1-3, N_c for $Al_{0.35}Ga_{0.65}As$ is 3.72×10^{17} cm^{-3}. n/N_c = 2/3.72 = 0.54. From Fig. 1-21, we locate 0.54 from the y-axis. The corresponding x-axis value is - 0.4. Therefore, $E_f - E_c = -0.4 \times 0.0258 = -0.01$ eV. Therefore, $\Phi_N = 0.01$ eV. For GaAs, $N_v = 4.7 \times 10^{18}$ cm^{-3}. $p/N_v = 5 / 4.7 = 1.06$. From Fig. 1-21, we locate 1.06 from the y-axis. The corresponding x-axis value is 0.4. Therefore, $E_v - E_f = 0.4 \times 0.0258 = 0.01$ eV. Therefore, $\Phi_p = -0.01$ eV. From Eq. (2-14), $\phi_{bi} = 1.668$ eV; $\phi_{No} = 1.6115$ V; $\phi_{p0} = 0.0564$ V; $X_{No} = 0.104$ µm; $X_{po} = 4.15 \times 10^{-3}$ µm.



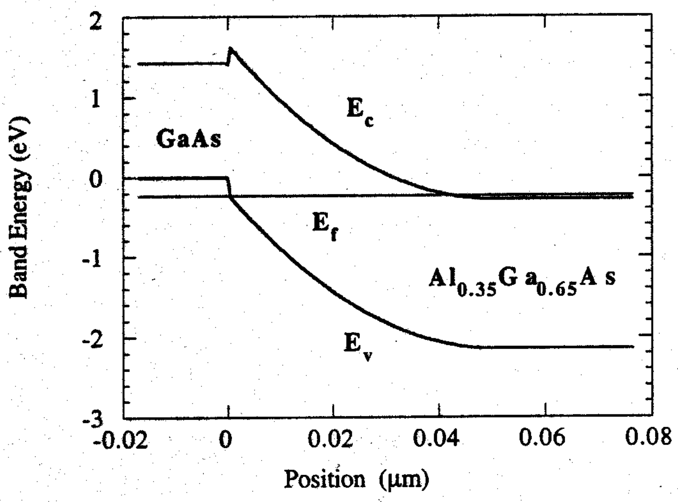
For the (N) InP/ (p) $In_{0.53}Ga_{0.47}As$ heterojunction:

We shall use the following parameters for InGaAs: $E_g = 0.75$ eV, $N_v = 6 \times 10^{18}$ cm^{-3}; ϵ_r = 14; and for InP: $E_g = 1.35$ eV, $N_c = 5.8 \times 10^{17}$ cm^{-3}; $\epsilon_r = 12.6$. The valence band discontinuity is 0.37 eV. For the InP, n/N_c = 2/5.8 = 0.345. From Fig. 1-21, we locate 0.345 from the y-axis. The corresponding x-axis value is - 0.9. Therefore, $E_f - E_c = -0.9 \times 0.0258 = -0.023$ eV. Therefore, $\Phi_N = 0.023$ eV. For the InGaAs, $p/N_v = 5 / 6 = 0.83$. From Fig. 1-21, we locate 0.83 from the y-axis. The corresponding x-axis value is 0.2. Therefore, $E_v - E_f = 0.2 \times 0.0258 = 0.005$ eV. Therefore, $\Phi_p = -0.005$ eV. From Eq. (2-14), $\phi_{bi} = 0.962$ V; $\phi_{No} = 0.9285$ V; $\phi_{p0} = 0.0335$ V; $X_{No} = 0.0805$ µm; $X_{po} = 3.22 \times 10^{-3}$ µm.



(b) For the (*N*) $Al_{0.35}Ga_{0.65}As$/ (*p*) GaAs heterojunction:

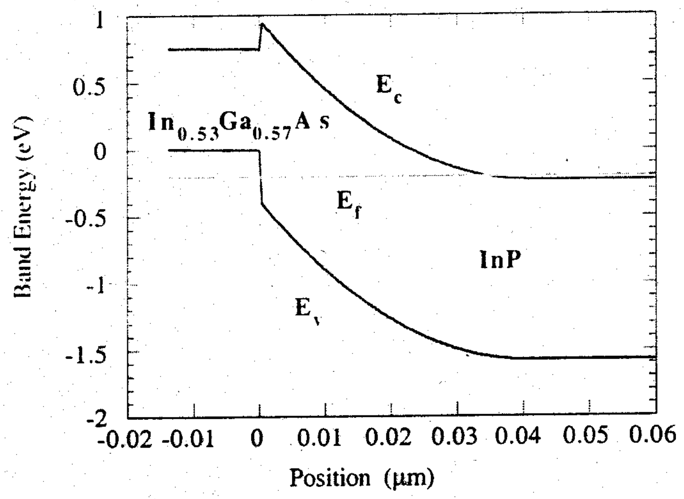
For the AlGaAs, n/N_c = 10/3.72 = 2.7. From Fig. 1-21, we locate 2.7 from the *y*-axis. The corresponding *x*-axis value is 2. Therefore, $E_f - E_c = 2 \times 0.0258 = 0.0516$ eV. Therefore, $\Phi_N = -0.0516$ eV. For GaAs, p/N_v = 100 / 4.7 = 21.3. From Fig. 1-21, we locate 21.3 from the *y*-axis. The corresponding *x*-axis value is 9.2. Therefore, $E_v - E_f = 9.2 \times 0.0258 = 0.237$ eV. Therefore, $\Phi_p = -0.237$ eV. From Eq. (2-14), $\phi_{bi} = 1.9566$ V; $\phi_{No} = 1.9388$ V; $\phi_{p0} = 0.01788$ V; $X_{No} = 0.0509$ μm; $X_{po} = 5.09 \times 10^{-4}$ μm.



