

1) For a 2 dimensional electron gas in GaAs with a density of 10^{11} cm^{-2} , calculate the Fermi energy and the Fermi wavelength. (This will be important when we discuss quantum dots and quantum point contacts.)

The answers are: 3.5 meV and 80 nm.

2) Calculate the change in the Gibbs free energy for a two-island circuit with no gates. (I.e. three tunnel junctions in series) for all possible electron transitions. (Left lead to island 1, island 1 to left lead, island 1 to island 2, island 2 to island 1, island 2 to right lead, right lead to island 2). Describe the conditions under which current can and cannot flow from the left lead to the right lead.

3) What is the Fermi wavelength of electrons in aluminum? Is it possible to fabricate 1d Al wires using photolithography? Is it possible to fabricate Al wires using electron beam lithography?

3.6 angstroms.

No.

4) Find the resistivity of pure copper at room temperature. Now, find the density of electrons in copper, assuming one free electron per atom. Now, calculate the scattering time and the mean free path of the electrons from the Drude model. Is it possible to fabricate copper wires in the ballistic limit using photolithography? Is it possible to fabricate copper wires in the ballistic limit using electron beam lithography?

$$10 \text{ microohm-cm} \quad 8.45 \cdot 10^{22} \text{ cm}^{-3} \quad \frac{1}{\rho} = \sigma = \frac{ne^2\tau}{m}$$

$$\tau = 4 \cdot 10^{-15} \text{ s} \quad l = v_{\text{Fermi}}\tau$$

$$v_{\text{Fermi}} = 1.6 \times 10^8 \text{ cm/s} \quad \text{so } l = 6.4 \text{ nm}$$

No

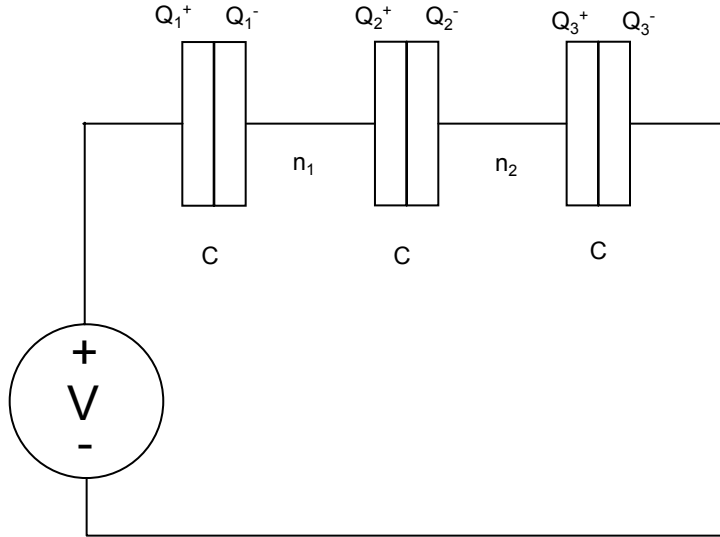
No

5) For a 2DEG in GaAs with $n=10^{11} \text{ cm}^{-2}$ and a mobility of $8,000 \text{ cm}^2/\text{V}\cdot\text{s}$ (typical of room temperature HEMT operation), calculate the scattering time and the mean free path from the Drude model. Is it possible to fabricate devices using lithography that are smaller than the mean free path? Remember you must use the effective mass of electrons for the Fermi energy, etc.

$$\mu = \frac{e\tau}{m^*} \quad \tau = 0.3 \text{ ps} \quad l = v_{\text{Fermi}}\tau \text{ so } l = 1.3 \cdot 10^5 \text{ m/s} \times 0.3 \text{ ps} = 40 \text{ nm}$$

Yes

Problem 4:



Definitions:

C is the capacitance of each capacitor.

(They are all the same.)

n_1 is the number of excess electrons on island 1.

n_2 is the number of excess electrons on island 2.

