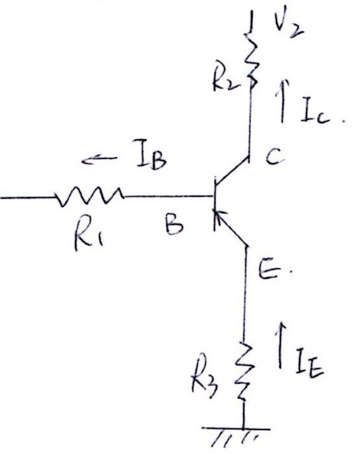
Homework 6 Solution.



1) Since $V_{BE} = 0.6V$, the transistor is <u>not</u> in forward active mode.

Assuming the transistor is in forward active mode:

$V_{EB} = 0.6V$. $\quad I_c = \beta I_B = 100 I_B \quad I_E = I_B + I_c = 101 I_B$.

KVL: $\quad -V_E + V_{EB} + V_{R_1} = -V_1 \quad (*)$.

$$\begin{cases} V_E = -R_3 I_E \\ V_{R_1} = I_B R_1 \\ V_{EB} = 0.6V \end{cases}$$

$(*) \Rightarrow \quad R_3 I_E + 0.6 + R_1 I_B = -V_1$

$$\left(\frac{R_1}{101} + R_3\right) I_E = -(V_1 + 0.6) \quad \Rightarrow \boxed{I_E = \frac{-101(V_1+0.6)}{R_1 + 101 R_3}}$$

$$\boxed{I_B = \frac{I_E}{101} = \frac{-(V_1+0.6)}{R_1+101R_3}} \qquad \boxed{I_c = 100 I_B = \frac{-100(V_1+0.6)}{R_1+101R_3}}$$

$$V_E = -I_E R_3 = \frac{-101 R_3 (V_1+0.6)}{R_1 + 101 R_3}$$

$$V_B = V_{BE} + V_E = V_E - 0.6 = \frac{-0.6 R_1 + 101 R_3 V_1}{R_1 + 101 R_3}$$

$$V_C = V_2 + R_2 I_c = V_2 - \frac{100 R_2 (0.6 + V_1)}{R_1 + 101 R_3}$$

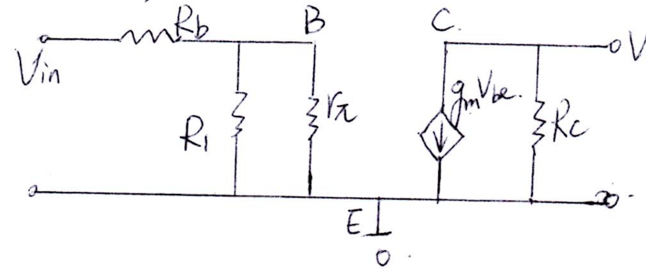
$$V_{BE} = -0.6V$$

$$V_{CE} = V_2 + R_2 I_c + R_3 I_E = V_2 - \frac{100 R_2 (0.6 + V_1)}{R_1 + 101 R_3} - \frac{101 R_3 (V_1 + 0.6)}{R_1 + 101 R_3}$$

$$V_{BC} = V_B - V_C = -I_E R_3 - 0.6 - V_2 - R_2 I_c$$

$$= \frac{(101 R_3 + 100 R_2) V_1 - (R_1 + 101 R_3) V_2 + (100 R_2 - R_1) \cdot 0.6}{R_1 + 101 R_3}$$

2. Hybrid Pi Model AC:



$$V_{out} = -g_m V_{be} R_c$$

$$V_{in} = \frac{R_b + R_1 // r_\pi}{R_1 // r_\pi} V_{be}$$

$$gain = \frac{V_{out}}{V_{in}} = \frac{-g_m V_{be} R_c}{\frac{R_b + R_1 // r_\pi}{R_1 // r_\pi} V_{be}}$$

$$R_1 // r_\pi = \frac{R_1 \beta / g_m}{R_1 + \beta / g_m} = \frac{R_1 \beta}{R_1 g_m + \beta} \Rightarrow gain = \frac{-g_m R_c R_1 \beta}{R_b(g_m R_1 + \beta) + R_1 \beta} \quad (*)$$