For the (*N*) InP/ (*p*) $In_{0.53}Ga_{0.47}As$ heterojunction:

For the InP, n/N_c = 10/5.8 = 1.72. From Fig. 1-21, we locate 1.72 from the *y*-axis. The corresponding *x*-axis value is 1.2. Therefore, $E_f - E_c = 1.2 \times 0.0258 = 0.031$ eV. Therefore, $\Phi_N = -0.031$ eV. For the InGaAs, p/N_v =100 / 6 = 16.7. From Fig. 1-21, we locate 16.7 from the *y*-axis. The corresponding *x*-axis value is 7.8. Therefore, $E_v - E_f = 7.8 \times 0.0258 = 0.20$ eV. Therefore, $\Phi_p = -0.20$ eV. From Eq. (2.14), $\phi_{bi} = 1.211$ V;

ϕ_{No}= 1.20 V; ϕ_{p0} = 0.0108 V; X_{No} = 0.0409 µm; X_{No} = 4.09 x 10^{-4} µm.



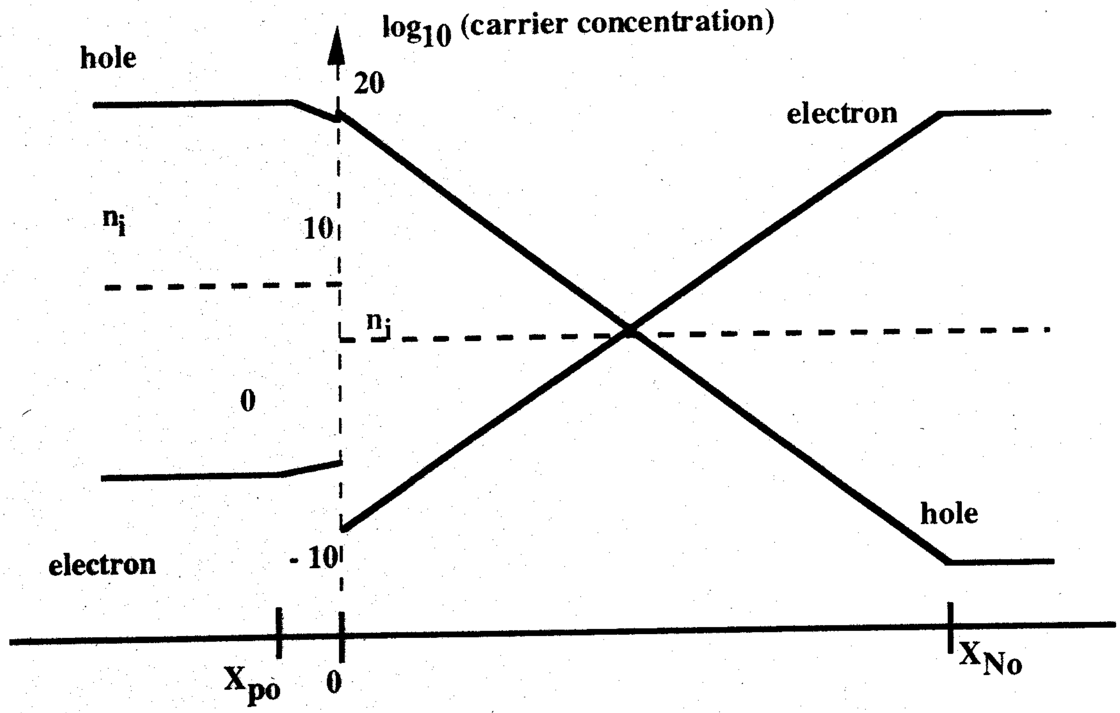
2. (a) <u>For the (*N*) $Al_{0.35}Ga_{0.65}As$/ (*p*) GaAs heterojunction:</u>

Although $pn = n_i^2$ holds only for non-degenerate semiconductors, we will continue to use it for all samples. According to Example 1-2, n_i for $Al_{0.35}Ga_{0.65}As$ is 3.6×10^2 cm^{-3}. Also, n_i for GaAs is 1.79×10^6 cm^{-3}. At $x = X_{N0}$, $n = 2 \times 10^{17}$ cm^{-3} and $p = n_i^2/n$ = $(3.6 \times 10^2)^2/(2 \times 10^{17}) = 6.5 \times 10^{-13}$ cm^{-3}. At $x = 0^+$, we use Eq. (2-17) to find n, knowing that ϕ_{No}= 1.6115 V. So, n at $x = 0^+$ is equal to $2 \times 10^{17} \times \exp(-1.6115/0.0258) = 1.5 \times 10^{-10}$ cm^{-3}. Further, we use Eq. (2-18) to find p at $x = 0^+$, which is equal to $6.5 \times 10^{-13} \times \exp(1.6115/0.0258) = 8.7 \times 10^{17}$ cm^{-3}.