$n_{1b} \equiv$ # excess electrons on island 1 before the tunnel event

$n_{1a} \equiv$ # excess electrons on island 1 after the tunnel event

$n_{2b} \equiv$ # excess electrons on island 2 before the tunnel event

$n_{2a} \equiv$ # excess electrons on island 2 after the tunnel event

$$Q_{before} \equiv \int_{-\infty}^{t_{before}} I(t') dt'$$

where $I(t')$ is the current flowing out of the positive terminal of the battery.

$$Q_{after} \equiv \int_{-\infty}^{t_{after}} I(t') dt'$$

where $I(t')$ is the current flowing out of the positive terminal of the battery.

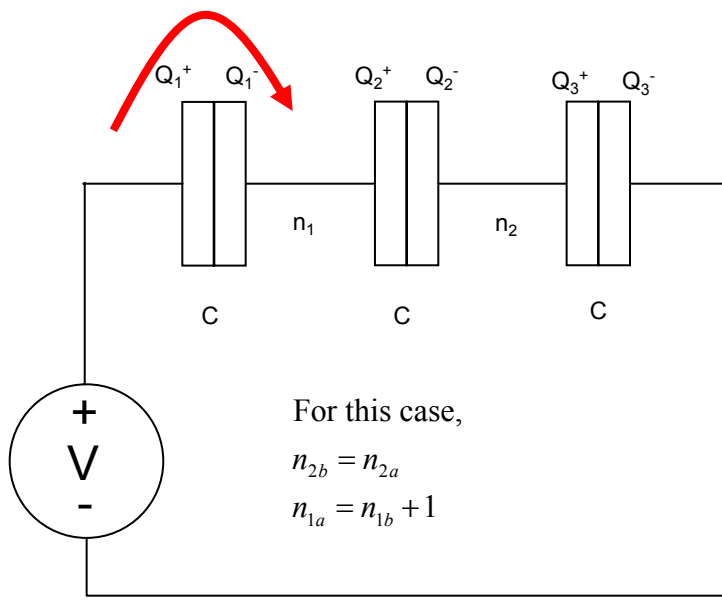
My calculations show that:

$$Q_1 = \frac{1}{3}(2en_1 + en_2 + CV)$$

$$Q_2 = \frac{1}{3}(-en_1 + en_2 + CV)$$

$$Q_3 = \frac{1}{3}(-en_1 - 2en_2 + CV)$$

$$E = \frac{Q_1^2}{2C} + \frac{Q_2^2}{2C} + \frac{Q_3^2}{2C} = \frac{1}{6C} [2e^2n_1^2 + 2e^2n_2^2 + (CV)^2 + e^2n_1n_2]$$



$$\Delta G = G_{\text{before}} - G_{\text{after}} = \{E_{\text{before}} - E_{\text{after}}\} - V \{Q_{\text{before}} - Q_{\text{after}}\}$$

$$\{E_{\text{before}} - E_{\text{after}}\}$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}]$$

$$= \frac{e^2}{6C} [2n_{1b}^2 + n_{1b} n_{2b}] - \frac{e^2}{6C} [2n_{1a}^2 + n_{1a} n_{2a}] = \frac{e^2}{6C} [2n_{1b}^2 + n_{1b} n_{2b} - 2(n_{1b} + 1)^2 - (n_{1b} + 1)n_{2b}]$$

$$= \frac{e^2}{6C} [2n_{1b}^2 + n_{1b} n_{2b} - 2(n_{1b}^2 + 2n_{1b} + 1) - (n_{1b} + 1)n_{2b}] = \frac{e^2}{6C} [2n_{1b}^2 + n_{1b} n_{2b} - 2n_{1b}^2 - 4n_{1b} - 2 - n_{1b} n_{2b} - n_{2b}]$$

$$= \frac{e^2}{6C} [-4n_{1b} - 2 - n_{2b}]$$

$$\{Q_{\text{before}} - Q_{\text{after}}\} = \int_{-\infty}^{t_{\text{before}}} I(t') dt' - \int_{-\infty}^{t_{\text{after}}} I(t') dt' = - \int_{t_{\text{before}}}^{t_{\text{after}}} I(t') dt'$$

