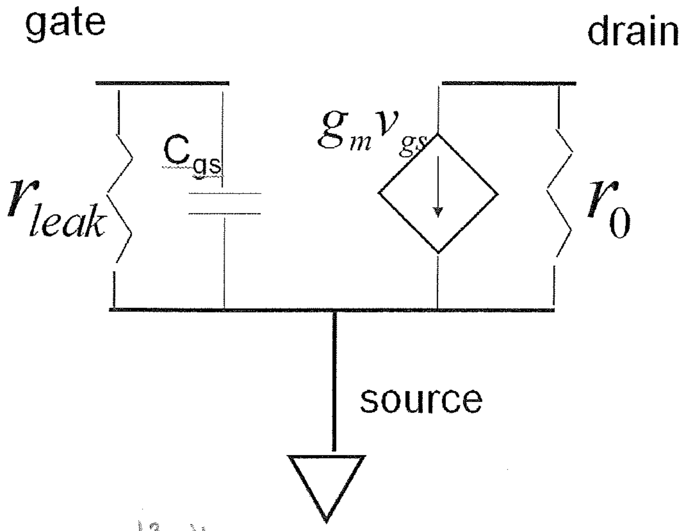
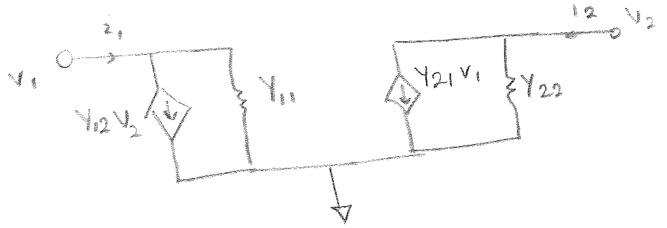
1) [40 points] Sometimes in a TFT, the gate is "leaky". We model that as a leakage resistance from gate to source below. For this small-signal circuit, find the Y-matrix and the cutoff frequency f_T and the f_{max}. Recall that f_T is the frequency at which the current gain h_{21} drops to unity, and f_{max} is the frequency at which Mason's unilateral power gain "U" drops to unity. (The f_{max} is a trick question. Why?)





$$Y_{12} = 0 \,,\quad Y_{11} = \frac{1}{r_{leak}} + j\omega C_{gs} \,,\quad Y_{21} = g_m \,,\quad Y_{22} = \frac{1}{r_o}$$

$$\Rightarrow [Y]_s = \begin{bmatrix} \frac{1}{r_{leak}} + j\omega C_{gs} & 0 \\ g_m & \frac{1}{r_o} \end{bmatrix}$$

To calculate f_T first we need to find the current gain:

$$i_1 = Y_{11} v_1$$
$$i_2 = Y_{21} v_1 + Y_{22} v_2$$
$$\Rightarrow \left.\frac{i_2}{i_1}\right|_{v_2=0} = \frac{Y_{21}}{Y_{11}} = \frac{g_m}{\left(\frac{1}{r_{leak}} + j\omega C_{gs}\right)} = \frac{g_m r_{leak}}{1 + j\omega r_{leak} C_{gs}}$$

$$\left|\frac{Y_{21}}{Y_{11}}\right|_{\omega=\omega_T} = 0 \text{ dB} \Rightarrow 1 + \omega_T^2 r_{leak}^2 C_{gs}^2 = g_m^2 r_{leak}^2 \Rightarrow \omega_T = \sqrt{\frac{g_m^2 r_{leak}^2 - 1}{r_{leak}^2 C_{gs}^2}} \approx \frac{g_m}{C_{gs}}$$

$$\Rightarrow f_T = \frac{1}{2\pi}\sqrt{\frac{g_m^2 r_{leak}^2 - 1}{r_{leak}^2 C_{gs}^2}} \approx \frac{g_m}{2\pi C_{gs}}$$

since $Y_{12} = 0$ the transistor is already unilateral:

$$U = \frac{|Y_{21} - Y_{12}|^2}{4\left[Re(Y_{11})Re(Y_{22}) - Re(Y_{12})Re(Y_{21})\right]}$$

$$\Rightarrow U = \frac{|g_m - 0|^2}{4\left[\frac{1}{r_{leak}} \cdot \frac{1}{r_o} - 0\right]} = \frac{g_m^2}{4 \times \frac{1}{r_{leak}} \cdot \frac{1}{r_o}} = 4 g_m^2 r_o r_{leak}$$

U is independant of frequency $\Rightarrow f_{max}$ can not be defined.