Now, let us find the p-side. At $x = -X_{p0}$, $p = 5 \times 10^{18}$ cm^{-3} and $n = n_i^2/p = (1.79 \times 10^6)^2/(5 \times 10^{18}) = 6.2 \times 10^{-7}$ cm^{-3}. At $x = 0^-$, we use Eq. (2-19) to find p, knowing that ϕ_{p0}= 0.0564 V. So, p at $x = 0^-$ is equal to $5 \times 10^{18} \times \exp(-0.0564/0.0258) = 5.6 \times 10^{17}$ cm^{-3}. Finally, we are ready to find n at $x = 0^-$. We use $n = n_i^2/p = (1.79 \times 10^6)^2/(5.6 \times 10^{17}) = 5.7 \times 10^{-6}$ cm^{-3}.

We tabulate the results:

	$n\|_{x=0+}$	$p\|_{x=0+}$	$n\|_{x=XNo}$	$p\|_{x=XNo}$	n_i (AlGaAs)	$p\|_{x=0-}$	$n\|_{x=0-}$	$p\|_{x=-Xpo}$	$n\|_{x=-Xpo}$	n_i (GaAs)
(cm^{-3})	1.5E-10	8.7E17	2E17	6.5E-13	3.6E2	5.6E17	5.7E-6	5E18	6.2E-7	1.79E6
$\log_{10}$	-9.8	17.94	17.3	-12.1	2.56	17.75	-5.24	18.7	-6.2	6.25



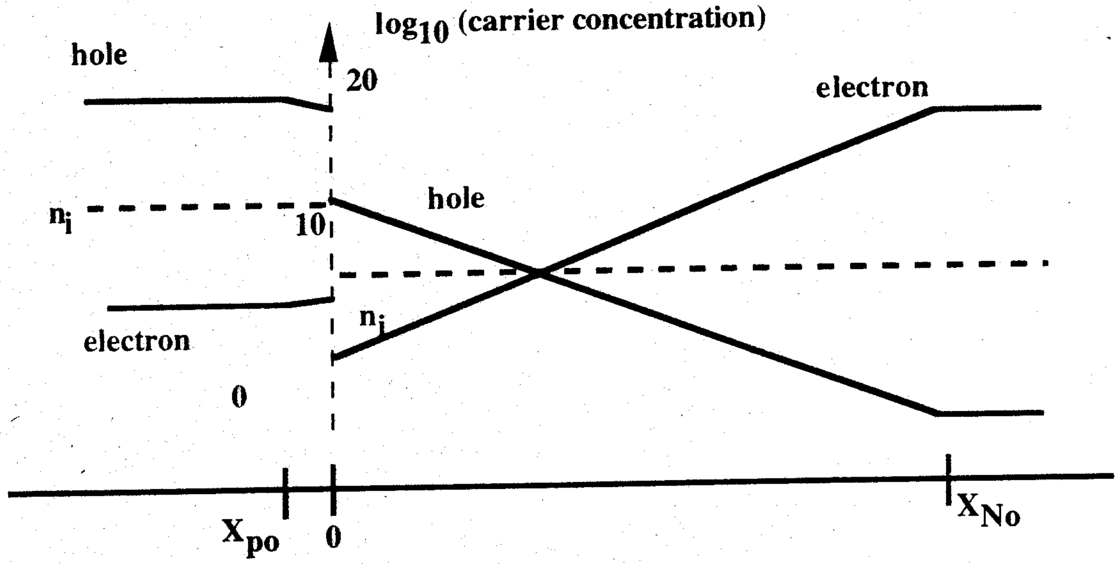
For the (*N*) InP/ (*p*) $In_{0.53}Ga_{0.47}As$ heterojunction:

n_i for InP is 1.05×10^7 cm^{-3}. Also, n_i for InGaAs is 6.31×10^{11} cm^{-3}. At $x = X_{NO}$, $n = 2 \times 10^{17}$ cm^{-3} and $p = n_i^2/n = 5.5 \times 10^{-4}$ cm^{-3}. At $x = 0^+$, we use Eq. (2-17) to find n, knowing that ϕ_{No}= 0.9285 V. So, n at $x = 0^+$ is equal to $2 \times 10^{17} \times \exp(-0.9285/0.0258) = 4.7 \times 10^1$ cm^{-3}. Further, we use Eq. (2-18) to find p at $x = 0^+$, which is equal to $5.5 \times 10^{-4} \times \exp(0.9285/0.0258) = 2.3 \times 10^{12}$ cm^{-3}.

Now, let us find the *p*-side. At $x = -X_{p0}$, $p = 5 \times 10^{18}$ cm^{-3} and $n = n_i^2/p = (6.31 \times 10^{11})^2/(5 \times 10^{18}) = 8.0 \times 10^4$ cm^{-3}. At $x = 0^-$, we use Eq. (2-19) to find p, knowing that $\phi_{p0} = \phi_{bi} - \phi_{No} = 0.0335$ V. So, p at $x = 0^-$ is equal to $5 \times 10^{18} \times \exp(-0.0335/0.0258) = 1.4 \times 10^{18}$ cm^{-3}. Finally, we are ready to find n at $x = 0^-$. We use $n = n_i^2/p = (6.31 \times 10^{11})^2/(1.4 \times 10^{18}) = 2.8 \times 10^5$ cm^{-3}.