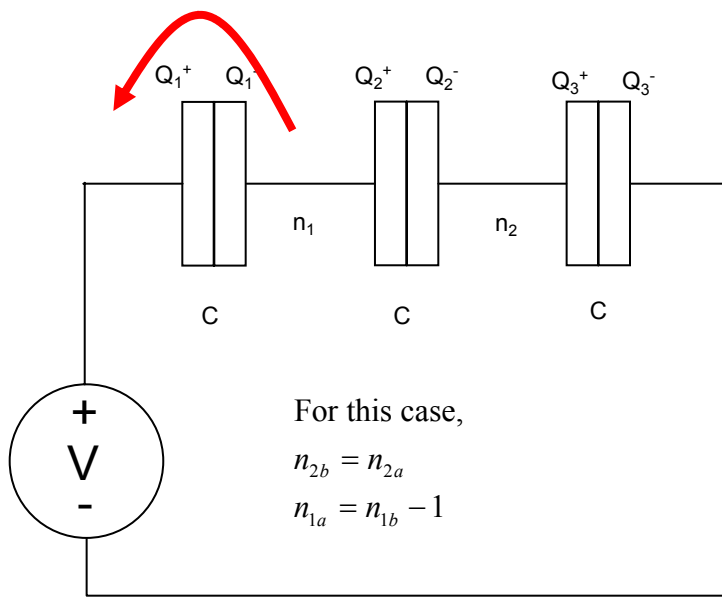
$$= - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{tunnel}}(t') dt' - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt'$$

$$= e - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt' = e + (Q_{1\text{before}} - Q_{1\text{after}})$$

$$(Q_{1\text{before}} - Q_{1\text{after}}) = \frac{1}{3} (2en_{1b} + en_{2b} + CV) - \frac{1}{3} (2en_{1a} + en_{2a} + CV) = \frac{1}{3} (2en_{1b}) - \frac{1}{3} (2en_{1a})$$

$$= \frac{2e}{3} (n_{1b} - n_{1a}) = \frac{2e}{3} (n_{1b} - (n_{1b} + 1)) = -\frac{2e}{3}$$

$$\Delta G = \frac{e^2}{6C} [-4n_{1b} - 2 - n_{2b}] - V \left\{ \frac{e}{3} \right\}$$



$$\Delta G = G_{\text{before}} - G_{\text{after}} = \{E_{\text{before}} - E_{\text{after}}\} - V \{Q_{\text{before}} - Q_{\text{after}}\}$$

$$\{E_{\text{before}} - E_{\text{after}}\}$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}]$$

$$= \frac{e^2}{6C} [2n_{1b}^2 + n_{1b} n_{2b}] - \frac{e^2}{6C} [2n_{1a}^2 + n_{1a} n_{2a}] = \frac{e^2}{6C} [2n_{1b}^2 + n_{1b} n_{2b} - 2(n_{1b} - 1)^2 - (n_{1b} - 1)n_{2b}]$$

$$= \frac{e^2}{6C} [2n_{1b}^2 + n_{1b} n_{2b} - 2(n_{1b}^2 - 2n_{1b} + 1) - (n_{1b} - 1)n_{2b}] = \frac{e^2}{6C} [2n_{1b}^2 + n_{1b} n_{2b} - 2n_{1b}^2 + 4n_{1b} - 2 - n_{1b} n_{2b} + n_{2b}]$$

$$= \frac{e^2}{6C} [+4n_{1b} - 2 + n_{2b}]$$

$$\{Q_{\text{before}} - Q_{\text{after}}\} = \int_{-\infty}^{t_{\text{before}}} I(t') dt' - \int_{-\infty}^{t_{\text{after}}} I(t') dt' = - \int_{t_{\text{before}}}^{t_{\text{after}}} I(t') dt'$$

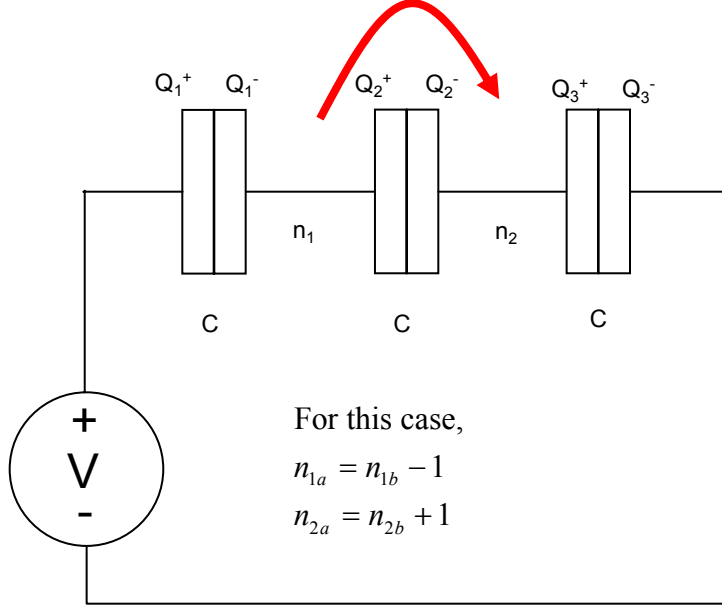
$$= - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{tunnel}}(t') dt' - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt'$$

