EECS 277C Nanotechnology HW #1

1. Consider a single electron in a box. Calculate the size of the box at which only the ground state is occupied at room temperature.
2. Same as 1. But use the effective mass of Si, GaAs, InSb (3 answers).
3. At what temperature would a 10 nm x 10 nm x 10 nm box have to be lowered to in order for only the lowest energy state to be occupied?
4. Now, consider many electrons in a box with Fermi energy of 10 eV. Find the total # of states in a box with size that you calculated in problem #1.

①

$$E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2\left(n_x^2 + n_y^2 + n_z^2\right)$$

$$E_{211} - E_{111} = \Delta E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 3 >> kT$$

$$\Rightarrow L << \sqrt{\frac{3\hbar^2}{2m}\pi^2\frac{1}{kT}}$$

$h = 6.6 \times 10^{-34}$ J-s

$= 4.1 \times 10^{-15}$ ~~J~~ eV-s

$K = 1.38\ 10^{-23}$ J/K

$\hbar = 10^{-34}$ J-s

$= 6.5 \times 10^{-16}$ eV-s

$m = 9.1 \times 10^{-31}$ kg

$$\Rightarrow \boxed{L << 6\,\text{nm}}$$

②

Si $m^* = 1.08\ m_o$

GaAs $m^* = 0.067\ m_o$

InSb $m^* = 0.013\ m_o$

$L \propto \sqrt{\frac{1}{m}}$

Si $L \ll 6\,nm$

GaAs $L \ll \frac{1}{\sqrt{0.067}}\, 6\,nm = 12\,nm$

InSb $L \ll \frac{1}{\sqrt{0.013}}\, 6\,nm = 52\,nm$

③

$$\Delta E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 3$$

$L = 10\,nm$

$T \ll \frac{\Delta E}{k_B} = 110$ Kelvin

④

$$E_F = \frac{\hbar^2}{2m}(3\pi^2)^{2/3}\, n^{2/3} \qquad n \equiv \frac{\#\text{electrons}}{\text{volume}}$$

$\# = n \times (10\,nm)^3 = 1.8 \times 10^5$

EECS 277C Nanotechnology HW #2

1. Estimate the gate capacitance of a modern transistor. Assume a parallel plate capacitor with k=10, d= 10 nm, L=W=0.1 microns. Now, calculate how much energy it costs to add one electron to the gate (e_2/C). Is this energy larger or smaller than a typical thermal energy (k_BT)?
2. Calculate the density of states in a 2 dimensional world.
3. Calculate the probability for an electron to tunnel through a 1 nm barrier that is 10 eV high. This is a good approximation for the tunnel junction shown in class. Use the formula below:

$$\boxed{T = \left[1 + \frac{V_0^2 \sinh^2 [ka]}{4E(V_0 - E)} \right]^{-1}}$$

$$\boxed{k = \sqrt{2m(V_0 - E)/\hbar^2}}$$

$$\boxed{V_0 = 10eV}$$

$$\boxed{E = 5eV}$$

4. A device shows Coulomb blockade at temperatures only well below 300 K. What is it's size? (i.e. what is the capacitance of the tunnel barrier?)

HW1 Problem 4; EECS 277C *Nanotechnology*

2 dimensions

$$N_k dk = ?$$

Volume of circular shell

$=2\pi kdk/4$

4 is for upper right quadrant

Number of states in area=
area x States/area

States/area = $1 / (\pi/L)^2$:

$$N_k dk = \left(2\pi kdk/4\right)\cdot\left(\frac{1}{(\pi/L)^2}\right)\cdot 2 = L^2\,\frac{kdk}{\pi}$$

$$\boxed{\rho_k dk \equiv \frac{N_k dk}{\text{area}} = \frac{kdk}{\pi}}$$

HW1 Problem 4; EECS 277C *Nanotechnology*

2 dimensions

$$\rho(E)dE = ?$$

We use:

$$\rho_k dk = \rho(E)dE$$

$$\rho_k dk = \frac{kdk}{\pi}$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}}$$

$$\boxed{\rho(E)dE = \frac{m}{\pi\hbar^2} dE}$$

HW1 Problem 6; EECS 277C *Nanotechnology*

Transmission prob:

$$\boxed{T = \left[1 + \frac{V_0^2 \sinh^2\left[ka\right]}{4E(V_0 - E)} \right]^{-1}}$$

$$\boxed{k = \sqrt{2m(V_0 - E)/\hbar^2}}$$

$$\boxed{V_0 = 10eV}$$

$$\boxed{E = 5eV}$$

$$\boxed{T = 1.5\ 10^{-10}}$$

①

$$C = \frac{\epsilon A}{d} = 88 \times 10^{-18}\,F$$

$$\frac{e^2}{C} = 1.8\ meV$$

less than $k_B T$

④

$$\frac{e^2}{C} = kT$$

$$\Rightarrow \quad C = \frac{e^2}{kT} = 5 \times 10^{-18}\,F$$