We tabulate the results:

	$n\|_{x=0+}$	$p\|_{x=0+}$	$n\|_{x=XNo}$	$p\|_{x=XNo}$	n_i (InP)	$p\|_{x=0-}$	$n\|_{x=0-}$	$p\|_{x=-Xpo}$	$n\|_{x=-Xpo}$	n_i (InGaAs
(cm^{-3})	4.7E1	2.3E12	2E17	5.5E-4	1.1E7	1.4E18	2.8E5	5E18	8E4	6.3E11
$\log_{10}$	1.67	12.4	17.3	-3.25	7.0	18.1	5.4	18.7	4.9	11.8



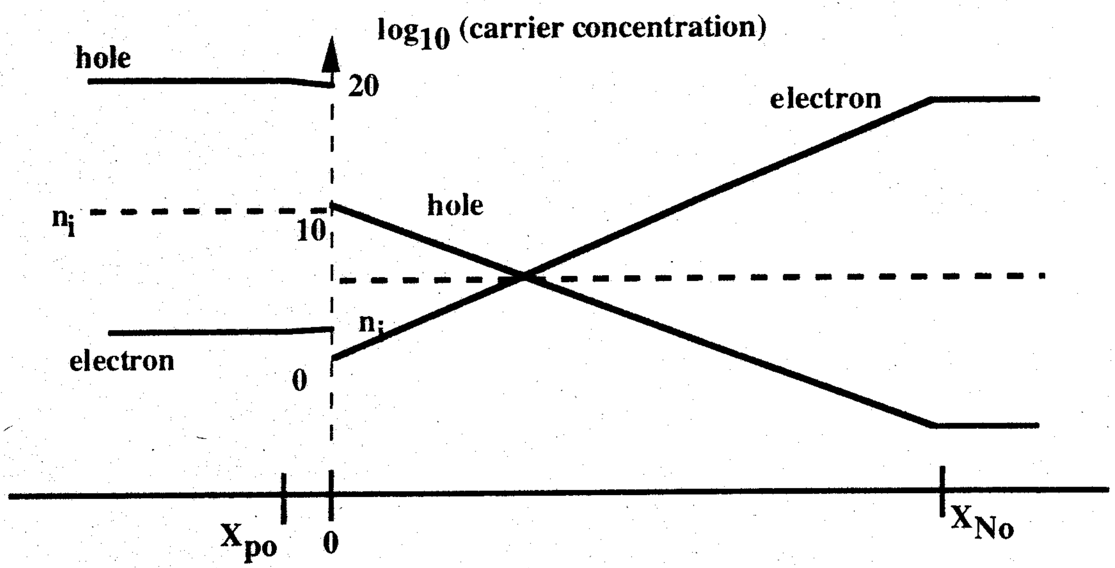
(b) <u>For the (*N*) $Al_{0.35}Ga_{0.65}As$/ (*p*) GaAs heterojunction:</u>

n_i for $Al_{0.35}Ga_{0.65}As$ is 3.6×10^2 cm^{-3}. Also, n_i for GaAs is 1.79×10^6 cm^{-3}. At $x = X_{NO}$, $n = 1 \times 10^{18}$ cm^{-3} and $p = n_i^2/n = (3.6 \times 10^2)^2/(1 \times 10^{18}) = 1.3 \times 10^{-13}$ cm^{-3}. At $x = 0^+$, we use Eq. (2-17) to find n, knowing that ϕ_{No}= 1.939 V. So, n at $x = 0^+$ is equal to $1 \times 10^{18} \times \exp(-1.939/0.0258) = 2.3 \times 10^{-15}$ cm^{-3}. Further, we use Eq. (2-18) to find p at $x = 0^+$ is $n_i^2/n = (3.6 \times 10^2)^2/(2.3 \times 10^{-15}) = 5.6 \times 10^{19}$ cm^{-3}.

Now, let us find the p-side. At $x = -X_{p0}$, $p = 1 \times 10^{20}$ cm^{-3} and $n = n_i^2/p = (1.79 \times 10^6)^2/(1 \times 10^{20}) = 3.2 \times 10^{-8}$ cm^{-3}. At $x = 0^-$, we use Eq. (2-19) to find p, knowing that ϕ_{p0}= 0.01778 V. So, p at $x = 0^-$ is equal to $1 \times 10^{20} \times \exp(-0.01778/0.0258) = 5.02 \times 10^{19}$ cm^{-3}. Finally, we are ready to find n at $x = 0^-$. We use $n = n_i^2/p = (1.79 \times 10^6)^2/(5.02 \times 10^{19}) = 6.38 \times 10^{-8}$ cm^{-3}.

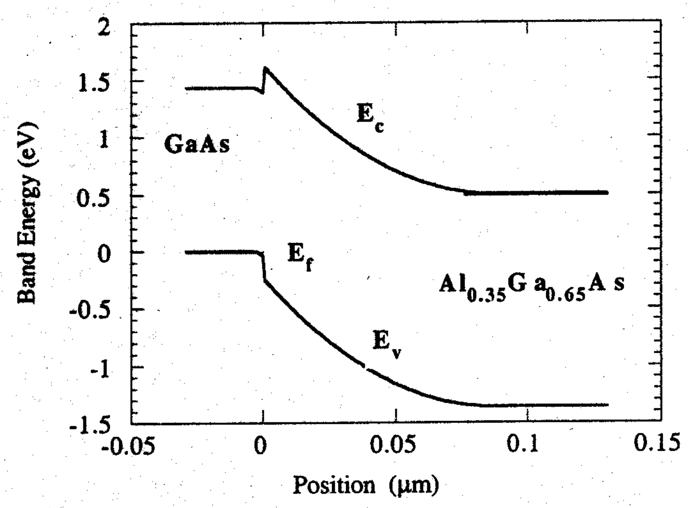
We tabulate the results:

	$n\|_{x=0+}$	$p\|_{x=0+}$	$n\|_{x=XNo}$	$p\|_{x=XNo}$	n_i (AlGaAs)	$p\|_{x=0-}$	$n\|_{x=0-}$	$p\|_{x=-Xpo}$	$n\|_{x=-Xpo}$	n_i (GaAs)
(cm^{-3})	2.3E-15	5.6E19	1E18	1.3E-13	3.6E2	5.02E19	6.38E-8	1E20	3.2E-8	1.79E6
$\log_{10}$	-14.6	19.75	18	-12.9	2.56	19.7	-7.2	20	-7.49	6.25

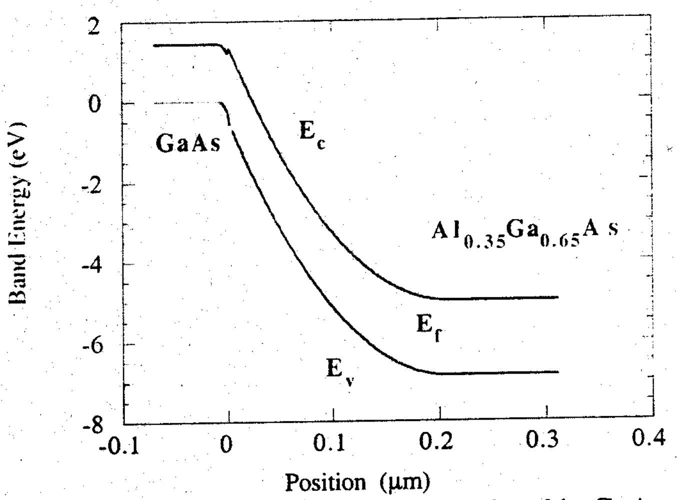


3. (a) For the (N) $Al_{0.35}Ga_{0.65}As$/ (p) GaAs heterojunction:

Let's calculate for $V_a = 0.5$ V first. From the solutions of Problem 1a, we see that $\Phi_N = 0.01$ eV and $\Phi_p = -0.01$ eV. V_{ap} is calculated to be 0.0177 V. $V_{aN} = V_a - V_{ap} = 0.4823$ V. The built-in voltages at the two sides of the junction, which are critical to the construction of the band diagram, are the following. $\phi_p = \phi_{po} - V_{ap} = 0.0413$ V; $\phi_N = \phi_{No} - V_{aN} = 1.127$ V. The depletion thicknesses, according to Eqs. (2-41) and (2-42) are, $X_p = 3.47 \times 10^{-7}$ cm and $X_N = 8.68 \times 10^{-6}$ cm.



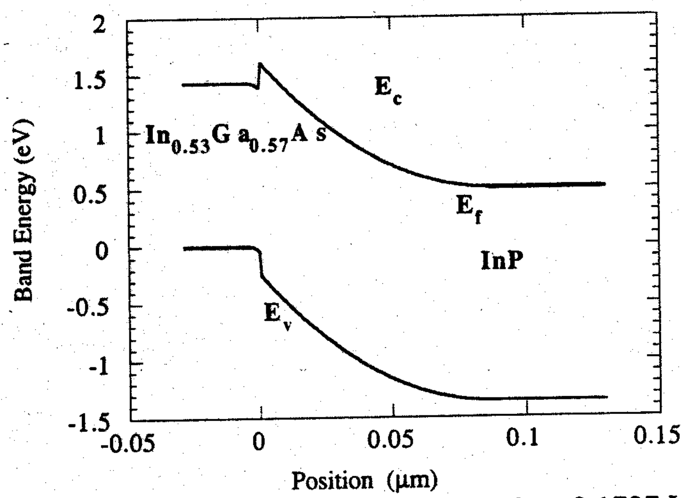
Let's calculate for $V_a = -5$ V. V_{ap} is calculated to be -0.177 V. $V_{aN} = V_a - V_{ap} = -4.823$ V. The built-in voltages at the two sides of the junction, which are critical to the construction of the band diagram, are the following. $\phi_p = \phi_{po} - V_{ap} = 0.236$ V; $\phi_N = \phi_{No} - V_{aN} = 6.432$ V. The depletion thicknesses, according to Eqs. (2-41) and (2-42) are, $X_p = 8.30 \times 10^{-7}$ cm and $X_N = 2.74 \times 10^{-5}$ cm.



The Fermi levels are nearly on top of the valence band in GaAs and the conduction band in AlGaAs.