$*$ $g_m = \frac{I_C}{V_T}$ $\quad I_C = \beta I_B$ $\quad 2 - R_1 I_B = V_{BE} = 0.6V$

$$I_B = \frac{2-0.6}{R_1} = \frac{1.4}{R_1} \Rightarrow I_C = \frac{140}{R_1} \qquad V_T = \frac{kT}{q} \ldots\ldots$$

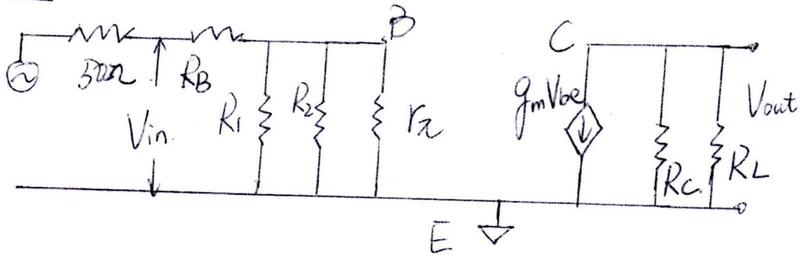
$$\Rightarrow \quad gain = \frac{-140 R_c}{1.4 R_b + V_T + R_1 V_T}$$

3. From $(*)$, $\Rightarrow \lim_{g_m \to \infty} gain = -\frac{g_m R_c R_1 \beta}{R_b (g_m R_1)} = -\beta \frac{R_c}{R_b}$.

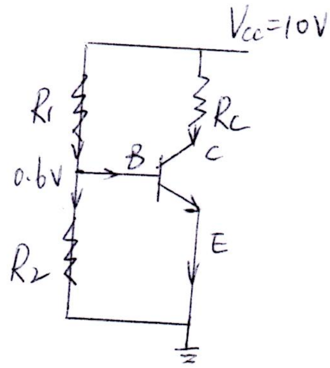
When $g_m \to \infty \Rightarrow \frac{kT}{q} \to 0 \Rightarrow T \to 0K$.

$$Gain = -\beta \frac{R_c}{R_b}.$$

4. AC:



DC:



$$V_{out} = -g_m V_{be} \cdot R_c // R_L.$$

$$V_{in} = \frac{R_B + R_1 // R_2 // r_\pi}{R_1 // R_2 // r_\pi} V_{be}.$$

$$Gain = \frac{V_{out}}{V_{in}} = \frac{-g_m R_c // R_L}{(R_B + R_1 // R_2 // r_\pi) / R_1 // R_2 // r_\pi} \quad (*)$$

$$KCL: I_B = I_{R_1} - I_{R_2} = \frac{10-0.6}{R_1} - \frac{0.6}{R_2}.$$

$$I_C = \beta I_B = \beta\left(\frac{9.4}{R_1} - \frac{0.6}{R_2}\right)$$

$$g_m = \frac{I_C}{V_T} = \frac{\beta}{V_T}\left(\frac{9.4}{R_1} - \frac{0.6}{R_2}\right)$$

$$r_\pi = \beta / g_m.$$

$$Gain = -g_m \frac{R_C R_L}{R_C+R_L} \left(\frac{\frac{R_1 R_2 r_\pi}{R_1R_2 + R_1 r_\pi + R_2 r_\pi}}{R_B + \frac{R_1 R_2 r_\pi}{R_1R_2 + R_1 r_\pi + R_2 r_\pi}} \right).$$

$$= \frac{-\beta}{V_T} \frac{R_C R_L}{R_C + R_L} \left(\frac{9.4}{R_1} - \frac{0.6}{R_2} \right) \cdot \left(\frac{\frac{R_1 R_2 r_\pi}{R_1 R_2 + R_1 r_\pi + R_2 r_\pi}}{R_B + \frac{R_1 R_2 r_\pi}{R_1 R_2 + R_1 r_\pi + R_2 r_\pi}} \right).$$ which $r_\pi = \beta / g_m$.

$$\lim_{\substack{V_T = 0 \\ R_L \to \infty}} Gain = \lim_{R_L \to \infty} -\beta \frac{R_L R_C}{R_b (R_L + R_C)} = -\beta \frac{R_C}{R_b}.$$

The condition we find: $\boxed{\begin{array}{l} V_T \to 0, \quad T \to 0\,k. \\ R_L \to +\infty \end{array}}$