$$= -e - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt' = -e + (Q_{1\text{before}} - Q_{1\text{after}})$$

$$(Q_{1\text{before}} - Q_{1\text{after}}) = \frac{1}{3} (2en_{1b} + en_{2b} + CV) - \frac{1}{3} (2en_{1a} + en_{2a} + CV) = \frac{1}{3} (2en_{1b}) - \frac{1}{3} (2en_{1a})$$

$$= \frac{2e}{3} (n_{1b} - n_{1a}) = \frac{2e}{3} (n_{1b} - (n_{1b} - 1)) = + \frac{2e}{3}$$

$$\Delta G = \frac{e^2}{6C} [+4n_{1b} - 2 + n_{2b}] - V \left\{ \frac{-e}{3} \right\}$$



$$\Delta G = G_{\text{before}} - G_{\text{after}} = \{E_{\text{before}} - E_{\text{after}}\} - V \{Q_{\text{before}} - Q_{\text{after}}\}$$

$$\{E_{\text{before}} - E_{\text{after}}\}$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}]$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 (n_{1b} - 1)^2 + 2e^2 (n_{2b} + 1)^2 + (CV)^2 + e^2 (n_{1b} - 1)(n_{2b} + 1)]$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 (n_{1b}^2 - 2n_{1b} + 1) + 2e^2 (n_{2b}^2 + 2n_{2b} + 1) + (CV)^2 + e^2 (n_{1b} n_{2b} + n_{1b} - n_{2b} - 1)]$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1b}^2 - 4e^2 n_{1b} + 2e^2 + 2e^2 n_{2b}^2 + 4e^2 n_{2b} + 2e^2 + (CV)^2 + e^2 n_{1b} n_{2b} + e^2 n_{1b} - e^2 n_{2b} - e^2]$$

$$= -\frac{1}{6C} [-4e^2 n_{1b} + 2e^2 + 4e^2 n_{2b} + 2e^2 + e^2 n_{1b} - e^2 n_{2b} - e^2]$$

$$= -\frac{1}{6C} [-3e^2 n_{1b} + 3e^2 + 3e^2 n_{2b}] = -\frac{3e^2}{6C} [-n_{1b} + 1 + n_{2b}] = -\frac{e^2}{2C} [-n_{1b} + 1 + n_{2b}] = \frac{e^2}{2C} [n_{1b} - n_{2b} - 1]$$

$$\{Q_{\text{before}} - Q_{\text{after}}\} = \int_{-\infty}^{t_{\text{before}}} I(t') dt' - \int_{-\infty}^{t_{\text{after}}} I(t') dt' = -\int_{t_{\text{before}}}^{t_{\text{after}}} I(t') dt'$$

$$= -\int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{tunnel}}(t') dt' - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt'$$