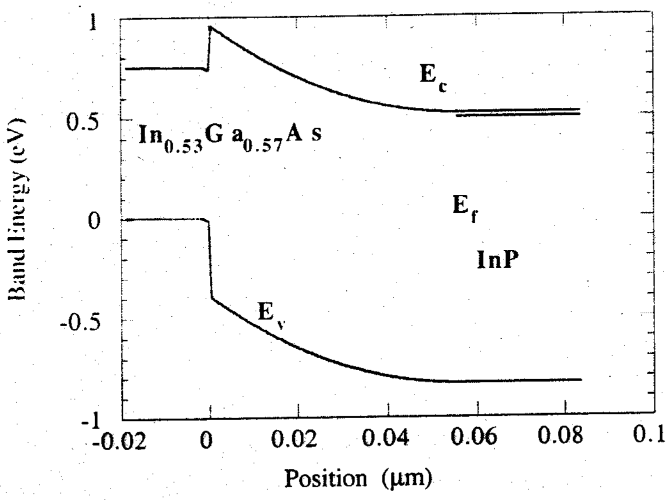
For the (*N*) InP/ (*p*) $In_{0.53}Ga_{0.47}As$ heterojunction:

Let's calculate for $V_a = 0.5$ V first. From the solutions of Problem 1a, we see that $\Phi_N = 0.023$ eV and $\Phi_p = -0.005$ eV. V_{ap} is calculated to be 0.01737 V. $V_{aN} = V_a - V_{ap} = 0.4826$ V. The built-in voltages at the two sides of the junction, which are critical to the construction of the band diagram, are the following. $\phi_p = \phi_{po} - V_{ap} = 0.01605$ V; $\phi_N = \phi_{No} - V_{aN} = 0.4459$ V. The depletion thicknesses, according to Eqs. (2-41) and (2-42) are, $X_p = 2.23 \times 10^{-7}$ cm and $X_N = 5.57 \times 10^{-6}$ cm.



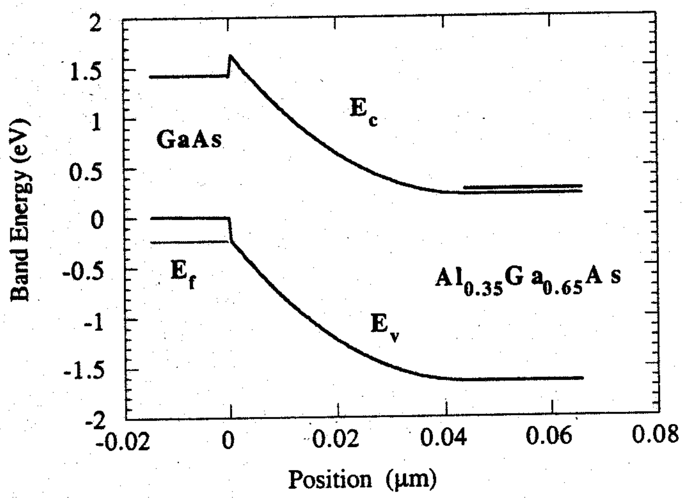
Let's calculate for $V_a = -5$ V. V_{ap} is calculated to be -0.1737 V. $V_{aN} = V_a - V_{ap} = -4.826$ V. The built-in voltages at the two sides of the junction, which are critical to the construction of the band diagram, are the following. $\phi_p = \phi_{po} - V_{ap} = 0.207$ V; $\phi_N = \phi_{No} - V_{aN} = 5.755$ V. The depletion thicknesses, according to Eqs. (2-41) and (2-42) are, $X_p =$

8.01×10^{-7} cm and $X_N = 2.00 \times 10^{-5}$ cm.

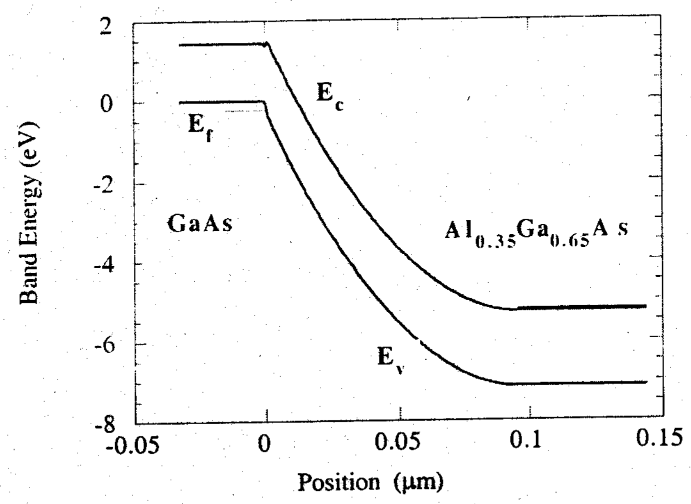


(b) For the (*N*) $Al_{0.35}Ga_{0.65}As$/ (*p*) GaAs heterojunction:

Let's calculate for $V_a = 0.5$ V first. From the solutions of Problem 1a, we see that $\Phi_N = -0.057$ eV and $\Phi_p = -0.237$ eV. V_{ap} is calculated to be 0.00454 V. $V_{aN} = V_a - V_{ap} = 0.4955$ V. The built-in voltages at the two sides of the junction, which are critical to the construction of the band diagram, are the following. $\phi_p = \phi_{po} - V_{ap} = 0.01324$ V; $\phi_N = \phi_{No} - V_{aN} = 1.4433$ V. The depletion thicknesses, according to Eqs. (2-41) and (2-42) are, $X_p = 4.39 \times 10^{-7}$ cm and $X_N = 4.39 \times 10^{-6}$ cm.