2) [40 points] Sketch the band diagram for an abrupt junction at thermal equilibrium for the following N-p junction: (N) $Al_{0.35}Ga_{0.65}As$ doped so that E_C-E_F = 0.2 eV; (p) GaAs doped so that E_F-E_V=0.1 eV. Indicated clearly (qualitative):

   a. Which side has the larger depletion region
   b. Which side has the larger potential drop

Indicate quantitatively:

   c. Label quantitatively the difference between E_C on both sides
   d. Label quantitatively the different between E_V on both sides

Other features only need to be drawn qualitatively.

$$\left.\begin{aligned} E_C - E_F &= 0.2 \\ E_F - E_V &= 0.1 \end{aligned}\right\} \Phi_p < \Phi_N \Rightarrow$$ p side is more heavily doped

⇒ N side has a larger depletion region

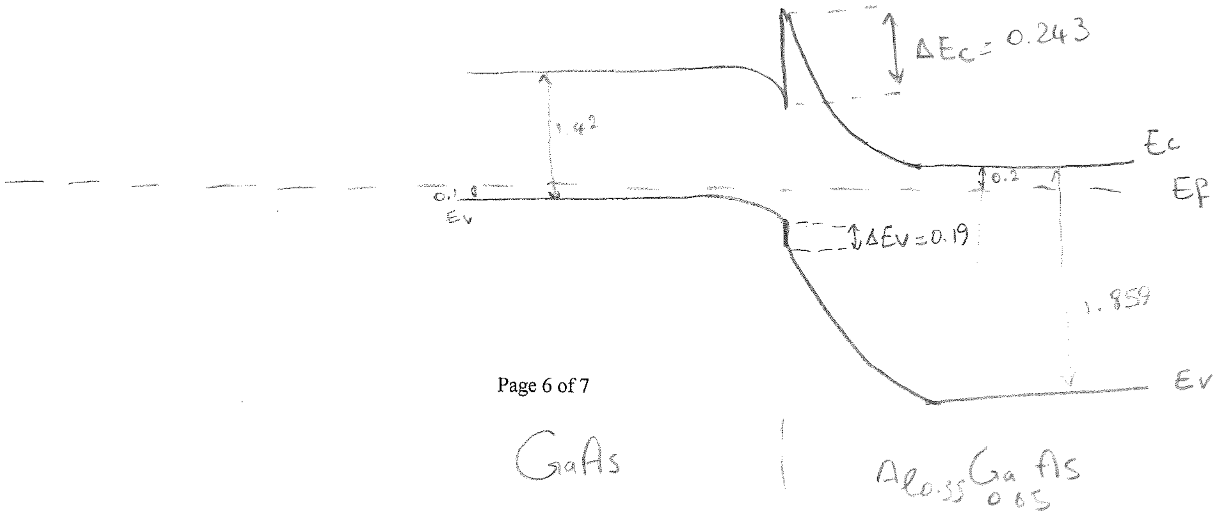
$$\frac{\varphi_{p_0}}{\varphi_{N_0}} \propto \frac{N_a X_{p_0}^2}{N_d X_{N_0}^2} \Leftarrow \frac{N_d}{N_a}\left(\frac{N_a^2 X_{p_0}^2}{N_d^2 X_{N_0}^2}\right)^{\nearrow 1} \doteq \frac{N_d}{N_a}, \quad N_d < N_a \Rightarrow \varphi_{p_0} < \varphi_{N_0}$$

⇒ N side has a larger potential drop.

$$\left.\begin{aligned} Eg_{GaAs} &= 1.424 \\ Eg_{Al_{0.35}Ga_{0.65}As} &= 1.42 + 1.247\zeta = 1.424 + 0.435 = 1.859 \end{aligned}\right\} \Rightarrow \Delta Eg = 0.435 \text{ (ev)}$$

$$\Delta E_v = 0.55\zeta = 0.55 \times 0.35 = 0.192 \text{ (ev)}$$

$$\Delta E_c = \Delta Eg - \Delta E_v = 0.435 - 0.192 = 0.243 \text{ (ev)}$$



3) [25 points] Describe the advantage of HBT over BT.

In BJTs in order to minimize the holes injected from base to emitter and get a large β, the ratio of base doping concentration to emitter doping concentration has to be small, ie the base doping concentration must be low and the emitter doping concentration must be high, so the base parasitic resistance and the Base-Emitter junction capacitor will be large and the transisor speed will be limited.

On the other hand, in HBTs the potential barrier of electron injection from emitter to base and hole injection from base to emitter are different, $\frac{I_{Bp}}{I_{En}} \propto e^{-\Delta E_g / kT}$, so, the hole injection from base to emitter is very small and a higher doping Level for base can be used $\Rightarrow$ HBT has a higher speed.