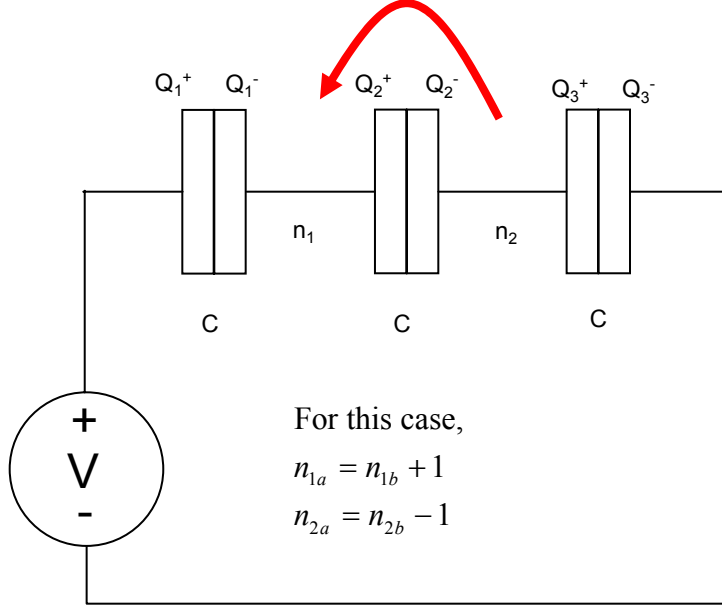
$$= 0 - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt' = (Q_{1\text{before}} - Q_{1\text{after}})$$

$$(Q_{1\text{before}} - Q_{1\text{after}}) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1a} + en_{2a} + CV) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2e(n_{1b} - 1) + e(n_{2b} + 1) + CV)$$

$$= \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1b} - 2e + en_{2b} + e + CV) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1b} - e + en_{2b} + CV)$$

$$= \frac{1}{3}(2en_{1b} + en_{2b} + CV - 2en_{1b} + e - en_{2b} - CV) = \frac{+e}{3}$$

$$\Delta G = \frac{e^2}{2C} [n_{1b} - n_{2b} - 1] - V \left\{ \frac{e}{3} \right\}$$



$$\Delta G = G_{before} - G_{after} = \{E_{before} - E_{after}\} - V \{Q_{before} - Q_{after}\}$$

$$\{E_{before} - E_{after}\}$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}]$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 (n_{1b} + 1)^2 + 2e^2 (n_{2b} - 1)^2 + (CV)^2 + e^2 (n_{1b} + 1)(n_{2b} - 1)]$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 (n_{1b}^2 + 2n_{1b} + 1) + 2e^2 (n_{2b}^2 - 2n_{2b} + 1) + (CV)^2 + e^2 (n_{1b} n_{2b} - n_{1b} + n_{2b} - 1)]$$

$$= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1b}^2 + 4e^2 n_{1b} + 2e^2 + 2e^2 n_{2b}^2 - 4e^2 n_{2b} + 2e^2 + (CV)^2 + e^2 n_{1b} n_{2b} - e^2 n_{1b} + e^2 n_{2b} - e^2]$$

$$= -\frac{1}{6C} [4e^2 n_{1b} + 2e^2 - 4e^2 n_{2b} + 2e^2 - e^2 n_{1b} + e^2 n_{2b} - e^2]$$

$$= -\frac{1}{6C} [4e^2 n_{1b} + 2e^2 - 4e^2 n_{2b} + 2e^2 - e^2 n_{1b} + e^2 n_{2b} - e^2] = -\frac{3e^2}{6C} [n_{1b} + 1 - n_{2b}] = -\frac{e^2}{2C} [n_{1b} + 1 - n_{2b}] = \frac{e^2}{2C} [-n_{1b} + n_{2b} - 1]$$

$$\{Q_{before} - Q_{after}\} = \int_{-\infty}^{t_{before}} I(t') dt' - \int_{-\infty}^{t_{after}} I(t') dt' = -\int_{t_{before}}^{t_{after}} I(t') dt'$$

$$= -\int_{t_{before}}^{t_{after}} I_{tunnel}(t') dt' - \int_{t_{before}}^{t_{after}} I_{polarization}(t') dt'$$

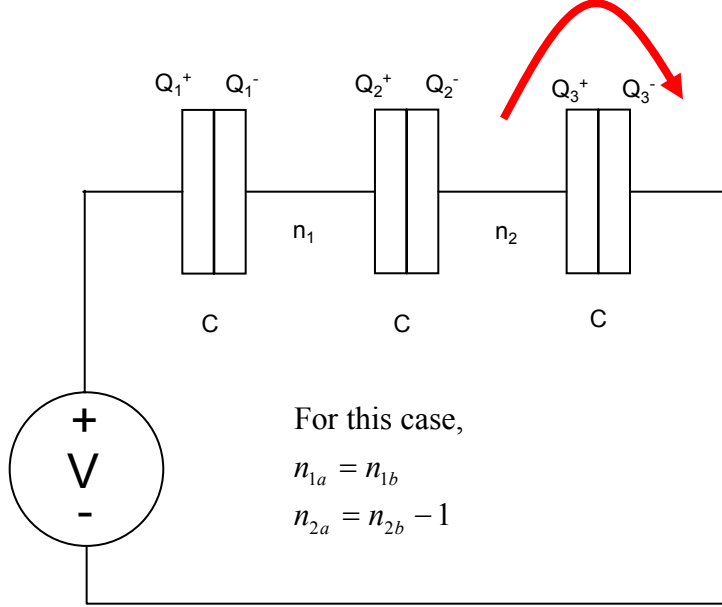
$$= 0 - \int_{t_{before}}^{t_{after}} I_{polarization}(t') dt' = (Q_{1before} - Q_{1after})$$

$$(Q_{1before} - Q_{1after}) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1a} + en_{2a} + CV) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2e(n_{1b} + 1) + e(n_{2b} - 1) + CV)$$

$$= \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1b} + 2e + en_{2b} - e + CV) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1b} + e + en_{2b} + CV)$$

$$= \frac{1}{3}(2en_{1b} + en_{2b} + CV - 2en_{1b} - e - en_{2b} - CV) = \frac{-e}{3}$$

$$\Delta G = \frac{e^2}{2C} [-n_{1b} + n_{2b} - 1] + V \left\{ \frac{e}{3} \right\}$$



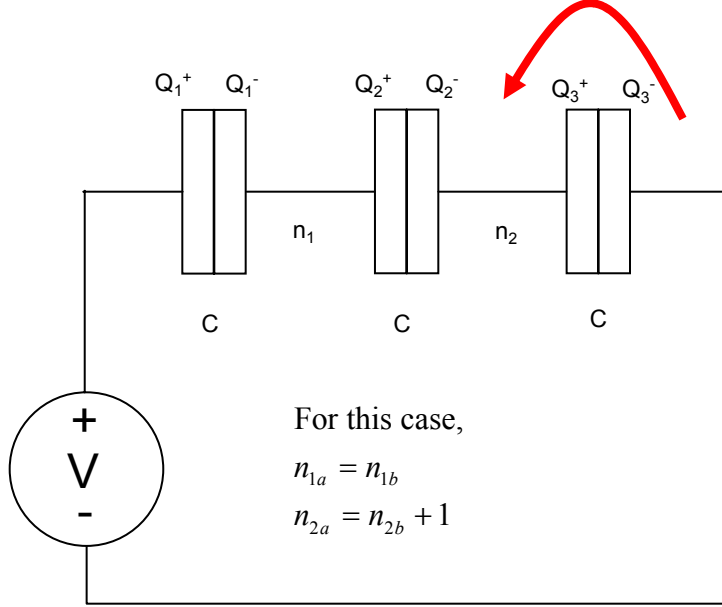
$$\Delta G = G_{\text{before}} - G_{\text{after}} = \{E_{\text{before}} - E_{\text{after}}\} - V \{Q_{\text{before}} - Q_{\text{after}}\}$$

$$\begin{aligned} & \{E_{\text{before}} - E_{\text{after}}\} \\ &= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}] \\ &= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 (n_{2b} - 1)^2 + (CV)^2 + e^2 n_{1b} (n_{2b} - 1)] \\ &= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 (n_{2b}^2 - 2n_{2b} + 1) + (CV)^2 + e^2 n_{1b} n_{2b} - e^2 n_{1b}] \\ &= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 - 4e^2 n_{2b} + 2e^2 + (CV)^2 + e^2 n_{1b} n_{2b} - e^2 n_{1b}] \\ &= -\frac{1}{6C} [-4e^2 n_{2b} + 2e^2 + e^2 n_{1b} n_{2b} - e^2 n_{1b}] = \frac{e^2}{6C} [4n_{2b} - 2 - n_{1b} n_{2b} + n_{1b}] \end{aligned}$$

$$\begin{aligned} \{Q_{\text{before}} - Q_{\text{after}}\} &= \int_{-\infty}^{t_{\text{before}}} I(t') dt' - \int_{-\infty}^{t_{\text{after}}} I(t') dt' = -\int_{t_{\text{before}}}^{t_{\text{after}}} I(t') dt' \\ &= -\int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{tunnel}}(t') dt' - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt' \\ &= 0 - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt' = (Q_{1\text{before}} - Q_{1\text{after}}) \end{aligned}$$

$$\begin{aligned} (Q_{1\text{before}} - Q_{1\text{after}}) &= \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1a} + en_{2a} + CV) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1b} + e(n_{2b} - 1) + CV) \\ &= \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1b} + en_{2b} - e + CV) \\ &= \frac{1}{3}(2en_{1b} + en_{2b} + CV - 2en_{1b} + e - en_{2b} - CV) = \frac{+e}{3} \end{aligned}$$

$$\Delta G = \frac{e^2}{6C} [4n_{2b} - 2 - n_{1b} n_{2b} + n_{1b}] - V \left\{ \frac{e}{3} \right\}$$



$$\Delta G = G_{\text{before}} - G_{\text{after}} = \{E_{\text{before}} - E_{\text{after}}\} - V \{Q_{\text{before}} - Q_{\text{after}}\}$$

$$\begin{aligned} & \{E_{\text{before}} - E_{\text{after}}\} \\ &= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}] \\ &= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 (n_{2b} + 1)^2 + (CV)^2 + e^2 n_{1b} (n_{2b} + 1)] \\ &= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 (n_{2b}^2 + 2n_{2b} + 1) + (CV)^2 + e^2 n_{1b} n_{2b} + e^2 n_{1b}] \\ &= \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}] - \frac{1}{6C} [2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + 4e^2 n_{2b} + 2e^2 + (CV)^2 + e^2 n_{1b} n_{2b} + e^2 n_{1b}] \\ &= -\frac{1}{6C} [4e^2 n_{2b} + 2e^2 + e^2 n_{1b} n_{2b} + e^2 n_{1b}] = \frac{e^2}{6C} [-4n_{2b} - 2 - n_{1b} n_{2b} - n_{1b}] \end{aligned}$$

$$\begin{aligned} \{Q_{\text{before}} - Q_{\text{after}}\} &= \int_{-\infty}^{t_{\text{before}}} I(t') dt' - \int_{-\infty}^{t_{\text{after}}} I(t') dt' = -\int_{t_{\text{before}}}^{t_{\text{after}}} I(t') dt' \\ &= -\int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{tunnel}}(t') dt' - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt' \\ &= 0 - \int_{t_{\text{before}}}^{t_{\text{after}}} I_{\text{polarization}}(t') dt' = (Q_{1\text{before}} - Q_{1\text{after}}) \end{aligned}$$

$$\begin{aligned} (Q_{1\text{before}} - Q_{1\text{after}}) &= \frac{1}{3} (2en_{1b} + en_{2b} + CV) - \frac{1}{3} (2en_{1a} + en_{2a} + CV) = \frac{1}{3} (2en_{1b} + en_{2b} + CV) - \frac{1}{3} (2en_{1b} + e(n_{2b} + 1) + CV) \\ &= \frac{1}{3} (2en_{1b} + en_{2b} + CV) - \frac{1}{3} (2en_{1b} + en_{2b} + e + CV) \\ &= \frac{1}{3} (2en_{1b} + en_{2b} + CV - 2en_{1b} - e - en_{2b} - CV) = \frac{-e}{3} \end{aligned}$$

$$\Delta G = \frac{e^2}{6C} [-4n_{2b} - 2 - n_{1b} n_{2b} - n_{1b}] + V \left\{ \frac{e}{3} \right\}$$

For current to flow from left to right, we need the following transitions to be allowed, for example, if the initial state of the system is $(0,0)$, i.e. $n_1=0, n_2=0$:

$0,0 \rightarrow 1,0$

$1,0 \rightarrow 0,1$

$0,1 \rightarrow 0,0$

For the first, we require $V < -e/C$.

For the second, we require $V < 0$.

For the third, we require $V < e/C$.

However, all 3 transitions must be allowed, so the only way to satisfy this is if $V < -e/C$.

Thus, no current can flow in that direction unless $V < -e/C$.

By a similar consideration, you can show that no current can flow in the opposite direction unless $V > e/C$.