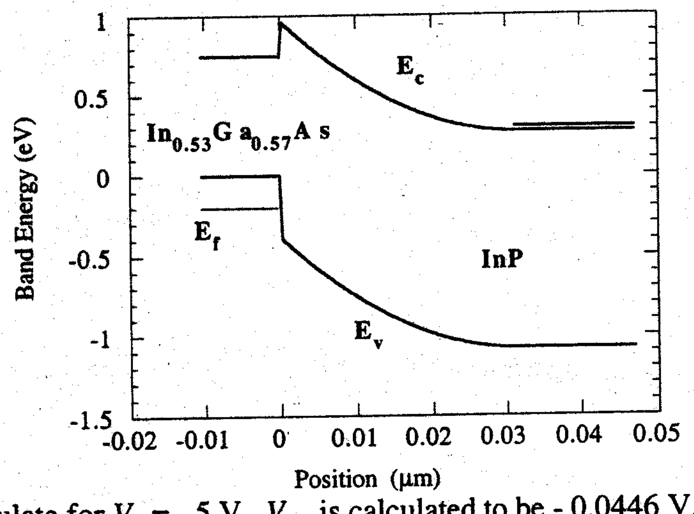


Let's calculate for $V_a = -5$ V. V_{ap} is calculated to be -0.0454 V. $V_{aN} = V_a - V_{ap} = -4.955$ V. The built-in voltages at the two sides of the junction, which are critical to the construction of the band diagram, are the following. $\phi_p = \phi_{po} - V_{ap} = 0.0632$ V; $\phi_N = \phi_{No} - V_{aN} = 6.893$ V. The depletion thicknesses, according to Eqs. (2-41) and (2-42) are, $X_p = 9.60 \times 10^{-8}$ cm and $X_N = 9.60 \times 10^{-6}$ cm.



<u>For the (N) InP/ (p) $In_{0.53}Ga_{0.47}As$ heterojunction:</u>

Let's calculate for $V_a = 0.5$ V first. From the solutions of Problem 1a, we see that $\Phi_N = -0.031$ eV and $\Phi_p = -0.20$ eV. V_{ap} is calculated to be 0.00446 V. $V_{aN} = V_a - V_{ap} = 0.4955$V. The built-in voltages at the two sides of the junction, which are critical to the construction of the band diagram, are the following. $\phi_p = \phi_{po} - V_{ap} = 0.00634$ V; $\phi_N = \phi_{No} - V_{aN} = 0.7047$ V. The depletion thicknesses, according to Eqs. (2-41) and (2-42) are, $X_p = 3.13 \times 10^{-7}$ cm and $X_N = 3.13 \times 10^{-6}$ cm.



Let's calculate for $V_a = -5$ V. V_{ap} is calculated to be -0.0446 V. $V_{aN} = V_a - V_{ap} = -4.955$ V. The built-in voltages at the two sides of the junction, which are critical to the construction of the band diagram, are the following. $\phi_p = \phi_{po} - V_{ap} = 0.0554$ V; $\phi_N = \phi_{No} - V_{aN} = 6.156$ V. *The depletion thicknesses,* according to Eqs. (2-41) and (2-42) are, $X_p = 9.26 \times 10^{-8}$ cm and $X_N = 9.26 \times 10^{-6}$ cm.

3. According to Eq. (3-26), and the fact that $J_{B,scr} = J_{B,p} \sim 0$:

$$\frac{1}{\beta} = \frac{J_{B,bulk}}{J_C} + 2\frac{K_{B,surf}}{J_C}\left(\frac{1}{W_E} + \frac{1}{L_E}\right)$$

When $W_E \times L_E = 10 \times 30\ \mu m^2$, $\beta = 30$. When $W_E \times L_E = 2 \times 30\ \mu m^2$, $\beta = 10$. Therefore,

$$0.0333 = \frac{J_{B,bulk}}{J_C} + 2\frac{K_{B,surf}}{J_C}\left(\frac{1}{10} + \frac{1}{30}\right)$$

$$0.1 = \frac{J_{B,bulk}}{J_C} + 2\frac{K_{B,surf}}{J_C}\left(\frac{1}{2} + \frac{1}{30}\right)$$

These two equations yield that, $J_{B,bulk}/J_C = 0.0111$and $K_{B,surf}/J_C = 0.0833$. Therefore, for the device with an area of $W_E \times L_E = 5 \times 30\ \mu m^2$, the current gain is:

$$\frac{1}{\beta} = 0.0111 + 2 \cdot 0.08333\left(\frac{1}{5} + \frac{1}{30}\right) = 0.05$$

That is, $\beta = 20$.

4. Let us call the bulk recombination current in the absence of the base field as $I_{B,bulk0}$, and the bulk recombination current in the presence of the base field as $I_{B,bulk1}$. According to the question, $I_{B,scr} = I_{B,bulk0}$. When there is the base field, the base electric field factor, according to Eq. (3-35), is,

$$\kappa = \frac{2 \times 10^4}{0.0258} 1000 \times 10^{-8} = 7.75$$

The factor $f(\kappa)$ evaluated from Eq. (3-37b) is,

$$f(\kappa) = \frac{2}{7.75}\left(1 - \frac{1}{7.75} + \frac{1}{7.75}e^{-7.75}\right) = 0.225$$

The base transit time, according to Eq. (3-37a), is proportional to $f(\kappa)$. Further, the base bulk recombination current is directly proportional to the base transit time. These dependencies lead to: $I_{B,bulk1} = 0.225 \times I_{B,bulk0}$.

The current gain in the absence of the base electric field is $I_C/(I_{B,scr} + I_{B,bulk0}) = I_C/(2 \cdot I_{B,bulk0})$. The current gain in the presence of the electric field is $I_C/(I_{B,scr} + I_{B,bulk1}) = I_C/(1.225 \cdot I_{B,bulk0})$. The change in the current gain is thus,

$$\frac{\Delta\beta}{\beta} = \left(\frac{1}{1.225} - \frac{1}{2}\right) \div \frac{1}{2} = 63\ \%$$

5. No, because the two test devices are both of large areas. The surface recombination is negligible compared to other base current components. In order to validate his claim, the graduate student needs to measure current gains from, for example, $4 \times 4\ \mu m^2$ and $100 \times 100\ \mu m^2$ devices.

6. (a) $n(x) = A \sinh(x/L_n) + B \cosh(x/L_n)$, and

$$n(0) = \frac{n_i^2}{N_B} e^{qV_{BE}/kT} \quad ; \ n(X_B) = 0 .$$

We note that the second boundary condition allow us to rewrite the original n(x) expression as, $n(x) = C \sinh[(X_B - x)/L_n]$. Now at $x = 0$, we then have,

$$n(0) = \frac{n_i^2}{N_B} e^{qV_{BE}/kT} = C \sinh\frac{X_B}{L_n}$$

From which, we obtain the expression for C as n(0)/ $\sinh(X_B/L_n)$. Thus,

$$n(x) = \frac{n(0)}{\sinh\left(\frac{X_B}{L_n}\right)} \sinh\left(\frac{X_B - x}{L_n}\right)$$

(b) Writing out the definition that sinh(y) = [exp(y) - exp(-y)]/2, we have,

$$n(x) = \frac{n(0)}{\exp\left(\frac{X_B}{L_n}\right) - \exp\left(-\frac{X_B}{L_n}\right)} \left[\exp\left(\frac{X_B - x}{L_n}\right) - \exp\left(-\frac{X_B - x}{L_n}\right)\right]$$

(c) The base charge is equal to

$$Q_B = \frac{qA_E n(0)}{\sinh\left(\frac{X_B}{L_n}\right)} \int_0^{X_B} \sinh\left(\frac{X_B - x}{L_n}\right) dx = \frac{qA_E\, n(0)\, L_n}{\sinh\left(\frac{X_B}{L_n}\right)} \left(-\cosh\left(\frac{X_B - x}{L_n}\right)\right)\Big|_0^{X_B}$$

$$Q_B = \frac{qA_E\, n(0)\, L_n}{\sinh\left(\frac{X_B}{L_n}\right)} \left(\cosh\left(\frac{X_B}{L_n}\right) - 1\right)$$

(d) $$I_C = qA_E D_n \frac{d\,n(x)}{dx}\Big|_{x = X_B} = \frac{qA_E D_n n(0)}{L_n \sinh\left(\frac{X_B}{L_n}\right)} \cosh\left(\frac{X_B - x}{L_n}\right)\Big|_{x = X_B}$$

$$I_C = \frac{qA_E D_n n(0)}{L_n \sinh\left(\frac{X_B}{L_n}\right)}$$

when $X_B/L_n \ll 1$,

$$I_C = \frac{qA_E D_n n(0)}{L_n \frac{X_B}{L_n}} = \frac{qA_E D_n n(0)}{X_B} = \frac{qA_E D_n}{X_B} \frac{n_i^2}{N_B} e^{qV_{BE}/kT}$$

(e) $$\tau_b = \frac{Q_B}{I_C} = \frac{qA_E\, n(0)\, L_n}{\sinh\left(\frac{X_B}{L_n}\right)} \left(\cosh\left(\frac{X_B}{L_n}\right) - 1\right) \times \frac{L_n \sinh\left(\frac{X_B}{L_n}\right)}{qA_E D_n n(0)} = \frac{L_n^2}{D_n} \left(\cosh\left(\frac{X_B}{L_n}\right) - 1\right)$$

We notice that,

$$\cosh(y) - 1 = \frac{e^y + e^{-y} - 2}{2} = 2 \cdot \left(\frac{e^{y/2} + e^{-y/2}}{2}\right)^2 = 2 \sinh\left(\frac{y}{2}\right)$$

Hence,

$$\tau_b = \frac{2L_n^2}{D_n} \sinh^2\left(\frac{X_B}{2L_n}\right)$$

when $X_B/L_n \ll 1$,

$$\tau_b \approx \frac{2L_n^2}{D_n} \left(\frac{X_B}{2L_n}\right)^2 = \frac{X_B^2}{2D_n}$$

(f) From Eq. (3-14),

$$\beta = \frac{\tau_n}{\tau_b} \approx \frac{D_n \tau_n}{2L_n^2} \frac{1}{\sinh^2\left(\frac{X_B}{2L_n}\right)} = \frac{1}{2 \sinh^2\left(\frac{X_B}{2L_n}\right)}$$

7. (a) $$I_E = qA_E D_n \frac{d\,n(x)}{dx}\Big|_{x = 0} = \frac{qA_E D_n n(0)}{L_n \sinh\left(\frac{X_B}{L_n}\right)} \cosh\left(\frac{X_B - x}{L_n}\right)\Big|_{x = 0}$$

$$I_E = \frac{qA_E D_n n(0)}{L_n} \coth\left(\frac{X_B}{L_n}\right)$$

(b) I_C was determined in part (d) of Problem 6. $I_B = I_E - I_C$ is given by: